Outline

1. Dynamic Games

2. Oligopoly: Stackelberg

3. General Equilibrium: Introduction

4. Edgeworth Box: Pure Exchange
1 Dynamic Games

• Nicholson, Ch. 8, pp. 255-266 (*better* than Ch. 15, pp. 449–454, 9th)

• Dynamic games: one player plays after the other

• Decision trees
  - Decision nodes
  - Strategy is a plan of action at each decision node
- Example: battle of the sexes game

<table>
<thead>
<tr>
<th></th>
<th>She \ He</th>
<th>Ballet</th>
<th>Football</th>
</tr>
</thead>
<tbody>
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<td>2, 1</td>
<td>0, 0</td>
<td></td>
</tr>
<tr>
<td>Football</td>
<td>0, 0</td>
<td>1, 2</td>
<td></td>
</tr>
</tbody>
</table>

- Dynamic version: she plays first
• **Subgame-perfect equilibrium.** At each node of the tree, the player chooses the strategy with the highest payoff, given the other players’ strategy

• Backward induction. Find optimal action in last period and then work backward

• Solution
- Example 2: Entry Game

<table>
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<th></th>
<th>Enter</th>
<th>Do not Enter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enter</td>
<td>$-1, -1$</td>
<td>$10, 0$</td>
</tr>
<tr>
<td>Do not Enter</td>
<td>$0, 5$</td>
<td>$0, 0$</td>
</tr>
</tbody>
</table>

- Exercise. Dynamic version.

- Coordination games solved if one player plays first
• Can use this to study finitely repeated games

• Suppose we play the prisoner’s dilemma game ten times.

\[
\begin{array}{c|ccc}
1 \backslash 2 & D & ND \\
D & -4, -4 & -1, -5 \\
ND & -5, -1 & -2, -2 \\
\end{array}
\]

• What is the subgame perfect equilibrium?
• The result differs if infinite repetition with a probability of terminating

• Can have cooperation

• Strategy of repeated game:
  – Cooperate (ND) as long as opponent always cooperate
  – Defect (D) forever after first defection

• Theory of repeated games: Econ. 104
2 Oligopoly: Stackelberg

- Nicholson, Ch. 15, pp. 543-545 (*better than* Ch. 14, pp. 423-424, 9th)

- Setting as in problem set

- 2 Firms

- Cost: \( c(y) = cy, \) with \( c > 0 \)

- Demand: \( p(Y) = a - bY, \) with \( a > c > 0 \) and \( b > 0 \)

- Difference: Firm 1 makes the quantity decision first

- Use subgame perfect equilibrium
Solution:

Solve first for Firm 2 decision as function of Firm 1 decision:

$$\max_{y_2} (a - by_2 - by_1^*) y_2 - cy_2$$

F.o.c.: $a - 2by_2^* - by_1^* - c = 0$

Firm 2 best response function:

$$y_2^* = \frac{a - c}{2b} - \frac{y_1^*}{2}.$$
• Firm 1 takes this response into account in the maximization:

\[
\max_{y_1} \left( a - by_1 - by_2^*(y_1) \right) y_1 - cy_1
\]

or

\[
\max_{y_1} \left( a - by_1 - b \left( \frac{a - c}{2b} - \frac{y_1}{2} \right) \right) y_1 - cy_1
\]

• F.o.c.:

\[
a - 2by_1 - \frac{(a - c)}{2} + by_1 - c = 0
\]

or

\[
y_1^* = \frac{a - c}{2b}
\]

and

\[
y_2^* = \frac{a - c}{2b} - \frac{y_1^*}{2} = \frac{a - c}{2b} - \frac{a - c}{4b} = \frac{a - c}{4b}.
\]
• Total production:

\[ Y_D^* = y_1^* + y_2^* = 3 \frac{a - c}{4b} \]

• Price equals

\[ p^* = a - b \left( \frac{3a - c}{4b} \right) = \frac{1}{4}a + \frac{3}{4}c \]

• Compare to monopoly:

\[ y_M^* = \frac{a - c}{2b} \]

and

\[ p_M^* = \frac{a + c}{2}. \]

• Compare to Cournot:

\[ Y_D^* = y_1^* + y_2^* = 2 \frac{a - c}{3b} \]

and

\[ p_D^* = \frac{1}{3}a + \frac{2}{3}c. \]
• Compare with Cournot outcome

• Firm 2 best response function:

\[ y_2^* = \frac{a - c}{2b} - \frac{y_1^*}{2} \]

• Firm 1 best response function:

\[ y_1^* = \frac{a - c}{2b} - \frac{y_2^*}{2} \]

• Intersection gives Cournot
• Stackelberg: Equilibrium is point on Best Response of Firm 2 that maximizes profits of Firm 1

• Plot iso-profit curve of Firm 1:

\[ \tilde{\Pi}_1 = (a - c) y_1 - by_1 y_2 - by_1^2 \]

• Solve for \( y_2 \) along iso-profit:

\[ y_2 = \frac{a - c}{b} - y_1 - \frac{\tilde{\Pi}_1}{by_1} \]

• Iso-profit curve is flat for

\[ \frac{dy_2}{dy_1} = -1 + \frac{\tilde{\Pi}}{b(y_1)^2} = 0 \]

or

\[ y_1 = \]
Figure
3 General Equilibrium: Introduction

• So far, we looked at consumers
  – Demand for goods
  – Choice of leisure and work
  – Choice of risky activities

• We also looked at producers:
  – Production in perfectly competitive firm
  – Production in monopoly
  – Production in oligopoly
- We also combined consumers and producers:
  - Supply
  - Demand
  - Market equilibrium

- Partial equilibrium: one good at a time

- General equilibrium: Demand and supply for all goods!
  - supply of young worker↑ \(\rightarrow\) wage of experienced workers?
  - minimum wage↑ \(\rightarrow\) effect on higher earners?
  - steel tariff↑ \(\rightarrow\) effect on car price
4 Edgeworth Box: Pure Exchange

• Nicholson, Ch. 13, pp. 441-444, 476-478 (Ch. 12, pp. 335–338, 369–370, 9th)

• 2 consumers in economy: $i = 1, 2$

• 2 goods, $x_1, x_2$

• Endowment of consumer $i$, good $j$: $\omega^i_j$

• Total endowment: $(\omega_1, \omega_2) = (\omega_1^1 + \omega_2^1, \omega_1^2 + \omega_2^2)$

• No production here. With production (as in book), $(\omega_1, \omega_2)$ are optimally produced
• Edgeworth box

• Draw preferences of agent 1

• Draw preferences of agent 2
• Consumption of consumer $i$, good $j$: $x^i_j$

• Feasible consumption:

$$x^1_i + x^2_i \leq \omega_i \text{ for all } i$$

• If preferences monotonic, $x^1_i + x^2_i = \omega_i \text{ for all } i$

• Can map consumption levels into box
5  Next lecture

- General Equilibrium

- Barter