Outline

1. Application 3: Altruism and charitable donations II

2. Introduction to probability

3. Expected Utility

4. Insurance
1 Altruism and Charitable Donations II

- Wendy maximizes

\[
\max_{c_M, D} u(c_W) + \alpha u(M_M + D) \\
s.t. \ c_W \leq M_W - D
\]

- Rewrite as:

\[
\max_D u(M_W - D) + \alpha u(M_M + D)
\]

- First order condition:

\[
-u'(M_W - D^*) + \alpha u'(M_M + D^*) = 0
\]

- Second order conditions:

\[
u''(M_W - D^*) + \alpha u''(M_M + D^*) < 0
\]
• Assume $\alpha = 1$.

  – Solution?

  – $u'(M_W - D) = u'(M_M + D^*)$

  – $M_W - D^* = M_M + D^*$ or $D^* = (M_W - M_M) / 2$

  – Transfer money so as to equate incomes!

  – Careful: $D < 0$ (negative donation!) if $M_M > M_W$

• Corrected maximization:

  $$\max_D u(M_W - D) + \alpha u(M_M + D)$$

  s.t. $D \geq 0$

• Solution ($\alpha = 1$):

  $$D^* = \begin{cases} 
  (M_W - M_M) / 2 & \text{if } M_W - M_M > 0 \\
  0 & \text{otherwise}
  \end{cases}$$
• Assume interior solution. \((D^* > 0)\)

• Comparative statics 1 (altruism):

\[
\frac{\partial D^*}{\partial \alpha} = -\frac{u'(M_M + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} > 0
\]

• Comparative statics 2 (income of donor):

\[
\frac{\partial D^*}{\partial M_W} = -\frac{-u''(M_W + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} > 0
\]

• Comparative statics 3 (income of recipient):

\[
\frac{\partial D^*}{\partial M_M} = -\frac{\alpha u''(M_M + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} < 0
\]
• A quick look at the evidence

• From Andreoni (2002)
2 Introduction to Probability

• So far deterministic world:
  – income given, known $M$
  – interest rate known $r$

• But some variables are unknown at time of decision:
  – future income $M_1$?
  – future interest rate $r_1$?

• Generalize framework to allow for uncertainty
  – Events that are truly unpredictable (weather)
  – Event that are very hard to predict (future income)
• Probability is the language of uncertainty

• Example:
  
  – Income $M_1$ at $t = 1$ depends on state of the economy
  
  – Recession ($M_1 = 20$), Slow growth ($M_2 = 25$), Boom ($M_3 = 30$)
  
  – Three probabilities: $p_1, p_2, p_3$

  – $p_1 = P(M_1) = P(\text{recession})$

• Properties:
  
  – $0 \leq p_i \leq 1$

  – $p_1 + p_2 + p_3 = 1$
• Mean income: \( EM = \sum_{i=1}^{3} p_i M_i \)

• If \((p_1, p_2, p_3) = (1/3, 1/3, 1/3)\),

\[
EM = \frac{1}{3} 20 + \frac{1}{3} 25 + \frac{1}{3} 30 = \frac{75}{3} = 25
\]

• Variance of income: \( V(M) = \sum_{i=1}^{3} p_i (M_i - EM)^2 \)

• If \((p_1, p_2, p_3) = (1/3, 1/3, 1/3)\),

\[
V(M) = \frac{1}{3} (20 - 25)^2 + \frac{1}{3} (25 - 25)^2 + \frac{1}{3} (30 - 25)^2
\]
\[
= \frac{1}{3} 5^2 + \frac{1}{3} 5^2 = 2/3 * 25
\]

• Mean and variance if \((p_1, p_2, p_3) = (1/4, 1/2, 1/4)\)?
3 Expected Utility

- Nicholson, Ch. 7, pp. 202-209 (Ch. 18, pp. 533–541, 9th)

- Consumer at time 0 asks: what is utility in time 1?

