Outline

1. Cost Curves II

2. One-step Profit Maximization

3. Second-Order Conditions

4. Introduction to Market Equilibrium

5. Aggregation

6. Market Equilibrium in the Short-Run
1 Cost Curves II

- Case 2. *Non-convex technology*. Plot production function, total cost, average and marginal. Supply function?
1.1 Supply Function

- Supply function: \( y^* = y^* (w, r, p) \)

- What happens to \( y^* \) as \( p \) increases?

- Is the supply function upward sloping?

- Remember f.o.c:
  \[
  p - c_y' (w, r, y) = 0
  \]

- Implicit function:
  \[
  \frac{\partial y^*}{\partial p} = -\frac{1}{-c_{yy}'' (w, r, y)} > 0
  \]
  as long as s.o.c. is satisfied.

- Yes! Supply function is upward sloping.
2 One-step Profit Maximization

• Nicholson, Ch. 11, pp. 374-380 (Ch. 9, pp. 265–270, 9th)

• One-step procedure: maximize profits

• Perfect competition. Price $p$ is given
  – Firms are small relative to market
  – Firms do not affect market price $p_M$

  – Will firm produce at $p > p_M$?
  – Will firm produce at $p < p_M$?
  – $\implies p = p_M$
• Revenue: \( py = pf(L, K) \)

• Cost: \( wL + rK \)

• Profit \( pf(L, K) - wL - rK \)
• Agent optimization:

$$\max_{L,K} pf(L, K) - wL - rK$$

• First order conditions:

$$pf'_L (L, K) - w = 0$$

and

$$pf'_K (L, K) - r = 0$$

• Second order conditions? $$pf''_{L,L} (L, K) < 0$$ and

$$|H| = \begin{vmatrix} pf''_{L,L} (L, K) & pf''_{L,K} (L, K) \\ pf''_{L,K} (L, K) & pf''_{K,K} (L, K) \end{vmatrix} = p^2 \left[ f''_{L,L} f''_{K,K} - (f''_{L,K})^2 \right] > 0$$

• Need $$f''_{L,K}$$ not too large for maximum
- Comparative statics with respect to $p, w,$ and $r$.

- What happens if $w$ increases?

\[
\frac{\partial L^*}{\partial w} = -\frac{|\begin{array}{cc} -1 & pf''_{L,K}(L,K) \\ 0 & pf''_{K,K}(L,K) \end{array}|}{pf''_{L,L}(L,K) pf''_{L,K}(L,K) pf''_{L,K}(L,K) pf''_{K,K}(L,K)} < 0
\]

and

\[
\frac{\partial L^*}{\partial r} =
\]

- Sign of $\partial L^*/\partial r$ depends on $f''_{L,K}$.
3 Second Order Conditions in P-Max: Cobb-Douglas

• How do the second order conditions relate for:
  – Cost Minimization
  – Profit Maximization?

• Check for Cobb-Douglas production function

\[ y = AK^\alpha L^\beta \]

• Cost Minimization. S.o.c.:

\[ c''_y (y^*, w, r) > 0 \]

• As we showed, for CD prod. ftn.,

\[
c''_y (y^*, w, r) = -\frac{1}{\alpha + \beta} \frac{1 - (\alpha + \beta)}{\alpha + \beta} \frac{B}{A^2} \left( \frac{y}{A} \right)^{\frac{1-2(\alpha+\beta)}{\alpha+\beta}}
\]

which is > 0 as long as \( \alpha + \beta < 1 \) (DRS)
• **Profit Maximization.** S.o.c.: \( pf_{L,L}''(L, K) < 0 \) and
\[
|H| = p^2 \left[ f_{L,L}'' f_{K,K}'' - (f_{L,K}'')^2 \right] > 0
\]

• As long as \( \beta < 1 \),
\[
pf_{L,L}''' = p\beta (\beta - 1) AK^\alpha L^{\beta - 2} < 0
\]

