Outline

1. Walrasian Equilibrium II

2. Example

3. Existence and Welfare Theorems

4. Asymmetric Information: Introduction
1 Walrasian Equilibrium

- **Walrasian Equilibrium.** \( (x_1^{1*}, x_2^{1*}), (x_1^{2*}, x_2^{2*}), p_1^*, p_2^* ) \) is a Walrasian Equilibrium if:

  - Each consumer maximizes utility subject to budget constraint:
    \[
    (x_1^{i*}, x_2^{i*}) = \arg \max_{x_1^i, x_2^i} u_i \left( x_1^i, x_2^i \right) \]
    
    \[\text{s.t. } p_1^* x_1^i + p_2^* x_2^i \leq p_1^* \omega_1^i + p_2^* \omega_2^i \]

  - All markets clear:
    \[x_1^{1*} + x_2^{1*} \leq \omega_1^1 + \omega_2^1 \text{ for all } j.\]

  \[x_1^{2*} + x_2^{2*} \leq \omega_1^2 + \omega_2^2 \text{ for all } j.\]
• Offer curve for consumer 1:

\[(x_1^1 (p_1, p_2, (\omega_1, \omega_2)), x_2^1 (p_1, p_2, (\omega_1, \omega_2)))\]

• Offer curve is set of points that maximize utility as function of prices \(p_1\) and \(p_2\).

• Then find offer curve for consumer 2:

\[(x_1^2 (p_1, p_2, (\omega_1, \omega_2)), x_2^2 (p_1, p_2, (\omega_1, \omega_2)))\]

• Figure
• *Step 2.* Find intersection(s) of two offer curves

• Walrasian Equilibrium is intersection of the two offer curves!
  
  – Both individuals maximize utility given prices
  
  – Total quantity demanded equals total endowment
• Relate Walrasian Equilibrium to barter equilibrium.

• Walrasian Equilibrium is a subset of barter equilibrium:
  – Does WE satisfy Individual Rationality condition?
  – Does WE satisfy the Pareto Efficiency condition?

• Walrasian Equilibrium therefore picks one (or more) point(s) on contract curve.
2 Example

- Consumer 1 has Leontieff preferences:

\[ u(x_1, x_2) = \min (x_1^1, x_2^1) \]

- Bundle demanded by consumer 1:

\[
x_1^* = x_2^* = x^* = \frac{p_1 \omega_1^1 + p_2 \omega_2^1}{p_1 + p_2} = \frac{\omega_1 + (p_2/p_1) \omega_2}{1 + (p_2/p_1)}
\]

- Graphically
• Comparative statics:

  - increase in $\omega$

  - increase in $p_2/p_1$:

    \[
    \frac{dx_1^{1\ast}}{dp_2/p_1} = \frac{\omega_2^{1\ast}(1 + (p_2/p_1))}{1 + (p_2/p_1)} = \frac{\omega_2^{1\ast} - \omega_1^{1\ast}}{(1 + (p_2/p_1))^2}
    \]

  - Effect depends on income effect through endowments:

    * A lot of good 2 $\rightarrow$ increase in price of good 2 makes richer

    * Little good 2 $\rightarrow$ increase in price of good 2 makes poorer

• Notice: Only ratio of prices matters (general feature)
• Consumer 2 has Cobb-Douglas preferences:

\[ u(x_1, x_2) = (x_1^2)^{.5} (x_2^2)^{.5} \]

• Demands of consumer 2:

\[ x_1^{2*} = \frac{.5 \left( p_1 \omega_1^1 + p_2 \omega_2^1 \right)}{p_1} = .5 \left( \frac{\omega_1^1 + \frac{p_2}{p_1} \omega_2^1}{p_1} \right) \]

and

\[ x_2^{2*} = \frac{.5 \left( p_1 \omega_1^1 + p_2 \omega_2^1 \right)}{p_2} = .5 \left( \frac{p_1 \omega_1^1 + \omega_2^1}{p_2} \right) \]
• Comparative statics:
  
  – increase in $\omega$ $\rightarrow$ Increase in final consumption
  
  – increase in $p_2/p_1$ $\rightarrow$ Unambiguous increase in $x_1^{2*}$ and decrease in $x_2^{2*}$
• Impose Walrasian equilibrium in market 1:

\[ x_1^{1*} + x_1^{2*} = \omega_1^1 + \omega_1^2 \]

