Outline

1. Price Changes

2. Expenditure Minimization

3. Slutsky Equation
1 Price changes

- Price of good $i$ decreases from $p_i$ to $p'_i > p_i$

- For example, decrease in price of good 2, $p'_2 < p_2$

- Budget line tilts:

  \[ x_2 = \frac{M}{p'_2} - x_1 \frac{p_1}{p'_2} \]

- New optimum?
• Demand curve: \( x_i^*(p_i) \): demand for good \( i \) as function of own price holding fixed \( p_j \) and \( M \)

• Odd convention of economists: plot price \( p_i \) on vertical axis and quantity \( x_i \) on horizontal axis. Better get used to it!
• Does $x_i^*$ decrease with $p_i$?
  
  – Yes. Most cases
  
  – No. Good $i$ is Giffen
  
  – Ex.: Potatoes in Ireland
  
  – Do not confuse with Veblen effect for luxury goods or informational asymmetries: these effects are real, but not included in current model
2 Expenditure minimization

- Nicholson, Ch. 4, pp. 127-132 (109–113, 9th)

- Solve problem **EMIN** (minimize expenditure):
  \[
  \min p_1 x_1 + p_2 x_2 \\
  \text{s.t. } u(x_1, x_2) \geq \bar{u}
  \]

- Choose bundle that attains utility $\bar{u}$ with minimal expenditure

- Ex.: You are choosing combination CDs/restaurant to make a friend happy

- If utility $u$ strictly increasing in $x_i$, can maximize s.t. equality

- Denote by $h_i(p_1, p_2, \bar{u})$ solution to EMIN problem

- $h_i(p_1, p_2, \bar{u})$ is Hicksian or compensated demand
• Graphically:
  
  – Fix indifference curve at level $\bar{u}$
  
  – Consider budget sets with different $M$
  
  – Pick budget set which is tangent to indifference curve

• Optimum coincides with optimum of Utility Maximization!

• Formally:

$$h_i(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p_1, p_2, \bar{u}))$$
• Expenditure function is expenditure at optimum

• \( e(p_1, p_2, \bar{u}) = p_1 h_1(p_1, p_2, \bar{u}) + p_2 h_2(p_1, p_2, \bar{u}) \)

• \( h_i(p_i) \) is *Hicksian or compensated demand* function

• Is \( h_i \) always decreasing in \( p_i \)? Yes!

• Graphical proof: moving along a convex indifference curve

• (For non-convex indifferent curves, still true)
• Using first order conditions:

\[ L(x_1, x_2, \lambda) = p_1 x_1 + p_2 x_2 - \lambda (u(x_1, x_2) - \bar{u}) \]

\[ \frac{\partial L}{\partial x_i} = p_i - \lambda u'_i(x_1, x_2) = 0 \]

• Write as ratios:

\[ \frac{u'_1(x_1, x_2)}{u'_2(x_1, x_2)} = \frac{p_1}{p_2} \]

• \( MRS \) = ratio of prices as in utility maximization!

• However: different constraint \( \Rightarrow \lambda \) is different
• Example 1: Cobb-Douglas utility

\[
\begin{align*}
\min p_1 x_1 + p_2 x_2 \\
\text{s.t. } x_1^\alpha x_2^{1-\alpha} \geq \bar{u}
\end{align*}
\]

• Lagrangean =

• F.o.c.:

• Solution: \( h_1^* = \), \( h_2^* = \)

• \( \partial h_i^*/\partial p_i < 0, \partial h_i^*/\partial p_j > 0, j \neq i \)
3 Slutsky Equation

- Nicholson, Ch. 5, pp. 155-158 (135–138, 9th)

- Now: go back to Utility Max. in case where $p_2$ increases to $p'_2 > p_2$

- What is $\partial x^*_2/\partial p_2$? Decompose effect:
  
  1. Substitution effect of an increase in $p_i$
     
     - $\partial h^*_2/\partial p_2$, that is change in EMIN point as $p_2$ decreases
     
     - Moving along an indifference curve
     
     - Certainly $\partial h^*_2/\partial p_2 < 0$
2. Income effect of an increase in $p_i$

- $\frac{\partial x_2^*}{\partial M}$, increase in consumption of good 2 due to increased income

- Shift out a budget line

- $\frac{\partial x_2^*}{\partial M} > 0$ for normal goods, $\frac{\partial x_2^*}{\partial M} < 0$ for inferior goods
• \( h_i(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p_1, p_2, \bar{u})) \)

• How does the Hicksian demand change if price \( p_i \) changes?

\[
\frac{dh_i}{dp_i} = \frac{\partial x_i^*(p, e)}{\partial p_i} + \frac{\partial x_i^*(p, e)}{\partial M} \frac{\partial e(p, \bar{u})}{\partial p_i}
\]

• What is \( \frac{\partial e(p, \bar{u})}{\partial p_i} \)? Envelope theorem:

\[
\frac{\partial e(p, \bar{u})}{\partial p_i} = \frac{\partial}{\partial p_i} \left[ p_1 h_1^* + p_2 h_2^* - \lambda (u(h_1^*, h_2^*, \bar{u}) - \bar{u}) \right] = h_i^*(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p, \bar{u}))
\]
• Therefore
\[
\frac{\partial h_i(p, \bar{u})}{\partial p_i} = \frac{\partial x_i^*(p, e)}{\partial p_i} + \frac{\partial x_i^*(p, e)}{\partial M} x_1^*(p_1, p_2, e)
\]

• Rewrite as
\[
\frac{\partial x_i^*(p, M)}{\partial p_i} = \frac{\partial h_i(p, v(p, M))}{\partial p_i} - x_1^*(p_1, p_2, M) \frac{\partial x_i^*(p, M)}{\partial M}
\]

• Important result! Allows decomposition into substitution and income effect
• Two effects of change in price:

1. Substitution effect negative: \( \frac{\partial h_i(p,v(p,M))}{\partial p_i} < 0 \)

2. Income effect: \(-x^*_1(p_1, p_2, M) \frac{\partial x^*_i(p,M)}{\partial M}\)
   
   - negative if good \( i \) is normal \( (\frac{\partial x^*_i(p,M)}{\partial M} > 0) \)
   
   - positive if good \( i \) is inferior \( (\frac{\partial x^*_i(p,M)}{\partial M} < 0) \)

• Overall, sign of \( \frac{\partial x^*_i(p,M)}{\partial p_i} \)?

   - negative if good \( i \) is normal

   - it depends if good \( i \) is inferior
- Example 1 (ctd.): Cobb-Douglas. Apply Slutsky equation

- \( x_i^* = \alpha M / p_i \)

- \( h_i^* = \)

- Derivative of Hicksian demand with respect to price:

\[
\frac{\partial h_i (p, u)}{\partial p_i} =
\]

- Rewrite \( h_i^* \) as function of \( m \): \( h_i (p, v(p, M)) \)

- Compute \( v(p, M) = \)
• Substitution effect:

\[ \frac{\partial h_i(p, v(p, M))}{\partial p_i} = \]

• Income effect:

\[ -x_i^*(p_1, p_2, M) \frac{\partial x_i^*(p, M)}{\partial M} = \]

• Sum them up to get

\[ \frac{\partial x_i^*(p, M)}{\partial p_i} = \]

• It works!
4 Next Lectures

- Complements and Substitutes

- Then moving on to applications:
  - Labor Supply
  - Intertemporal choice
  - Economics of Altruism