Economics 101A

(Lecture 17)

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Outline

1. Comparative Statics of Equilibrium
2. Elasticities
3. Response to Taxes
4. Producer Surplus
5. Consumer Surplus
1 Comparative statics of equilibrium

- Nicholson, Ch. 12, pp. 403-406 (Ch. 10, pp. 293-295, 9th)

- Supply and Demand function of parameter $\alpha$:
  
  - $Y^S_i (p_i, w, r, \alpha)$
  
  - $X^D_i (p, M, \alpha)$

- How does $\alpha$ affect $p^*$ and $Y^*$?

- Comparative statics with respect to $\alpha$

- Equilibrium:
  
  $Y^S_i (p_i, w, r, \alpha) = X^D_i (p, M, \alpha)$
• Can write equilibrium as implicit function:

\[ Y_i^S (p_i, w, r, \alpha) - X_i^D (p, M, \alpha) = 0 \]

• What is \( dp^*/d\alpha \)?

• Implicit function theorem:

\[ \frac{\partial p^*}{\partial \alpha} = -\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p} \]

• What is sign of denominator?

• Sign of \( \partial p^*/\partial \alpha \) is negative of sign of numerator
• Examples:

1. *Fad.* Good becomes more fashionable: $\frac{\partial X^D}{\partial \alpha} > 0 \implies \frac{\partial p^*}{\partial \alpha} > 0$

2. *Recession in Europe.* Negative demand shock for US firms: $\frac{\partial X^D}{\partial \alpha} < 0 \implies \frac{\partial p^*}{\partial \alpha} < 0$

3. *Oil shock.* Import prices increase: $\frac{\partial Y^S}{\partial \alpha} < 0 \implies \frac{\partial p^*}{\partial \alpha} > 0$

4. *Computerization.* Improvement in technology. $\frac{\partial Y^S}{\partial \alpha} > 0 \implies \frac{\partial p^*}{\partial \alpha} < 0$
2 Elasticities

• Nicholson, Ch.1, pp. 26-27 (pp. 27–28, 9th)

• How do we interpret magnitudes of $\frac{\partial p^*}{\partial \alpha}$?

• Result depends on units of measure.

• Can we write $\frac{\partial p^*}{\partial \alpha}$ in a unit-free way?

• Yes! Use elasticities.

• Elasticity of $x$ with respect to parameter $p$ is

$$\varepsilon_{x,p} = \frac{\partial x}{\partial p x}$$
• Interpretation: Percent response in $x$ to percent change in $p$:

$$
\varepsilon_{x,p} = \frac{\partial x}{\partial p} \frac{x(p + dp) - x(p)p}{x} = \lim_{dp \to 0} \frac{dx}{dp} \frac{dp}{p} = \lim_{dp \to 0} \frac{dx}{x} \frac{dp}{dp}$$

where $dx \equiv x(p + dp) - x(p)$.

• Now, show

$$
\varepsilon_{x,p} = \frac{\partial \ln x}{\partial \ln p}
$$

• Notice: This makes sense only for $x > 0$ and $p > 0$
• Proof. Consider function

\[ x = f(p) \]

• Rewrite as

\[ \ln(x) = \ln f(p) = \ln f(e^{\ln(p)}) \]

• Define \( \hat{x} = \ln(x) \) and \( \hat{p} = \ln(p) \)

• This implies

\[ \hat{x} = \ln f(e^{\hat{p}}) \]

• Get

\[
\frac{\partial \hat{x}}{\partial \hat{p}} = \frac{\partial \ln x}{\partial \ln p} = \frac{1}{f(e^{\hat{p}})} \frac{\partial f(e^{\hat{p}})}{\partial \hat{p}} e^{\hat{p}} = \frac{\partial x}{\partial p} \frac{p}{x}
\]
• Example with Cobb-Douglas utility function

\[ U(x, y) = x^\alpha y^{1-\alpha} \] implies solutions

\[ x^* = \alpha \frac{M}{p_x}, y^* = (1 - \alpha) \frac{M}{p_y} \]

• Elasticity of demand with respect to own price \( \varepsilon_{x,p_x} \):

\[ \varepsilon_{x,p_x} = \frac{\partial x^* \frac{p_x}{p_x}}{\partial p_x x^*} = -\frac{\alpha M}{(p_x)^2} \frac{p_x}{\alpha p_x} = -1 \]

• Elasticity of demand with respect to other price \( \varepsilon_{x,p_y} = 0 \)
• Go back to problem above:

\[
\frac{\partial p^*}{\partial \alpha} = -\frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha}
\]

\[
\frac{\partial p^*}{\partial \alpha} = \frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}
\]

