Outline

1. Example of General Equilibrium

2. Existence and Welfare Theorems

3. Asymmetric Information: Introduction

4. Hidden Action (Moral Hazard)
1 Example

• Consumer 1 has Leontieff preferences:
  \[ u(x_1, x_2) = \min (x_1^1, x_2^1) \]

• Bundle demanded by consumer 1:
  \[ x_1^{1*} = x_2^{1*} = x^{1*} = \frac{p_1 \omega_1^1 + p_2 \omega_2^1}{p_1 + p_2} = \]
  \[ = \frac{\omega_1^1 + (p_2/p_1) \omega_2^1}{1 + (p_2/p_1)} \]

• Graphically
• Comparative statics:
  
  – increase in $\omega$

  – increase in $p_2/p_1$:

  $$\frac{dx_1^{1*}}{dp_2/p_1} = \frac{\omega_2^1 (1 + (p_2/p_1))}{1 + (p_2/p_1)^2} = \frac{\omega_2^1 - \omega_1^1}{(1 + (p_2/p_1))^2}$$

  – Effect depends on income effect through endowments:

    * A lot of good 2 → increase in price of good 2 makes richer

    * Little good 2 → increase in price of good 2 makes poorer

• Notice: Only ratio of prices matters (general feature)
• Consumer 2 has Cobb-Douglas preferences:

\[ u(x_1, x_2) = (x_1^2)^{0.5} (x_2^2)^{0.5} \]

• Demands of consumer 2:

\[ x_1^{2*} = \frac{0.5 (p_1 \omega_1^1 + p_2 \omega_2^1)}{p_1} = 0.5 \left( \frac{\omega_1^1 + p_2 \omega_1^2}{p_1} \right) \]

and

\[ x_2^{2*} = \frac{0.5 (p_1 \omega_1^1 + p_2 \omega_2^1)}{p_2} = 0.5 \left( \frac{p_1 \omega_1^1 + \omega_2^1}{p_2} \right) \]
Comparative statics:

- increase in $\omega \rightarrow$ Increase in final consumption
- increase in $p_2/p_1 \rightarrow$ Unambiguous increase in $x_{1*}^2$ and decrease in $x_{2*}^2$
• Impose Walrasian equilibrium in market 1:

\[ x_1^* + x_2^* = \omega_1 + \omega_2 \]

This implies

\[
\frac{\omega_1 + (p_2/p_1) \omega_2}{1 + (p_2/p_1)} + \frac{.5 \left( \omega_1 + \frac{p_2}{p_1} \omega_2 \right)}{1 + (p_2/p_1)} = \omega_1 + \omega_2
\]

or

\[
.5 - .5 \left( \frac{p_2}{p_1} \right) \omega_1 + \frac{.5 \left( \frac{p_2}{p_1} \right) + .5 \left( \frac{p_2}{p_1} \right)^2 - 1}{1 + (p_2/p_1)} \omega_2 = 0
\]

or

\[
(\omega_1 - 2\omega_2) + (\omega_1 + \omega_2) \left( \frac{p_2}{p_1} \right) + \omega_2 \left( \frac{p_2}{p_1} \right)^2 = 0
\]
• Solution for $p_2/p_1$:

$$
\frac{p_2}{p_1} = \frac{-\left(\omega_1^1 - 2\omega_2^1\right) + \sqrt{\frac{\left(\omega_1^1 + \omega_2^1\right)^2}{-4 \left(\omega_1^1 - 2\omega_2^1\right) \omega_2^1}}}{2 \left(\omega_1^1 - 2\omega_2^1\right)}
$$

• Some complicated solution!

• Problem set has solution that is easier to compute (and interpret)
2 Existence and Welfare Theorems

- Does Walrasian Equilibrium always exist? In general, yes, as long as preference convex

- Is Walrasian Equilibrium always unique? Not necessarily

- Is Walrasian Equilibrium efficient? Yes.
• **First Fundamental Welfare Theorem.** All Walrasian Equilibria are on Contract Curve (and therefore are Pareto Efficient).

