Outline

1. Walrasian Equilibrium

2. Example

3. Summary of General Equilibrium
1 Walrasian Equilibrium

- Prices $p_1, p_2$

- Consumer 1 faces a budget set:

$$p_1 x_1^1 + p_2 x_2^1 \leq p_1 \omega_1^1 + p_2 \omega_2^1$$

- How about consumer 2?

- Budget set of consumer 2:

$$p_1 x_1^2 + p_2 x_2^2 \leq p_1 \omega_1^2 + p_2 \omega_2^2$$

or (assuming $x_i^1 + x_i^2 = \omega_i$)

$$p_1 (\omega_1 - x_1^1) + p_2 (\omega_1 - x_2^1) \leq p_1 (\omega_1 - \omega_1^1) + p_2 (\omega_2 - \omega_2^1)$$

or

$$p_1 x_1^1 + p_2 x_2^1 \geq p_1 \omega_1^1 + p_2 \omega_2^1$$
Walrasian Equilibrium. \(((x_1^1, x_2^1), (x_1^2, x_2^2), p^*_1, p^*_2)\) is a Walrasian Equilibrium if:

- Each consumer maximizes utility subject to budget constraint:

\[
(x_1^{i*}, x_2^{i*}) = \arg \max_{x_1^i, x_2^i} u_i(x_1^i, x_2^i) \\
\text{s.t. } p^*_1 x_1^i + p^*_2 x_2^i \leq p^*_1 \omega_1^i + p^*_2 \omega_2^i
\]

- All markets clear:

\[
x_j^{1*} + x_j^{2*} \leq \omega_j^1 + \omega_j^2 \text{ for all } j.
\]
• Compare with partial (Marshallian) equilibrium:
  – each consumer maximizes utility
  – market for good \( i \) clears.
  – (no requirement that all markets clear)

• How do we find the Walrasian Equilibria?
• **Graphical method.**

1. Compute first for each consumer set of utility-maximizing points as function of prices

2. Check that market-clearing condition holds

• **Step 1.** Compute optimal points as prices $p_1$ and $p_2$ vary

• Start with Consumer 1. Find points of tangency between budget sets and indifference curves

• Figure
• **Offer curve** for consumer 1:

\[ (x_1^{*1} (p_1, p_2, (\omega_1, \omega_2)), x_1^{*2} (p_1, p_2, (\omega_1, \omega_2))) \]

• Offer curve is set of points that maximize utility as function of prices \( p_1 \) and \( p_2 \).

• Then find offer curve for consumer 2:

\[ (x_1^{*2} (p_1, p_2, (\omega_1, \omega_2)), x_2^{*2} (p_1, p_2, (\omega_1, \omega_2))) \]

• Figure
• **Step 2.** Find intersection(s) of two offer curves

• Walrasian Equilibrium is intersection of the two offer curves!
  
  – Both individuals maximize utility given prices
  
  – Total quantity demanded equals total endowment
• Relate Walrasian Equilibrium to barter equilibrium.

• Walrasian Equilibrium is a subset of barter equilibrium:
  – Does WE satisfy Individual Rationality condition?
  – Does WE satisfy the Pareto Efficiency condition?

• Walrasian Equilibrium therefore picks one (or more) point(s) on contract curve.
2 Example

- Consumer 1 has Leontieff preferences:

\[ u(x_1, x_2) = \min \left( x_1^1, x_2^1 \right) \]

- Bundle demanded by consumer 1:

\[
\begin{align*}
    x_1^{1*} &= x_2^{1*} = x^{1*} = \frac{p_1 \omega_1^1 + p_2 \omega_2^1}{p_1 + p_2} = \\
    &= \frac{\omega_1^1 + (p_2/p_1) \omega_2^1}{1 + (p_2/p_1)}
\end{align*}
\]

- Graphically
• Comparative statics:

  – increase in $\omega$

  – increase in $p_2/p_1$:

  \[
  \frac{dx_1^*}{dp_2/p_1} = \frac{\omega_2^1 (1 + (p_2/p_1))}{(1 + (p_2/p_1))^2} = \frac{\omega_2^1}{(1 + (p_2/p_1))^2} - \frac{(\omega_1^1 + (p_2/p_1) \omega_2^1)}{(1 + (p_2/p_1))^2} = \frac{\omega_2^1 - \omega_1^1}{(1 + (p_2/p_1))^2}
  \]

  – Effect depends on income effect through endowments:

    * A lot of good 2 $\rightarrow$ increase in price of good 2 makes richer

    * Little good 2 $\rightarrow$ increase in price of good 2 makes poorer

• Notice: Only ratio of prices matters (general feature)
• Consumer 2 has Cobb-Douglas preferences:

$$u(x_1, x_2) = (x_1^2)^{.5}(x_2^2)^{.5}$$

• Demands of consumer 2:

$$x_{1*}^2 = \frac{.5 \left(p_1\omega_1^1 + p_2\omega_2^1\right)}{p_1} = .5 \left(\frac{\omega_1^1}{p_1} + \frac{p_2\omega_2^1}{p_1}\right)$$

and

$$x_{2*}^2 = \frac{.5 \left(p_1\omega_1^1 + p_2\omega_2^1\right)}{p_2} = .5 \left(\frac{p_1\omega_1^1}{p_2} + \omega_2^1\right)$$
• Impose Walrasian equilibrium in market 1:

\[ x_1^* + x_2^* = \omega_1 + \omega_2 \]

This implies

\[ \frac{\omega_1 + (p_2/p_1) \omega_2}{1 + (p_2/p_1)} + 0.5 \left( \frac{\omega_1 + p_2 \omega_2}{p_1} \right) = \omega_1 + \omega_2 \]

or

\[ 0.5 - 0.5 \left( \frac{p_2}{p_1} \right) + 0.5 \left( \frac{p_2}{p_1} \right) + 0.5 \left( \frac{p_2}{p_1} \right)^2 \omega_2 = 0 \]

or

\[ (\omega_1 - 2\omega_2) + (\omega_1 + \omega_2) \left( \frac{p_2}{p_1} \right) + \omega_2 \left( \frac{p_2}{p_1} \right)^2 = 0 \]
• Solution for $p_2/p_1$:

$$\frac{p_2}{p_1} = \frac{-\left(\omega_1^1 - 2\omega_2^1\right) + \sqrt{\left(\omega_1^1 + \omega_2^1\right)^2 - 4\left(\omega_1^1 - 2\omega_2^1\right)\omega_2^1}}{2\left(\omega_1^1 - 2\omega_2^1\right)}$$

• Some complicated solution!

• Problem set has solution that is much easier to compute (and interpret)
3 Summary of General Equilibrium

- Does Walrasian Equilibrium always exist? In general, yes, as long as preference convex

- Is Walrasian Equilibrium always unique? Not necessarily

- Is Walrasian Equilibrium efficient? Yes.
• **First Fundamental Welfare Theorem.** All Wal-rasian Equilibria are on Contract Curve (and therefore are Pareto Efficient).

• Smithian Invisible Hand. Market leads to an allocation that is Pareto Efficient.

  – **BUT:** problems with externalities and public good

  – **BUT:** what about distribution?
4 Next and last lecture

• Empirical Economics

• Some examples of Empirical Economics
  – House insurance
  – Save More Tomorrow
  – Fox News