• Use elasticities to rewrite response of \( p \) to change in \( \alpha \):

\[
\frac{\partial p^*\alpha}{\partial \alpha} p = -\frac{\left( \frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha} \right) \alpha}{\left( \frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p} \right) \frac{p}{Y}}
\]

or (using fact that \( X^{D*} = Y^{s*} \))

\[
\varepsilon_{p,\alpha} = -\frac{\varepsilon_{S,\alpha} - \varepsilon_{D,\alpha}}{\varepsilon_{S,p} - \varepsilon_{D,p}}
\]

• We are likely to know elasticities from empirical studies
3 Response to taxes

• Nicholson, Ch. 12, pp. 423-426 (Ch. 11, pp. 322–323, 9th)

• Per-unit tax $t$

• Write price $p_i$ as price including tax

• Supply: $Y_i^S (p_i - t, w, r)$

• Demand: $X_i^D (p, M)$

\[ Y_i^S (p_i - t, w, r) - X_i^D (p, M) = 0 \]

• What is $dp^*/dt$?
• Comparative statics:

\[
\frac{\partial p^*}{\partial t} = - \frac{\partial Y^S}{\partial t} = \frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p} = \frac{-\frac{\partial Y^S}{\partial p} p}{X} = \frac{\varepsilon_{S,p}}{\varepsilon_{S,p} - \varepsilon_{D,p}}
\]

• How about price received by suppliers \( p^* - t \)?

\[
\frac{\partial (p^* - t)}{\partial t} = \frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p} - 1 = \frac{\varepsilon_{D,p}}{\varepsilon_{S,p} - \varepsilon_{D,p}}
\]
• *Inflexible Supply.* (Capacity is fixed) Supply curve vertical ($\varepsilon_{S,p} = 0$)

• Producers bear burden of tax

• *Flexible Supply.* (Constant Returns to Scale) Supply curve horizontal ($\varepsilon_{S,p} \rightarrow \infty$)

• Consumers bear burden of tax
• Inflexible demand. Demand curve vertical ($\varepsilon_{D,p} = 0$)?

• Consumers bear burden

• General lesson: Least elastic side bears larger part of burden

• What happens with a subsidy ($t < 0$)?

• What happens to quantity sold?

• Use demand curve:

$$\frac{\partial X^D}{\partial t} = \frac{\partial X^D}{\partial p^*} \frac{\partial p^*}{\partial t}$$

and use expression for $\partial p^*/\partial t$ above
4 Welfare: Producer Surplus

- Nicholson, Ch. 11, pp. 371-374 (Ch. 9, pp. 261–263, 9th)

- Producer Surplus is easier to define:

\[ \pi(p, y_0) = py_0 - c(y_0). \]

- Can give two graphical interpretations:

- **Interpretation 1.** Rewrite as

\[ \pi(p, y_0) = y_0 \left[ p - \frac{c(y_0)}{y_0} \right]. \]

- Profit equals rectangle of quantity times \((p - \text{Av. Cost})\)
• **Interpretation 2.** Remember:

\[ f(x) = f(0) + \int_{0}^{x} f_x'(s) \, ds. \]

• Rewrite profit as

\[
\left[ p \times 0 + p \int_{0}^{y_0} 1 \, dy \right] - \left[ c(0) + \int_{0}^{y_0} c'_y(y) \, dy \right] = \\
= \int_{0}^{y_0} \left( p - c'_y(y) \right) \, dy - c(0).
\]

• Producer surplus is area between price and marginal cost (minus fixed cost)
5 Welfare: Consumer Surplus

- Nicholson, Ch. 5, pp. 165-169 (Ch. 5, pp. 145–149, 9th)

- Welfare effect of price change from $p_0$ to $p_1$

- Proposed measure:

$$e(p_0, u) - e(p_1, u)$$

- Can rewrite expression above as

$$e(p_0, u) - e(p_1, u) = \left( e(0, u) + \int_0^{p_0} \frac{\partial e(p, u)}{\partial p} dp \right) - \left( e(0, u) + \int_0^{p_1} \frac{\partial e(p, u)}{\partial p} dp \right)$$

$$= \int_{p_1}^{p_0} \frac{\partial e(p, u)}{\partial p} dp$$

- What is $\frac{\partial e(p, u)}{\partial p}$?
• Remember envelope theorem...

• Result:

\[ \frac{\partial e(p, u)}{\partial p} = h(p, u) \]

• Welfare measure is integral of area to the side of H Hick-sian compensated demand

• Graphically,
• Example of welfare effects: Imposition of Tax

• Welfare before tax

• Welfare after tax
6 Next Lecture

- Trade
- Market Equilibrium in the Long-Run
- Then: Market Power
- Monopoly
- Price Discrimination