Outline

1. Properties of Preferences

2. From Preferences to Utility (and viceversa)

3. Common Utility Functions

4. Utility maximization
1 Properties of Preferences

• Nicholson, Ch. 3, pp. 87-88 (69-70, 9th)

• Commodity set $X$ (apples vs. strawberries, work vs. leisure, consume today vs. tomorrow)

• Preference relation $\succeq$ over $X$

• A preference relation $\succeq$ is *rational* if

  1. It is *complete*: For all $x$ and $y$ in $X$, either $x \succeq y$, or $y \succeq x$ or both

  2. It is *transitive*: For all $x$, $y$, and $z$, $x \succeq y$ and $y \succeq z$ implies $x \succeq z$

• Preference relation $\succeq$ is *continuous* if for all $y$ in $X$, the sets $\{ x : x \succeq y \}$ and $\{ x : y \succeq x \}$ are closed sets.
• Example: $X = R^2$ with map of indifference curves

• Counterexamples:

1. Incomplete preferences. Dominance rule.

2. Intransitive preferences. Quasi-discernible differences.

3. Discontinuous preferences. Lexicographic order
- Indifference relation $\sim$: $x \sim y$ if $x \geq y$ and $y \geq x$

- Strict preference: $x \succ y$ if $x \geq y$ and not $y \geq x$

- Exercise. If $\geq$ is rational,
  - $\succ$ is transitive
  - $\sim$ is transitive
  - Reflexive property of $\geq$. For all $x$, $x \geq x$. 
• Other features of preferences

• Preference relation $\succeq$ is:

  – *monotonic* if $x \geq y$ implies $x \succeq y$.

  – *strictly monotonic* if $x \geq y$ and $x_j > y_j$ for some $j$ implies $x \succ y$.

  – *convex* if for all $x, y, z$ in $X$ such that $x \succeq z$ and $y \succeq z$, then $tx + (1 - t)y \succeq z$ for all $t$ in $[0, 1]$
2 From preferences to utility

- Nicholson, Ch. 3

- Economists like to use utility functions $u : X \rightarrow R$

- $u(x)$ is ‘liking’ of good $x$

- $u(a) > u(b)$ means: I prefer $a$ to $b$.

- **Def.** Utility function $u$ represents preferences $\succeq$ if, for all $x$ and $y$ in $X$, $x \succeq y$ if and only if $u(x) \geq u(y)$.

- **Theorem.** If preference relation $\succeq$ is rational and continuous, there exists a continuous utility function $u : X \rightarrow R$ that represents it.
• [Skip proof]

• Example:

\[(x_1, x_2) \succeq (y_1, y_2) \text{ iff } x_1 + x_2 \geq y_1 + y_2\]

• Draw:

• Utility function that represents it: \( u(x) = x_1 + x_2 \)

• But... Utility function representing \( \succeq \) is not unique

• Take \( 3u(x) \) or \( \exp(u(x)) \)

• \( u(a) > u(b) \iff \exp(u(a)) > \exp(u(b)) \)
• If $u(x)$ represents preferences $\succeq$ and $f$ is a strictly increasing function, then $f(u(x))$ represents $\succeq$ as well.

• If preferences are represented from a utility function, are they rational?
  
  – completeness

  – transitivity
• Indifference curves: \( u(x_1, x_2) = \bar{u} \)

• They are just implicit functions! \( u(x_1, x_2) - \bar{u} = 0 \)

\[
\frac{dx_2}{dx_1} = -\frac{U'_x_1}{U'_x_2} = MRS
\]

• Indifference curves for:
  
  – monotonic preferences;

  – strictly monotonic preferences;

  – convex preferences
3 Common utility functions

- Nicholson, Ch. 3, pp. 100-104 (82-86, 9th)

1. Cobb-Douglas preferences: \( u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha} \)

   - \( MRS = -\alpha x_1^{\alpha-1} x_2^{1-\alpha} / (1-\alpha) x_1^\alpha x_2^{-\alpha} = \frac{\alpha}{1-\alpha} \frac{x_2}{x_1} \)

2. Perfect substitutes: \( u(x_1, x_2) = \alpha x_1 + \beta x_2 \)

   - \( MRS = -\alpha / \beta \)
3. Perfect complements: $u(x_1, x_2) = \min(\alpha x_1, \beta x_2)$

- $MRS$ discontinuous at $x_2 = \frac{\alpha}{\beta} x_1$

4. Constant Elasticity of Substitution: $u(x_1, x_2) = \left(\alpha x_1^\rho + \beta x_2^\rho\right)^{1/\rho}$

- $MRS = -\frac{\alpha}{\beta} \left(\frac{x_1}{x_2}\right)^{\rho-1}$

- if $\rho = 1$, then...

- if $\rho = 0$, then...

- if $\rho \to -\infty$, then...
4 Utility Maximization

- Nicholson, Ch. 4, pp. 114–124 (94–105, 9th)

- $X = \mathbb{R}_+^2$ (2 goods)

- Consumers: choose bundle $x = (x_1, x_2)$ in $X$ which yields highest utility.

- Constraint: income $= M$

- Price of good 1 $= p_1$, price of good 2 $= p_2$

- Bundle $x$ is feasible if $p_1x_1 + p_2x_2 \leq M$

- Consumer maximizes

$$\max_{x_1, x_2} u(x_1, x_2)$$

s.t. $p_1x_1 + p_2x_2 \leq M$

$x_1 \geq 0, \ x_2 \geq 0$
• Maximization subject to inequality. How do we solve that?

• Trick: $u$ strictly increasing in at least one dimension. ($\geq$ strictly monotonic)

• Budget constraint always satisfied with equality

• Ignore temporarily $x_1 \geq 0, x_2 \geq 0$ and check afterwards that they are satisfied for $x_1^*$ and $x_2^*$. 
Problem becomes

\[
\max_{x_1, x_2} u(x_1, x_2)
\]
\[
s.t. \ p_1 x_1 + p_2 x_2 - M = 0
\]
5 Next Class

- Utility Maximization (ctd)

- Utility Maximization – tricky cases

- Indirect Utility Function