• Figure
• **Second Fundamental Welfare theorem.** Given convex preferences, for every Pareto efficient allocation \(((x_1^1, x_1^1), (x_1^2, x_2^2))\) there exists some endowment \((\omega_1, \omega_2)\) such that \(((x_1^1, x_1^1), (x_1^2, x_2^2))\) is a Walrasian Equilibrium for endowment \((\omega_1, \omega_2)\).

• Figure
• Significance of these results:

  – First Theorem: Smithian Invisible Hand. Market leads to an allocation that is Pareto Efficient.

  – BUT: problems with externalities and public good

  – BUT: what about distribution?

  – Second Theorem: Can redistribute endowments to achieve any efficient outcome as a WE.

  – But redistribution is hard to implement, and distortive.
3 Asymmetric Information: Introduction

- Nicholson, Ch. 18, pp. 627-632 \[NOT in 9th Ed.\]

- Common economic relationship

- Contract between two parties:
  - Principal
  - Agent

- Two parties have asymmetric information
  - Principal offers a contract to the agent
  - Agent chooses an action
  - Action of agent (or his type) is not observed by principle
• Example 1: *Manager and worker*
  – Manager employs worker and offers wage
  – Worker exerts effort (not observed)
  – Manager pays worker as function of output

• Example 2: *Car Insurance*
  – Car insurance company offers insurance contract
  – Driver chooses quality of driving (not observed)
  – Insurance company pays for accidents

• Example 3: *Shareholders and CEO*
  – Shareholders choose compensation for CEO
  – CEO puts effort
  – CEO paid as function of stock price
• In all of these cases (and many more!), common structure
  – Principal would like to observe effort (of worker, of CEO, of driver)
  – Unfortunately, this is not observable
  – Only a related, noisy proxy is observable: output, accident, success
  – Contract offered by principal is function of this proxy

• This means that occasionally an agent that put a lot of effort but has bad luck is ‘punished’

• Also, agents that shirked may instead be compensated

• These principle-agent problems are called hidden action or moral hazard
• Second category (next lecture): *hidden type* or *adverse selection*

• Example 1: *Manager and worker*
  – Manager employs worker and offers wage
  – Worker can be hard-working or lazy

• Example 2: *Car Insurance*
  – Car insurance company offers insurance contract
  – Drivers ex ante can be careful or careless

• Example 3: *Shareholders and CEO*
  – Shareholders choose compensation for CEO
  – CEO is high-quality or thief
• Problem is similar (action is not observed), but with a twist
  
  – *Hidden action*: principal can convince agent to exert high effort with the appropriate incentives

  – *Hidden type*: agent’s behavior is not affected by incentives, but by her type

• Different task for principal:
  
  – *Hidden action*: Principal wants to incentivize agent to work hard

  – *Hidden type*: Principal wants to make sure to recruit ‘good’ agent, not ‘bad’ one

• Two look similar, but analysis is different

• Start from *Hidden Action*
4 Hidden Action (Moral Hazard)

- Nicholson, Ch. 18, pp. 632-637 [NOT in 9th Ed.]

- Example 3: Shareholders and CEO
  - Division of ownership and control

- Shareholders (owners of firm):
  - Have capital, but do not have time to run company themselves
  - Want firm run so as to maximize profits

- CEO (manager)
  - Has time and managerial skill
  - Does not have capital to own the firm
• If CEO owns the company (private enterprises), problem is solved → Infeasible in large companies

• Agent chooses effort $e$ (unobserved)
  
  – Induces output $y = e + \varepsilon$, where $\varepsilon$ is a noise term, with $E(\varepsilon) = 0$
  
  – Example: Despite putting effort, investment project did not succeed

• Principal pays a salary $w$ to the agent
  
  – Salary is a function of output $y$: $w = w(y)$
  
  – Remember: Salary cannot be function of effort $e$
• Principal maximizes expected profits

\[ E [\pi] = E [y - w(y)] = e - E [w(y)] \]