This implies

\[ \frac{\omega_1^1 + (p_2/p_1) \omega_1^2}{1 + (p_2/p_1)} + 0.5 \left( \omega_1^1 + \frac{p_2 \omega_1^2}{p_1} \right) = \omega_1^1 + \omega_1^2 \]

or

\[ \frac{0.5 - 0.5 (p_2/p_1)}{1 + (p_2/p_1)} \omega_1^1 + \frac{0.5 (p_2/p_1) + 0.5 (p_2/p_1)^2 - 1}{1 + (p_2/p_1)} \omega_2^1 = 0 \]

or

\[ (\omega_1^1 - 2\omega_2^1) + (\omega_1^1 + \omega_2^1) (p_2/p_1) + \omega_2^1 (p_2/p_1)^2 = 0 \]
• Solution for $p_2/p_1$:

$$\frac{p_2}{p_1} = \frac{-(\omega_1^1 - 2\omega_2^1) + \sqrt{-4(\omega_1^1 - 2\omega_2^1)^2 + (\omega_1^1 + \omega_2^1)^2}}{2(\omega_1^1 - 2\omega_2^1)}$$

• Some complicated solution!

• Problem set has solution that is much easier to compute (and interpret)
3 Existence and Welfare Theorems

- Does Walrasian Equilibrium always exist? In general, yes, as long as preference convex

- Is Walrasian Equilibrium always unique? Not necessarily

- Is Walrasian Equilibrium efficient? Yes.
• **First Fundamental Welfare Theorem.** All Walrasian Equilibria are on Contract Curve (and therefore are Pareto Efficient).

• Figure
• **Second Fundamental Welfare theorem.** Given convex preferences, for every Pareto efficient allocation \( ((x_1^1, x_1^1), (x_1^2, x_2^2)) \) there exists some endowment \((\omega_1, \omega_2)\) such that \( ((x_1^1, x_1^1), (x_1^2, x_2^2)) \) is a Walrasian Equilibrium for endowment \((\omega_1, \omega_2)\).

• Figure
• Significance of these results:

  – First Theorem: Smithian Invisible Hand. Market leads to an allocation that is Pareto Efficient.
  – BUT: problems with externalities and public good
  – BUT: what about distribution?

  – Second Theorem: Can redistribute endowments to achieve any efficient outcome as a WE.
  – But redistribution is hard to implement, and distortive.
4 Asymmetric Information: Introduction

- Nicholson, Ch. 18, pp. 627-632 [NOT in 9th Ed.]

- Common economic relationship

- Contract between two parties:
  - Principal
  - Agent

- Two parties have asymmetric information
  - Principal offers a contract to the agent
  - Agent chooses an action
  - Action of agent (or his type) is not observed by principle
• Example 1: *Manager and worker*
  
  – Manager employs worker and offers wage  
  – Worker exerts effort (not observed)  
  – Manager pays worker as function of output  

• Example 2: *Car Insurance*
  
  – Car insurance company offers insurance contract  
  – Driver chooses quality of driving (not observed)  
  – Insurance company pays for accidents  

• Example 3: *Shareholders and CEO*
  
  – Shareholders choose compensation for CEO  
  – CEO puts effort  
  – CEO paid as function of stock price
In all of these cases (and many more!), common structure

- Principal would like to observe effort (of worker, of CEO, of driver)
- Unfortunately, this is not observable
- Only a related, noisy proxy is observable: output, accident, success
- Contract offered by principal is function of this proxy

This means that occasionally an agent that put a lot of effort but has bad luck is ‘punished’

Also, agents that shirked may instead be compensated

These principle-agent problems are called hidden action or moral hazard
• Second category (next lecture): *hidden type* or *adverse selection*

• Example 1: *Manager and worker*
  
  – Manager employs worker and offers wage
  
  – Worker can be hard-working or lazy

• Example 2: *Car Insurance*
  
  – Car insurance company offers insurance contract
  
  – Drivers ex ante can be careful or careless

• Example 3: *Shareholders and CEO*
  
  – Shareholders choose compensation for CEO
  
  – CEO is high-quality or thief
• Problem is similar (action is not observed), but with a twist
  
  – *Hidden action*: principal can convince agent to exert high effort with the appropriate incentives
  
  – *Hidden type*: agent’s behavior is not affected by incentives, but by her type

• Different task for principal:
  
  – *Hidden action*: Principal wants to incentivize agent to work hard
  
  – *Hidden type*: Principal wants to make sure to recruit ‘good’ agent, not ‘bad’ one

• Two look similar, but analysis is different

• Start from *Hidden Action*
5 Next lecture

• Asymmetric Information

• Moral Hazard