Outline

1. Slutsky Equation II

2. Complements and substitutes

3. Do utility functions exist?
1 Slutsky Equation II

- Slutsky Equation

\[
\frac{\partial x^*_i(p, M)}{\partial p_i} = \frac{\partial h_i(p, v(p, M))}{\partial p_i} - x^*_1(p_1, p_2, M) \frac{\partial x^*_i(p, M)}{\partial M}
\]

- Important result! Allows decomposition into substitution and income effect
• Example 1 (ctd.): Cobb-Douglas. Apply Slutsky equation

• \( x_i^* = \alpha M/p_i \)

• \( h_i^* = \)

• Derivative of Hicksian demand with respect to price:

\[
\frac{\partial h_i(p, \bar{u})}{\partial p_i} = \)

• Rewrite \( h_i^* \) as function of \( m \):

\( h_i(p, v(p, M)) \)

• Compute \( v(p, M) = \)
• Substitution effect:
\[
\frac{\partial h_i (\mathbf{p}, v(\mathbf{p}, M))}{\partial p_i} = \\
\]

• Income effect:
\[
-x^*_i (p_1, p_2, M) \frac{\partial x^*_i (\mathbf{p}, M)}{\partial M} = \\
\]

• Sum them up to get
\[
\frac{\partial x^*_i (\mathbf{p}, M)}{\partial p_i} = \\
\]

• It works!
2 Complements and substitutes

- Nicholson, Ch. 6, pp. 182-187 (161–166, 9th)

- How about if price of another good changes?

- Generalize Slutsky equation

- Slutsky Equation:

\[
\frac{\partial x^*_i(p, M)}{\partial p_j} = \frac{\partial h_i(p, v(p, M))}{\partial p_j} - x^*_j(p_1, p_2, M) \frac{\partial x^*_i(p, M)}{\partial M}
\]
• Substitution effect

\[
\frac{\partial h_i(p, v(p, M))}{\partial p_j} > 0
\]

for \( n = 2 \) (two goods). Ambiguous for \( n > 2 \).

• Income effect:

\[
-x_j^* (p_1, p_2, M) \frac{\partial x_i^* (p, M)}{\partial M}
\]

- negative if good \( i \) is normal
- positive if good \( i \) is inferior

• How do we define complements and substitutes?
• Def. 1. Goods $i$ and $j$ are **gross substitutes** at price $p$ and income $M$ if

$$\frac{\partial x_i^*(p, M)}{\partial p_j} > 0$$

• Def. 2. Goods $i$ and $j$ are **gross complements** at price $p$ and income $M$ if

$$\frac{\partial x_i^*(p, M)}{\partial p_j} < 0$$

• Example 1 (ctd.): $x_1^* = \alpha M/p_1$, $x_2^* = \beta M/p_2$.

• Gross complements or gross substitutes? Neither!

• Notice: $\frac{\partial x_i^*(p, M)}{\partial p_j}$ is usually different from $\frac{\partial x_j^*(p, M)}{\partial p_i}$
• Better definition.

• Def. 3. Goods $i$ and $j$ are **net substitutes** at price $p$ and income $M$ if

$$\frac{\partial h_i^* (p, v(p, M))}{\partial p_j} = \frac{\partial h_j^* (p, v(p, M))}{\partial p_i} > 0$$

• Def. 4. Goods $i$ and $j$ are **net complements** at price $p$ and income $M$ if

$$\frac{\partial h_i^* (p, v(p, M))}{\partial p_j} = \frac{\partial h_j^* (p, v(p, M))}{\partial p_i} < 0$$

• Example 1 (ctd.): $h_1^* = \bar{u} \left( \frac{\alpha p_2}{1-\alpha p_1} \right)^{1-\alpha}$

• Net complements or net substitutes? Net substitutes!
3 Do utility functions exist?

- Preferences and utilities are theoretical objects
- Many different ways to write them
- How do we tie them to the world?
- Use actual choices – revealed preferences approach
• Typical economists’ approach. Compromise of:
  
  – realism

  – simplicity

• Assume a class of utility functions (CES, Cobb-Douglas...) with free parameters

• Estimate the parameters using the data
4 Labor Supply

• Nicholson Ch. 16, pp. 573-581 (477–484, 9th)

• Labor supply decision: how much to work in a day.

• Goods: consumption good $c$, hours worked $h$

• Price of good $p$, hourly wage $w$

• Consumer spends $24 - h = l$ hours in units of leisure

• Utility function: $u(c, l)$
• Budget constraint?

• Income of consumer: \( M + wh = M + w(24 - l) \)

• Budget constraint: \( pc \leq M + w(24 - l) \) or
\[ pc + wl \leq M + 24w \]

• Notice: leisure \( l \) is a consumption good with price \( w \). Why?

• General category: **opportunity cost**

• Instead of enjoying one hour of TV, I could have worked one hour and gained wage \( w \).

• You should value the marginal hour of TV \( w \)!
• Opportunity costs are very important!

• Example 2. CostCo has a warehouse in SoMa

• SoMa used to have low cost land, adequate for warehouses

• Price of land in SoMa triples in 10 years.

• Should firm relocate the warehouse?
• Did costs of staying in SoMa go up?

• No.

• Did the opportunity cost of staying in SoMa go up?

• Yes!

• Firm can sell at high price and purchase land in cheaper area.
• Let’s go back to labor supply

• Maximization problem is

\[
\max u(c, l) \\
\text{s.t. } pc + wl \leq M + 24w
\]

• Standard problem (except for 24w)

• First order conditions

• Assume utility function Cobb-Douglas:

\[
u(c, l) = c^\alpha l^{1-\alpha}
\]
• Solution is

\[ c^* = \alpha \frac{M + 24w}{p} \]

\[ l^* = (1 - \alpha) \left( 24 + \frac{M}{w} \right) \]

• Both \( c \) and \( l \) are normal goods

• Unlike in standard Cobb-Douglas problems, \( c^* \) depends on price of other good \( w \)

• Why? Agents are endowed with \( M \) AND 24 hours of \( l \) in this economy

• Normally, agents are only endowed with \( M \)
5  Next Lectures

• Applications:
  – Labor Supply
  – Intertemporal Choice
  – Economics of Altruism