- At \( t = 1 \) consumer maximizes

\[
\max U(c^1) \\
\text{s.t. } c^1_i \leq M^1_i + (1 + r)(M^0 - c^0) \\
\text{with } i = 1, 2, 3.
\]

- What is utility at optimum at \( t = 1 \) if \( U' > 0 \)?

- Assume for now \( M^0 - c^0 = 0 \)

- Utility \( U(M^1_i) \)

- This is uncertain, depends on which \( i \) is realized!
• How do we evaluate future uncertain utility?

• Expected utility

\[ EU = \sum_{i=1}^{3} p_i U \left( M_i^1 \right) \]

• In example:

\[ EU = \frac{1}{3} U(20) + \frac{1}{3} U(25) + \frac{1}{3} U(30) \]

• Compare with \( U(EC) = U(25) \).

• Agents prefer riskless outcome \( EM \) to uncertain outcome \( M \) if

\[
1/3U(20) + 1/3U(25) + 1/3U(30) < U(25) \text{ or } \\
1/3U(20) + 1/3U(30) < 2/3U(25) \text{ or } \\
1/2U(20) + 1/2U(30) < U(25)
\]
• Picture
• Depends on sign of $U''$, on concavity/convexity

• Three cases:

  - $U''(x) = 0$ for all $x$. (linearity of $U$)
    
    * $U(x) = a + bx$
    
    * $1/2U(20) + 1/2U(30) = U(25)$

  - $U''(x) < 0$ for all $x$. (concavity of $U$)
    
    * $1/2U(20) + 1/2U(30) < U(25)$

  - $U''(x) > 0$ for all $x$. (convexity of $U$)
    
    * $1/2U(20) + 1/2U(30) > U(25)$
- If $U''(x) = 0$ (linearity), consumer is indifferent to uncertainty

- If $U''(x) < 0$ (concavity), consumer dislikes uncertainty

- If $U''(x) > 0$ (convexity), consumer likes uncertainty

- Do consumers like uncertainty?

- Do you like uncertainty?
• **Theorem. (Jensen’s inequality)** If a function $f(x)$ is concave, the following inequality holds:

$$f(Ex) \geq Ef(x)$$

where $E$ indicates expectation. If $f$ is strictly concave, we obtain

$$f(Ex) > Ef(x)$$

• Apply to utility function $U$.

• Individuals dislike uncertainty:

$$U(Ex) \geq EU(x)$$

• Jensen’s inequality then implies $U$ concave ($U'' \leq 0$)

• Relate to diminishing marginal utility of income
4 Insurance

• Nicholson, Ch. 7, pp. 216–221 (18, pp. 545–551, 9th) Notice: different treatment than in class

• Individual has:
  
  – wealth \( w \)
  
  – utility function \( u \), with \( u' > 0 \), \( u'' < 0 \)

• Probability \( p \) of accident with loss \( L \)

• Insurance offers coverage:
  
  – premium \( q \) for each $1 paid in case of accident
  
  – units of coverage purchased \( \alpha \)
• Individual maximization:

\[
\max_{\alpha} (1 - p) u (w - q\alpha) + pu (w - q\alpha - L + \alpha)
\]
\[s.t. \alpha \geq 0\]

• Assume \( \alpha^* \geq 0 \), check later

• First order conditions:

\[
0 = -q (1 - p) u' (w - q\alpha) + (1 - q) pu' (w - q\alpha - L + \alpha)
\]

or

\[
\frac{u' (w - q\alpha)}{u' (w - q\alpha - L + \alpha)} = \frac{1 - q}{q} \frac{p}{1 - p}.
\]

• Assume first \( q = p \) (insurance is fair)

• Solution for \( \alpha^* = ? \)
• $\alpha^* > 0$, so we are ok!

• What if $q > p$ (insurance needs to cover operating costs)?

• Insurance will be only partial (if at all): $\alpha^* < L$

• Exercise: Check second order conditions!
5 Next Lectures

- Risk aversion

- Applications:
  - Portfolio choice
  - Consumption choice II