Outline

1. Application 2: Intertemporal choice

2. Application 3: Altruism and charitable donations
1 Intertemporal choice

- Nicholson Ch. 17, pp. 597-601 (502–506, 9th)

- So far, we assumed people live for one period only

- Now assume that people live for two periods:
  - $t = 0$ – people are young
  - $t = 1$ – people are old

- $t = 0$: income $M_0$, consumption $c_0$ at price $p_0 = 1$

- $t = 1$: income $M_1 > M_0$, consumption $c_1$ at price $p_1 = 1$

- Credit market available: can lend or borrow at interest rate $r$
• Budget constraint in period 1?

• Sources of income:
  
  − $M_1$
  
  − $(M_0 - c_0) \times (1 + r)$ (this can be negative)

• Budget constraint:

$$c_1 \leq M_1 + (M_0 - c_0) \times (1 + r)$$

or

$$c_0 + \frac{1}{1 + r} c_1 \leq M_0 + \frac{1}{1 + r} M_1$$
• Utility function?

• Assume

\[ u(c_0, c_1) = U(c_0) + \frac{1}{1 + \delta} U(c_1) \]

• \( U' > 0, \ U'' < 0 \)

• \( \delta \) is the discount rate

• Higher \( \delta \) means higher impatience

• Elicitation of \( \delta \) through hypothetical questions

• Person is indifferent between 1 hour of TV today and 1 + \( \delta \) hours of TV next period
• Maximization problem:

\[
\max U(c_0) + \frac{1}{1 + \delta} U(c_1)
\]

s.t. \( c_0 + \frac{1}{1 + r} c_1 \leq M_0 + \frac{1}{1 + r} M_1 \)

• Lagrangean

• First order conditions:

• Ratio of f.o.c.s:

\[
\frac{U''(c_0)}{U''(c_1)} = \frac{1 + r}{1 + \delta}
\]
• Case $r = \delta$

  - $c_0^* = c_1^*$?

  - Substitute into budget constraint using $c_0^* = c_1^* = c^*$:

    $$\frac{2 + r c^*}{1 + r} = \left[ M_0 + \frac{1}{1 + r} M_1 \right]$$

    or

    $$c^* = \frac{1 + r}{2 + r} M_0 + \frac{1}{2 + r} M_1$$

  - We solved problem virtually without any assumption on $U$!

  - Notice: $M_0 < c^* < M_1$

• Case $r > \delta$

  - $c_0^* = c_1^*$?
• Comparative statics with respect to income $M_0$

• Rewrite ratio of f.o.c.s as

$$U'(c_0) - \frac{1 + r}{1 + \delta} U'(c_1) = 0$$

• Substitute $c_1$ in using $c_1 = M_1 + (M_0 - c_0)(1 + r)$ to get

$$U'(c_0) - \frac{1 + r}{1 + \delta} U'(M_1 + (M_0 - c_0)(1 + r)) = 0$$

• Apply implicit function theorem:

$$\frac{\partial c_0^*(r, M)}{\partial M_0} = -\frac{-\frac{1 + r}{1 + \delta} U''(c_1)(1 + r)}{U''(c_0) - \frac{1 + r}{1 + \delta} U''(c_1) * (- (1 + r))}$$
• Denominator is always negative

• Numerator is positive

• $\partial c^*_0 (r, M) / \partial M_0 > 0$ — consumption at time 0 is a normal good.

• Can also show $\partial c^*_0 (r, M) / \partial M_1 > 0$
• Comparative statics with respect to interest rate $r$

• Apply implicit function theorem:

$$\frac{\partial c^*_0 (r, M)}{\partial r} = \frac{-\frac{1}{1+\delta} U'(c_1)}{U''(c_0) - \frac{1+r}{1+\delta} U''(c_1) * (- (1 + r))} - \frac{-\frac{1+r}{1+\delta} U''(c_1) * (M_0 - c_0)}{U''(c_0) - \frac{1+r}{1+\delta} U''(c_1) * (- (1 + r))}$$

• Denominator is always negative

• Numerator: First term negative (substitution eff.)

• Numerator: Second term (income effect:)
  
  – positive if $M_0 > c_0$

  – negative if $M_0 < c_0$
2 Altruism and Charitable Donations

• Maximize utility = satisfy self-interest?

• No, not necessarily

• 2-person economy:
  – Mark has income $M_M$ and consumes $c_M$
  – Wendy has income $M_W$ and consumes $c_W$

• One good: $c$, with price $p = 1$
• Utility function: \( u(c) \), with \( u' > 0 \), \( u'' < 0 \)

• Wendy is altruistic: she maximizes \( u(c_W) + \alpha u(c_M) \) with \( \alpha > 0 \)

• Mark simply maximizes \( u(c_M) \)

• Wendy can give a donation of income \( D \) to Mark.
• Wendy computes the utility of Mark as a function of the donation $D$

• Mark maximizes

$$\max_{c_M} u(c_M)$$
$$s.t. \ c_M \leq M_M + D$$

• Solution: $c_M^* = M_M + D$

• Wendy maximizes

$$\max_{c_M, D} u(c_W) + \alpha u(M_M + D)$$
$$s.t. \ c_W \leq M_W - D$$
• Rewrite as:

\[
\max_D u(M_W - D) + \alpha u(M_M + D)
\]

• First order condition:

\[
-u'(M_W - D^*) + \alpha u'(M_M + D^*) = 0
\]

• Second order conditions:

\[
u''(M_W - D^*) + \alpha u''(M_M + D^*) < 0
\]
• Assume $\alpha = 1$.

- Solution?

- $u'(M_W - D) = u'(M_M + D^*)$

- $M_W - D^* = M_M + D^*$ or $D^* = (M_W - M_M) / 2$

- Transfer money so as to equate incomes!

- Careful: $D < 0$ (negative donation!) if $M_M > M_W$

• Corrected maximization:

$$\max_D u(M_W - D) + \alpha u(M_M + D)$$

$$s.t. D \geq 0$$

• Solution ($\alpha = 1$):

$$D^* = \begin{cases} 
(M_W - M_M) / 2 & \text{if } M_W - M_M > 0 \\
0 & \text{otherwise} 
\end{cases}$$
• Assume interior solution. \((D^* > 0)\)

• Comparative statics 1 (altruism):

\[
\frac{\partial D^*}{\partial \alpha} = -\frac{u'(M_M + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} > 0
\]

• Comparative statics 2 (income of donor):

\[
\frac{\partial D^*}{\partial M_W} = -\frac{-u''(M_W + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} > 0
\]

• Comparative statics 3 (income of recipient):

\[
\frac{\partial D^*}{\partial M_M} = -\frac{\alpha u''(M_M + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} < 0
\]
• A quick look at the evidence

• From Andreoni (2002)
3 Next Lectures

- Introduction to Probability
- Risk Aversion
- Coefficient of risk aversion