Outline

1. Oligopoly: Bertrand

2. Second-price Auction

3. Auctions: eBay Evidence
1 Oligopoly: Bertrand

- Cournot oligopoly: firms choose quantities

- Bertrand oligopoly: firms first choose prices, and then produce quantity demanded by market

- Market demand function \( Y(p) \)

- 2 firms

- Profits:

\[
\pi_i(p_i, p_{-i}) = \begin{cases} 
(p_i - c) Y(p_i) & \text{if } p_i < p_{-i} \\
(p_i - c) Y(p_i) / 2 & \text{if } p_i = p_{-i} \\
0 & \text{if } p_i > p_{-i}
\end{cases}
\]
• First show that $p_1 = c = p_2$ is Nash Equilibrium

• Does any firm have a (strict) incentive to deviate?

• Check profits for Firm 1

• Symmetric argument for Firm 2
• Second, show that this equilibrium is unique.

• For each of the next 5 cases at least one firm has a profitable deviation

• Case 1. \( p_1 > p_2 > c \)

• Case 2. \( p_1 = p_2 > c \)

• Case 3. \( p_1 > c \geq p_2 \)
• Case 4. \( c > p_1 \geq p_2 \)

• Case 5. \( p_1 = c > p_2 \)

• Only Case 6 remains: \( p_1 = c = p_2 \), which is Nash Equilibrium

• It is unique!
• Notice:

• To show that something is an equilibrium $\rightarrow$ Show that there is *no* profitable deviation

• To show that something is *not* an equilibrium $\rightarrow$ Show that there is *one* profitable deviation
• Surprising result of Bertrand Competition

• Marginal cost pricing

• Two firms are enough to guarantee perfect competition!

• Realistic? Price wars between PC makers
2 Second-price Auction

- Sealed-bid auction
- Highest bidder wins object
- Price paid is second highest price

- Two individuals: $I = 2$
- Strategy $s_i$ is bid $b_i$
- Each individual knows value $v_i$
• Payoff for individual $i$ is

$$u_i(b_i, b_{-i}) = \begin{cases} 
    v_i - b_i & \text{if } b_i > b_{-i} \\
    \frac{(v_i - b_{-i})}{2} & \text{if } b_i = b_{-i} \\
    0 & \text{if } b_i < b_{-i}
\end{cases}$$

• Show: weakly dominant to set $b_i^* = v_i$

• To show:

$$u_i(v_i, b_{-i}) \geq u_i(b_i, b_{-i})$$

for all $b_i$, for all $b_{-i}$, and for $i = 1, 2$. 
1. Assume $b_{-i} > v_i$

   - $u_i(v_i, b_{-i}) = 0 = u_i(b_i, b_{-i})$ for any $b_i < b_{-i}$
   - $u_i(b_{-i}, b_{-i}) = (v_i - b_{-i}) / 2 < 0$
   - $u_i(b_i, b_{-i}) = (b_i - b_{-i}) < 0$ for any $b_i > b_{-i}$

2. Assume now $b_{-i} = v_i$
3. Assume now $b_{-i} < v_i$. 
3 Auctions: Evidence from eBay

- In second-price auction, optimal strategy is to bid one’s own value

- Is this true?

- eBay has proxy system: If you have highest bid, you pay bid of second-highest bidder

- eBay is essentially a second-price auction

- Two deviations:
  1. People bid multiple times – they should not in this theory
  2. People may overbid
An example:
eBay Bidding for a Board Game

- Bidding environment with clear boundary for rational willingness to pay ("buy-it-now price").
- Empirical environment unaffected by common-value arguments (presumably bidding for private use; in addition "buy-it-now" price).
- Still non-negligible amount ($100-$200).

→ Is there evidence of overbidding?
→ If so, can we detect determinants of overbidding?
The Object
The Data

• Cashflow 101: board game with the purpose of finance/accounting education.
• Retail price: $195 plus shipping cost ($10.75) from manufacturer (www.richdad.com).
• Two ways to purchase Cashflow 101 on eBay
  – Auction (quasi-second price proxy bidding)
  – Buy-it-now
• Hand-collected data of all auctions and Buy-it-now transactions of Cashflow 101 on eBay from 2/19/2004 to 9/6/2004.
Sample

- **Listings**
  - 206 by individuals (187 auctions only, 19 auctions with buy-it-now option)
  - 493 by two retailers (only buy-it-now)

- **Remove non-US$, terminated, unsold items and items without simultaneous professional buy-it-now listing.** → 169 auctions

