Outline

1. Cost Curves and Supply Function II

2. One-step Profit Maximization

3. Second-Order Conditions

4. Introduction to Market Equilibrium

5. Aggregation

6. Market Equilibrium in the Short-Run
1 Cost Curves II

- Assume only 1 input (expenditure minimization is trivial)

- Case 1. Production function. $y = L^\alpha$
  
  - Cost function? (cost of input is $w$):
    \[ c(w, y) = wL^*(w, y) = wy^{1/\alpha} \]

  - Marginal cost?
    \[ \frac{\partial c(w, y)}{\partial y} = \frac{1}{\alpha}wy(1-\alpha)/\alpha \]

  - Average cost $c(w, y)/y$?
    \[ \frac{c(w, y)}{y} = \frac{wy^{1/\alpha}}{y} = wy(1-\alpha)/\alpha \]
• **Case 1a.** $\alpha > 1$. Plot production function, total cost, average and marginal. Supply function?

• **Case 1b.** $\alpha = 1$. Plot production function, total cost, average and marginal. Supply function?

• **Case 1c.** $\alpha < 1$. Plot production function, total cost, average and marginal. Supply function?
• **Case 2.** *Non-convex technology.* Plot production function, total cost, average and marginal. Supply function?

• **Case 3.** *Technology with setup cost.* Plot production function, total cost, average and marginal. Supply function?
1.1 Supply Function

- Supply function: \( y^* = y^* (w, r, p) \)

- What happens to \( y^* \) as \( p \) increases?

- Is the supply function upward sloping?

- Remember f.o.c:
  \[
  p - c'_y (w, r, y) = 0
  \]

- Implicit function:
  \[
  \frac{\partial y^*}{\partial p} = -\frac{1}{-c''_{y,y} (w, r, y)} > 0
  \]
  as long as s.o.c. is satisfied.

- Yes! Supply function is upward sloping.
2 One-step Profit Maximization

• Nicholson, Ch. 11, pp. 374-380 (Ch. 9, pp. 265–270, 9th)

• One-step procedure: maximize profits

• Perfect competition. Price $p$ is given
  – Firms are small relative to market
  – Firms do not affect market price $p_M$

  – Will firm produce at $p > p_M$?
  – Will firm produce at $p < p_M$?
  – $\implies p = p_M$
• Revenue: $py = pf (L, K)$

• Cost: $wL + rK$

• Profit $pf (L, K) - wL - rK$
• Agent optimization:

\[
\max_{L,K} pf(L, K) - wL - rK
\]

• First order conditions:

\[
pf_L'(L, K) - w = 0
\]
and

\[
pf_K'(L, K) - r = 0
\]

• Second order conditions? \( pf''_{L,L} (L, K) < 0 \) and

\[
|H| = \begin{vmatrix}
 pf''_{L,L} (L, K) & pf''_{L,K} (L, K) \\
 pf''_{L,K} (L, K) & pf''_{K,K} (L, K)
\end{vmatrix}
= p^2 \left[ f''_{L,L} f''_{K,K} - \left( f''_{L,K} \right)^2 \right] > 0
\]

• Need \( f''_{L,K} \) not too large for maximum
- Comparative statics with respect to to $p$, $w$, and $r$.

- What happens if $w$ increases?

$$\frac{\partial L^*}{\partial w} = - \frac{\begin{vmatrix} -1 & pf''_{L,K} (L, K) \\ 0 & pf''_{K,K} (L, K) \end{vmatrix}}{\begin{vmatrix} pf''_{L,L} (L, K) & pf''_{L,K} (L, K) \\ pf''_{L,K} (L, K) & pf''_{K,K} (L, K) \end{vmatrix}} < 0$$

and

$$\frac{\partial L^*}{\partial r} =$$

- Sign of $\partial L^*/\partial r$ depends on $f''_{L,K}$. 
3 Second Order Conditions in P-Max: Cobb-Douglas

- How do the second order conditions relate for:
  - Cost Minimization
  - Profit Maximization?

