Outline

1. Insurance II

2. Investment in Risky Asset

3. Risk Aversion and Lottery

4. Measures of Risk Aversion

5. Mid-Term Feedback
1 Insurance II

- Individual maximization:
  \[
  \max_{\alpha} (1 - p) u (w - q\alpha) + pu (w - q\alpha - L + \alpha) \\
  s.t. \alpha \geq 0
  \]

- Assume \( \alpha^* \geq 0 \), check later

- First order conditions:
  \[
  0 = -q (1 - p) u' (w - q\alpha) + (1 - q) pu' (w - q\alpha - L + \alpha)
  \]
  or
  \[
  \frac{u' (w - q\alpha)}{u' (w - q\alpha - L + \alpha)} = \frac{1 - q}{q} \frac{p}{1 - p}.
  \]

- Assume first \( q = p \) (insurance is fair)

- Solution for \( \alpha^* =? \)
• $\alpha^* > 0$, so we are ok!

• What if $q > p$ (insurance needs to cover operating costs)?

• Insurance will be only partial (if at all): $\alpha^* < L$

• Exercise: Check second order conditions!
2 Investment in Risk Asset

• Individual has:
  - wealth $w$
  - utility function $u$, with $u' > 0$

• Two possible investments:
  - Asset B (bond) yields return 1 for each dollar
  - Asset S (stock) yields uncertain return $(1 + r)$:
    * $r = r_+ > 0$ with probability $p$
    * $r = r_- < 0$ with probability $1 - p$
    * $Er = pr_+ + (1 - p)r_- > 0$

• Share of wealth invested in stock $S = \alpha$
- Individual maximization:

\[
\max_{\alpha} (1 - p) u(w [(1 - \alpha) + \alpha (1 + r_-)]) + \\
+pu (w [(1 - \alpha) + \alpha (1 + r_+)])
\]

\[
s.t. 0 \leq \alpha \leq 1
\]

- Case of risk neutrality: \( u(x) = a + bx, \ b > 0 \)

- Assume \( a = 0 \) (no loss of generality)

- Maximization becomes

\[
\max_{\alpha} b (1 - p) (w [1 + \alpha r_-]) + bp (w [1 + \alpha r_+])
\]

or

\[
\max_{\alpha} bw + \alpha bw [(1 - p) r_- + pr_+]
\]

- Sign of term in square brackets? Positive!

- Set \( \alpha^* = 1 \)
• Case of risk aversion: \( u'' < 0 \)

• Assume \( 0 \leq \alpha^* \leq 1 \), check later

• First order conditions:

\[
0 = (1 - p)(wr_-)u'(w[1 + \alpha r_-]) + p(wr_+)u'(w[1 + \alpha r_+])
\]

• Can \( \alpha^* = 0 \) be solution?

• Solution is \( \alpha^* > 0 \) (positive investment in stock)

• Exercise: Check s.o.c.
3 Risk aversion and Lottery

• Risk aversion:
  – individuals dislike uncertainty
  – $u$ concave, $u'' < 0$

• Implications?
  – purchase of insurance (possible accident)
  – investment in risky asset (risky investment)
  – choice over time (future income uncertain)
• Experiment — Are you risk-averse?
4 Measures of Risk Aversion

• Nicholson, Ch. 7, pp. 209-213 (Ch. 18, pp. 541–545, 9th)

• How risk averse is an individual?

• Two measures:
  - Absolute Risk Aversion $r_A$:
    \[ r_A = - \frac{u''(x)}{u'(x)} \]
  - Relative Risk Aversion $r_R$:
    \[ r_R = - \frac{u''(x)}{u'(x)} x \]

• Examples in the Problem Set
5 Mid-Term Feedback

- Thanks for the feedback!
6 Next lecture and beyond

- Time Inconsistency

- Production Function