- **Buy-it-now offers of the two retailers**
  - Continuously present for all but six days. (Often individual buy-it-now offers present as well; they are often lower.)
  - 100% and 99.9% positive feedback scores.
  - Same prices $129.95 until 07/31/2004; $139.95 since 08/01/2004.
  - Shipping cost $9.95; other retailer $10.95.
  - New items (with bonus tapes/video).
<table>
<thead>
<tr>
<th>Title</th>
<th>Price</th>
<th>Quantity</th>
<th>Delivery Time</th>
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<tbody>
<tr>
<td>Rich Dad's Cashflow Quadrant, Rich dad ...</td>
<td>$12.50</td>
<td>4</td>
<td>1d 00h 14m</td>
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<tr>
<td>Rich Dad's Cashflow Quadrant by Robert T. ...</td>
<td>$9.00</td>
<td>9</td>
<td>1d 00h 43m</td>
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<tr>
<td>Real Estate Investment Cashflow Software $$!</td>
<td>$10.49</td>
<td>2</td>
<td>1d 04h 36m</td>
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<tr>
<td>CASHFLOW® 101 202 Robert Kiyosaki Best Pak $</td>
<td>$207.96</td>
<td></td>
<td>1d 06h 47m</td>
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<tr>
<td>TRY IT TODAY, WITH ABSOLUTELY NO RISK,</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>CASHFLOW® 101 Robert Kiyosaki Plus Bonuses!</td>
<td>$129.95</td>
<td></td>
<td>1d 08h 02m</td>
</tr>
<tr>
<td>Your satisfaction is GUARANTEED, 100% $ back</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MINT Cashflow 101 *Robert Kiyosaki Game NR!</td>
<td>$140.00</td>
<td>13</td>
<td>1d 08h 04m</td>
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<tr>
<td>It's easy to be rich. Brand New. Still sealed</td>
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<td></td>
<td></td>
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<tr>
<td>cashflow Hard Money Funding 101 real estate</td>
<td>$14.99</td>
<td></td>
<td>1d 09h 28m</td>
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<tr>
<td>BRANDNEW RICHDAD CASHFLOW FOR KIDS E-GAME</td>
<td>$20.00</td>
<td>1</td>
<td>1d 13h 54m</td>
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<tr>
<td>CASHFLOW® 101 202 Robert Kiyosaki Best Pak $</td>
<td>$207.96</td>
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<td>1d 15h 47m</td>
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<tr>
<td>TRY IT TODAY, WITH ABSOLUTELY NO RISK,</td>
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<td></td>
</tr>
</tbody>
</table>
Listing Example – Magnified
Example of the description

CASHFLOW 101 Game, video & Cassette Tapes by Rich Dad Robert Kiyosaki

- Like-new CASHFLOW 101 game - $195 retail. Opened but in mint condition.
- A 3 audio cassette album titled "It's Easy to Be Rich ... if you have a strong financial education."
- A video tape titled "The Secrets of the Rich" that gives you information about the game.
- Game sheets, playing cards, cash, instructions, etc. Everything that comes with the game is included
- CASHFLOW® 101 is an educational program that teaches accounting, finance, and investing at the same time and makes learning fun. Learn how to get out of the rat race and onto the fast track where your money works for you instead of you working hard for your money.

CASHFLOW 101 is recommended for adults and children age 10 and older.
Bidding history of an item

<table>
<thead>
<tr>
<th>User ID</th>
<th>Bid Amount</th>
<th>Date of bid</th>
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<tbody>
<tr>
<td>breezeybugs</td>
<td>US $152.50</td>
<td>Aug-11-04 09:51:21 PDT</td>
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<tr>
<td>mlb4-hall</td>
<td>US $150.00</td>
<td>Aug-11-04 06:39:53 PDT</td>
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<tr>
<td>breezeybugs</td>
<td>US $140.00</td>
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<td>sj_orbit</td>
<td>US $130.01</td>
<td>Aug-08-04 25:42:02 PDT</td>
</tr>
<tr>
<td>successbroker</td>
<td>US $110.00</td>
<td>Aug-08-04 19:56:26 PDT</td>
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<tr>
<td>successbroker</td>
<td>US $105.00</td>
<td>Aug-06-04 17:13:21 PDT</td>
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<td>US $102.50</td>
<td>Aug-06-04 17:11:31 PDT</td>
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<tr>
<td>002la</td>
<td>US $100.00</td>
<td>Aug-05-04 15:41:40 PDT</td>
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<td>US $96.00</td>
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<td>US $40.00</td>
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<td>bearnbulles22</td>
<td>US $31.00</td>
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<td>75clin</td>
<td>US $30.00</td>
<td>Aug-04-04 19:45:54 PDT</td>
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<td>bearnbulles22</td>
<td>US $28.00</td>
<td>Aug-05-04 06:40:26 PDT</td>
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<tr>
<td>bearnbulles22</td>
<td>US $25.00</td>
<td>Aug-05-04 06:40:01 PDT</td>
</tr>
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</table>

If you and another bidder placed the same bid amount, the earlier bid takes priority.
Hypotheses

Given the information on the listing website:

• (H1) An auction should never end at a price above the concurrently available purchase price.

• (H2) Mentioning of higher outside prices should not affect bidding behavior.
Figure 1. Starting Price (*startprice*)

- 45% below $20; mean=$46; SD=43.88
- only 6 auctions with first *bid* (not price) above buy-it-now
Figure 2. Final Price (finalprice)

41% are above “buy-it-now” (mean $132; SD 16.83)
Figure 4. Total Price (incl. shipping cost)

⇒ 51% are above “buy-it-now” plus its shipping cost (mean=$144.20; SD=15.00)
Who are the overbidders?

• Less experienced?
  – No. (Not significantly, see last table.)
• Repeated bids?
  – Sometimes.
  – Data to be completed.
• What about the first bid?
  – First bid overall in auction typically below “buy-it-now”.
  – First bid of a bidder in an ongoing auction not necessarily below “buy-it-now” though typically inducing a price below “buy-it-now”.
Survey Results

• 52% log on to watch the last-minute bidding
• 40% admit that they sometimes bid more than they originally wanted. Why?
  – “Competition”
  – “Last minute bidding frenzy for highly desired item”
  – “I really wanted the item and my price wasn't that far from the original (about 2 dollars)”
• No concerns about buy-it-now features (if anything marginally better experience).
The Other Lesson?

Some unsolicited eBay advice.

• Can make money by selling “Cashflow 101” to those who aspire to become financially smart, and overpay for the board game!

• Sellers : add exaggerated retail price, pay 20 cents extra (now 40 cents) for 10 day listing!

• Buyers : check out the “buy-it-now” price before you bid!
4 Next lecture

- Dynamic Games

- Stackelberg duopoly