- Check for Cobb-Douglas production function
  \[ y = AK^\alpha L^\beta \]

- **Cost Minimization.** S.o.c.:
  \[ c_y''(y^*, w, r) > 0 \]

- As we showed, for CD prod. ftn.,
  \[ c_y''(y^*, w, r) = - \frac{1}{\alpha + \beta} \frac{1 - (\alpha + \beta)}{\alpha + \beta} B \left( \frac{y}{A} \right)^{\frac{1-2(\alpha+\beta)}{\alpha+\beta}} \]

  which is > 0 as long as \( \alpha + \beta < 1 \) (DRS)
• **Profit Maximization.** S.o.c.: \( p f''_{L,L} (L, K) < 0 \)
and
\[
|H| = p^2 \left[ f''_{L,L} f''_{K,K} - \left( f''_{L,K} \right)^2 \right] > 0
\]

• As long as \( \beta < 1 \),
\[
p f''_{L,L} = p \beta (\beta - 1) AK^\alpha L^\beta - 2 < 0
\]

• Then,
\[
|H| = p^2 \left[ f''_{L,L} f''_{K,K} - \left( f''_{L,K} \right)^2 \right] = \\
= p^2 \left[ \beta (\beta - 1) AK^\alpha L^{\beta - 2} \right] = \\
= p^2 \left[ \alpha (\alpha - 1) AK^{\alpha - 2} L^{\beta - 1} \right] = \\
= p^2 A^2 K^{2 \alpha - 2} L^{2 \beta - 2} \alpha \beta [1 - \alpha - \beta]
\]

• Therefore, \( |H| > 0 \) iff \( \alpha + \beta < 1 \) (DRS)

• The two conditions coincide
4 Introduction to Market Equilibrium

• Nicholson, (Ch. 10, pp. 279–295, 9th)

• Two ways to analyze firm behavior:
  – Two-Step Cost Minimization
  – One-Step Profit Maximization

• What did we learn?
  – Optimal demand for inputs $L^*, K^*$ (see above)
  – Optimal quantity produced $y^*$
• **Supply function.** \( y = y^* (p, w, r) \)

  – From profit maximization:
    \[
    y = f (L^* (p, w, r), K^* (p, w, r))
    \]

  – From cost minimization:
    \[
    MC \text{ curve above } AC
    \]

  – Supply function is increasing in \( p \)

• Market Equilibrium. Equate demand and supply.

• Aggregation?

• Industry supply function!
5 Aggregation

5.1 Producers aggregation

- $J$ companies, $j = 1, \ldots, J$, producing good $i$

- Company $j$ has supply function

  $$y_j^i = y_j^i(p_i, w, r)$$

- Industry supply function:

  $$Y_i(p_i, w, r) = \sum_{j=1}^{J} y_j^i(p_i, w, r)$$

- Graphically,
5.2 Consumer aggregation

- Nicholson, (Ch. 10, pp. 279–282)

- *One-consumer economy*

- Utility function $u(x_1, \ldots, x_n)$

- Prices $p_1, \ldots, p_n$

- Maximization $\Rightarrow$

  $x_1^* = x_1^*(p_1, \ldots, p_n, M),$

  $\vdots$

  $x_n^* = x_n^*(p_1, \ldots, p_n, M).$
Focus on good $i$. Fix prices $p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n$ and $M$

**Single-consumer demand function:**

$$x_i^* = x_i^* (p_i | p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n, M)$$

What is sign of $\partial x_i^*/\partial p_i$?

- Negative if good $i$ is normal
- Negative or positive if good $i$ is inferior
• Aggregation: $J$ consumers, $j = 1, \ldots, J$

• Demand for good $i$ by consumer $j$:

\[ x_{j}^{i*} = x_{j}^{i*} \left( p_{1}, \ldots, p_{n}, M^{j} \right) \]

• Market demand $X_{i}$:

\[
X_{i} \left( p_{1}, \ldots, p_{n}, M^{1}, \ldots, M^{J} \right) = \sum_{j=1}^{J} x_{j}^{i*} \left( p_{1}, \ldots, p_{n}, M^{j} \right)
\]

• Graphically,
• Notice: market demand function depends on distribution of income $M^J$

• Market demand function $X_i$:
  
  – Consumption of good $i$ as function of prices $p$
  
  – Consumption of good $i$ as function of income distribution $M^j$
6 Market Equilibrium in the Short-Run

- Nicholson, (Ch. 14, pp. 368–382, 9th)

- What is equilibrium price $p_i$?

- Magic of the Market...

- Equilibrium: No excess supply, No excess demand

- Prices $p^*$ equates demand and supply of good $i$:

\[ Y^* = Y_i^S (p_i^*, w, r) = X_i^D (p_1^*, ..., p_n^*, M^1, ..., M^J) \]
• Graphically,

• Notice: in short-run firms can make positive profits
• Comparative statics exercises with endogenous price $p_i$:
  
  – increase in wage $w$ or interest rate $r$:

  – change in income distribution
7 Next Lecture

- Market Equilibrium
- Comparative Statics of Equilibrium
- Elasticities
- Taxes and Subsidies