• Then,
\[
|H| = p^2 \left[ f_{L,L}'' f_{K,K}'' - (f_{L,K}'')^2 \right] =
\[
= p^2 \left[ \frac{\beta(\beta - 1) AK^\alpha L^{\beta - 2}}{(\alpha \beta AK^{\alpha - 1} L^{\beta - 1})^2} \right] =
\[
= p^2 A^2 K^{2\alpha - 2} L^{2\beta - 2} \alpha \beta [1 - \alpha - \beta]
\]

• Therefore, \( |H| > 0 \) iff \( \alpha + \beta < 1 \) (DRS)

• The two conditions coincide
4 Introduction to Market Equilibrium

• Nicholson, (Ch. 10, pp. 279–295, 9th)

• Two ways to analyze firm behavior:
  – Two-Step Cost Minimization
  – One-Step Profit Maximization

• What did we learn?
  – Optimal demand for inputs $L^*, K^*$ (see above)
  – Optimal quantity produced $y^*$
• **Supply function.** \( y = y^* (p, w, r) \)
  
  – From profit maximization:
  \[
  y = f (L^* (p, w, r), K^* (p, w, r))
  \]
  
  – From cost minimization:
  
  \[
  MC \text{ curve above } AC
  \]
  
  – Supply function is increasing in \( p \)

• Market Equilibrium. Equate demand and supply.

• Aggregation?

• Industry supply function!
5 Aggregation

5.1 Producers aggregation

• $J$ companies, $j = 1, ..., J$, producing good $i$

• Company $j$ has supply function

\[ y_i^j = y_i^{j*} (p_i, w, r) \]

• Industry supply function:

\[ Y_i (p_i, w, r) = \sum_{j=1}^{J} y_i^{j*} (p_i, w, r) \]

• Graphically,
5.2 Consumer aggregation

- Nicholson, (Ch. 10, pp. 279–282)

- One-consumer economy

- Utility function $u(x_1, \ldots, x_n)$

- Prices $p_1, \ldots, p_n$

- Maximization $\Rightarrow$

  \[
  x_1^* = x_1^*(p_1, \ldots, p_n, M),
  \]

  \[
  \vdots
  \]

  \[
  x_n^* = x_n^*(p_1, \ldots, p_n, M).
  \]
• Focus on good $i$. Fix prices $p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n$ and $M$

• **Single-consumer demand function:**

$$x_i^* = x_i^* (p_i|p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n, M)$$

• What is sign of $\partial x_i^*/\partial p_i$?

• Negative if good $i$ is normal

• Negative or positive if good $i$ is inferior
• **Aggregation:** \( J \) consumers, \( j = 1, \ldots, J \)

• Demand for good \( i \) by consumer \( j \):
  \[
x_{i}^{j*} = x_{i}^{j*} \left( p_{1}, \ldots, p_{n}, M^{j} \right)
  \]

• Market demand \( X_{i} \):
  \[
  X_{i} \left( p_{1}, \ldots, p_{n}, M^{1}, \ldots, M^{J} \right) \\
  = \sum_{j=1}^{J} x_{i}^{j*} \left( p_{1}, \ldots, p_{n}, M^{j} \right)
  \]

• Graphically,
• Notice: market demand function depends on distribution of income $M^J$

• Market demand function $X_i$:
  
  – Consumption of good $i$ as function of prices $p$
  
  – Consumption of good $i$ as function of income distribution $M^j$
6 Market Equilibrium in the Short-Run

- Nicholson, (Ch. 14, pp. 368–382, 9th)

- What is equilibrium price $p_i$?

- Magic of the Market...

- Equilibrium: No excess supply, No excess demand

- Prices $p^*$ equates demand and supply of good $i$:

$$Y^* = Y_i^S (p_i^*, w, r) = X_i^D (p_1^*, ..., p_n^*, M^1, ..., M^J)$$
• Graphically,

• Notice: in short-run firms can make positive profits
• Comparative statics exercises with endogenous price $p_i$:

  – increase in wage $w$ or interest rate $r$:

  – change in income distribution
7 Next Lecture

- Comparative Statics of Equilibrium
- Elasticities
- Taxes and Subsidies