• Agent is risk averse and maximizes

\[ E [U (w(e + \varepsilon))] - c(e) \]

- \( c(e) \) is cost of effort: assume \( c'(e) > 0 \) and \( c''(e) > 0 \) for all \( e \)

- Utility function \( U \) satisfies \( U' > 0 \) and \( U'' < 0 \)

- Notice: Agent is risk-averse, Principal is risk-neutral

• Assume \( U(w) = -e^{-\gamma w} \) and \( \varepsilon \sim N(0, \sigma^2) \)

• Can solve explicitly for \( EU(w) \):

\[ EU(w) = -\frac{1}{\sqrt{2\pi}} \int e^{-\gamma w} e^{-\frac{1}{2} \frac{w-\mu_w}{\sigma_w^2}} dw = \mu_w - \frac{\gamma}{2} \sigma_w^2 \]

[Take this for granted]
• Expected utility of agent is $EU (w) = \mu_w - \frac{\gamma}{2} \sigma^2_w$

• Note: $\mu_w$ is average salary and $\sigma^2_w$ is variance of salary
  
  – Agent likes high mean salary $\mu_w$
  
  – Agent dislikes variance in salary $\sigma^2_w$
  
  – Dislike for variance increases in risk aversion $\gamma$

• Assume that contract is linear: $w = a + by = a + be + b\varepsilon$

  – Compute $\mu_w = E (w) = E [a + be + b\varepsilon] = a + be + bE [\varepsilon] = a + be$

  – Compute $\sigma^2_w = Var [a + be + b\varepsilon] = b^2 \sigma^2$

• Rewrite expected utility as

  $$EU (w) = a + be - \frac{\gamma}{2} b^2 \sigma^2$$
- Back to Principal-Agent problem

- Solve problem in three Steps, starting from last stage (backward induction)

  - **Step 1** (*Effort Decision*). Given contract \( w(y) \), what effort \( e^* \) is agent going to put in?

  - **Step 2.** (*Individual Rationality*) Given contract \( w(y) \) and anticipating to put in effort \( e^* \), does agent accept the contract?

  - **Step 3.** (*Profit Maximization*) Anticipating that the effort of the agent \( e^* \) (and the acceptance of the contract) will depend on the contract, what contract \( w(y) \) does principal choose to maximize profits?
• **Step 1.** Solve effort maximization of agent:

\[ Max_{ea} + be - \frac{\gamma}{2} b^2 \sigma^2 - c(e) \]

• Solution:

\[ c'(e) = b \]

• If assume \( c(e) = ce^2/2 \rightarrow e^* = b/c \)

• Check comparative statics

  – With respect to \( b \rightarrow \) What happens with more pay-for-performance?

  – With respect to \( c \rightarrow \) What happens with higher cost of effort?
• **Step 2.** Agent needs to be willing to work for principal

• *Individual rationality* condition:

\[
EU (w(e^*)) - c(e^*) \geq 0
\]

• Substitute in the solution for \(e^*\) and obtain

\[
a + be^* - \frac{\gamma}{2} b^2 \sigma^2 - c(e^*) \geq 0
\]

• Will be satisfied with equality: \(a^* = -be^* + \frac{\gamma}{2} b^2 \sigma^2 + c(e^*)\)
• **Step 3:** Owner maximizes expected profits

\[
\max_{a,b} E[\pi] = e - E[w(y)] = e - a - be
\]

• Substitute in the two constraints: \( c'(e) = b \) (Step 1) and \( a^* = -be^* + \frac{\gamma}{2} b^2 \sigma^2 + c(e^*) \) (Step 2)

• Obtain

\[
E[\pi] = e - \left( -be + \frac{\gamma}{2} b^2 \sigma^2 + c(e) \right) - c'(e) e \\
= e + be - \frac{\gamma}{2} b^2 \sigma^2 - c(e^*) - c'(e) e \\
= e + c'(e) e - \frac{\gamma}{2} \left( c'(e) \right)^2 \sigma^2 - c(e^*) - c'(e) e \\
= e - \frac{\gamma}{2} \left( c'(e) \right)^2 \sigma^2 - c(e^*)
\]

• Profit maximization yields f.o.c.

\[
1 - \gamma c'(e) \sigma^2 c''(e) - c'(e) = 0
\]
and hence
\[ c'(e^*) = \frac{1}{1 + \gamma\sigma^2 c''(e^*)} \]

- Notice: This implies \( c'(e^*) < 1 \)

- Substitute \( c(e) = ce^2/2 \) to get
\[ e^* = \frac{1}{c} \frac{1}{1 + \gamma\sigma^2 c} \]

- Comparative Statics:
  - Higher risk aversion \( \gamma \rightarrow \)
  - Higher variance of output \( \sigma \rightarrow \)
  - Higher effort cost \( c \rightarrow \)
• Also, remember \( b^* = c'(e^*) = ce^* \) and hence

\[
b^* = ce^* = c \frac{1}{1 + \gamma \sigma^2 c} = \frac{1}{1 + \gamma \sigma^2 c}
\]

• Notice \( 0 < b^* < 1 \):

  – Agent gets paid increasing function of output to incentivize

  – Does not get paid one-on-one \((b = 1)\) because that would pass on too much risk to agent

  – (Remember \( w^* = a^* + b^* y = a^* + b^* e + b^* \varepsilon \))

  – Comparative Statics: what happens to \( b^* \) if \( \gamma = 0 \) or \( \sigma = 0 \)? Interpret
• Compare this solution to solution when effort is observable

• This is so-called **first best** since it eliminates the uncertainty involved in connecting pay to performance (as opposed to effort)

  – Principal offers a flat wage $w = a$ as long as agent works $e^*$

  – Agent accepts job if

    $$a - c(e^*) \geq 0$$

  – Principal wants to pay minimal necessary and hence sets $a^* = c(e^*)$

  – Substitute into profit of principal

    $$\max_{a,b} E[\pi] = e - E[w(y)] = e - a^* = e - c(e)$$
- Solution for $e^*$: $c'(e^*) = 1$ or
  
  $e^*_{FB} = 1/c$

- Compare $e^*$ above and $e^*_{FB}$ in first best

- $\rightarrow$ With observable effort (first best) agent works harder
• Summary of hidden-action solution with risk-averse agent:

• **Risk-incentive trade-off:**
  
  – Agent needs to be incentivized \((b^* > 0)\) or will not put in effort \(e\)
  
  – Cannot give too much incentive \((b^* too high)\) because of risk-aversion
  
  – Trade-off solved if
    
    * Action \(e\) observable OR
    
    * No risk aversion \((\gamma = 0)\) OR
    
    * No noise in outcome \((\sigma^2 = 0)\)

  – Otherwise, effort \(e^*\) in equilibrium is sub-optimal

• Same trade-off applies to other cases
• Example 2: *Insurance* (Not fully solved)

  - Two states of the world: Loss and No Loss
  - Probability of Loss is $\pi(e)$, with $\pi'(e) < 0$
    * Example: Careful driving (Car Insurance)
    * Example: Maintaining your house better (House insurance)
  - Agent chooses quantity of insurance $\alpha$ purchased
  - Agent risk averse: $U(c)$ with $U' > 0$ and $U'' < 0$
• Qualitative solution:
  
  – No hidden action $\Rightarrow$ Full insurance: $\alpha^* = L$

  – Hidden action $\Rightarrow$
    
    * Trade-off risk-incentives $\Rightarrow$ Only Partial insurance $0 < \alpha^* < L$

    * Need to make agent partially responsible for accident to incentivize

    * Do not want to make too responsible because of risk-aversion
5 Next lecture

- Asymmetric Information
- Moral Hazard