Outline

1. Oligopoly: Cournot

2. Oligopoly: Bertrand

3. Second-price Auction

4. Auctions: eBay Evidence
1 Oligopoly: Cournot

- Nicholson, Ch. 14, pp. 524-530 (*better* than Ch. 14, pp. 418–419, 421–422, 9th)

- Back to oligopoly maximization problem

- Assume 2 firms, cost $c_i(y_i) = cy_i$, $i = 1, 2$

- Firms choose simultaneously quantity $y_i$

- Firm $i$ maximizes:

$$\max_{y_i} p (y_i + y_{-i}) y_i - cy_i.$$  

- First order condition with respect to $y_i$:

$$p_Y' \left( y_i^* + y_{-i}^* \right) y_i^* + p - c = 0, \quad i = 1, 2.$$
• Nash equilibrium:
  - $y_1$ optimal given $y_2$;
  - $y_2$ optimal given $y_1$.

• Solve equations:
  
  $p_0 Y (y_1^* + y_2^*) y_1^* + p - c = 0$ and 
  $p_0 Y (y_2^* + y_1^*) y_2^* + p - c = 0$.

• Cournot -> Pricing above marginal cost
2 Oligopoly: Bertrand

- Cournot oligopoly: firms choose quantities

- Bertrand oligopoly: firms first choose prices, and then produce quantity demanded by market

- Market demand function $Y(p)$

- 2 firms

- Profits:

$$\pi_i(p_i, p_{-i}) = \begin{cases} 
(p_i - c) Y(p_i) & \text{if } p_i < p_{-i} \\
(p_i - c) Y(p_i) / 2 & \text{if } p_i = p_{-i} \\
0 & \text{if } p_i > p_{-i}
\end{cases}$$
• First show that $p_1 = c = p_2$ is Nash Equilibrium

• Does any firm have a (strict) incentive to deviate?

• Check profits for Firm 1

• Symmetric argument for Firm 2
• Second, show that this equilibrium is unique.

• For each of the next 5 cases at least one firm has a profitable deviation

• Case 1. \( p_1 > p_2 > c \)

• Case 2. \( p_1 = p_2 > c \)

• Case 3. \( p_1 > c \geq p_2 \)
• Case 4. $c > p_1 \geq p_2$

• Case 5. $p_1 = c > p_2$

• Only Case 6 remains: $p_1 = c = p_2$, which is Nash Equilibrium

• It is unique!
• Notice:

• To show that something is an equilibrium $\rightarrow$ Show that there is *no* profitable deviation

• To show that something is *not* an equilibrium $\rightarrow$ Show that there is *one* profitable deviation
• Surprising result of Bertrand Competition

• Marginal cost pricing

• Two firms are enough to guarantee perfect competition!

• Realistic? Price wars between PC makers
3 Second-price Auction

- Nicholson, Ch. 18, pp. 659–66 [Not in old book]

- Sealed-bid auction

- Highest bidder wins object

- Price paid is second highest price

- Two individuals: $I = 2$

- Strategy $s_i$ is bid $b_i$

- Each individual knows value $v_i$
• Payoff for individual $i$ is

$$u_i(b_i, b_{-i}) = \begin{cases} 
    v_i - b_i & \text{if } b_i > b_{-i} \\
    (v_i - b_{-i}) / 2 & \text{if } b_i = b_{-i} \\
    0 & \text{if } b_i < b_{-i}
\end{cases}$$

• Show: weakly dominant to set $b_i^* = v_i$

• To show:

$$u_i(v_i, b_{-i}) \geq u_i(b_i, b_{-i})$$

for all $b_i$, for all $b_{-i}$, and for $i = 1, 2$. 
1. Assume $b_{-i} > v_i$

   - $u_i(v_i, b_{-i}) = 0 = u_i(b_i, b_{-i})$ for any $b_i < b_{-i}$
   - $u_i(b_{-i}, b_{-i}) = (v_i - b_{-i})/2 < 0$
   - $u_i(b_i, b_{-i}) = (v_i - b_{-i}) < 0$ for any $b_i > b_{-i}$

2. Assume now $b_{-i} = v_i$
3. Assume now $b_{-i} < v_i$.
4 Auctions: Evidence from eBay

• In second-price auction, optimal strategy is to bid one’s own value

• Is this true?

• eBay has proxy system: If you have highest bid, you pay bid of second-highest bidder

• eBay is essentially a second-price auction

• Two deviations:

  1. People bid multiple times – they should not in this theory

  2. People may overbid
An example:
eBay Bidding for a Board Game

- Bidding environment with clear boundary for rational willingness to pay ("buy-it-now price").
- Empirical environment unaffected by common-value arguments (presumably bidding for private use; in addition "buy-it-now" price).
- Still non-negligible amount ($100-$200).

→ Is there evidence of overbidding?
→ If so, can we detect determinants of overbidding?
The Object

Awaken Your Financial Genius...

Cashflow

How to Get Out of the Rat Race

The More You Play this Game the Richer You Become

You will Discover

△ Consider This △

△ You will Discover △
The Data

- Cashflow 101: board game with the purpose of finance/accounting education.
- Retail price: $195 plus shipping cost ($10.75) from manufacturer (www.richdad.com).
- Two ways to purchase Cashflow 101 on eBay
  - Auction (quasi-second price proxy bidding)
  - Buy-it-now
Sample

- **Listings**
  - 206 by individuals (187 auctions only, 19 auctions with buy-it-now option)
  - 493 by two retailers (only buy-it-now)

- **Remove non-US$, terminated, unsold items and items without simultaneous professional buy-it-now listing.** 169 auctions

- **Buy-it-now offers of the two retailers**
  - Continuously present for all but six days. (Often individual buy-it-now offers present as well; they are often lower.)
  - 100% and 99.9% positive feedback scores.
  - Same prices $129.95 until 07/31/2004; $139.95 since 08/01/2004.
  - Shipping cost $9.95; other retailer $10.95.
  - New items (with bonus tapes/video).
# Listing Example (02/12/2004)

<table>
<thead>
<tr>
<th>Item</th>
<th>Price</th>
<th>Quantity</th>
<th>Shipping Time</th>
</tr>
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<tbody>
<tr>
<td>Rich Dad's Cashflow Quadrant, Rich dad ...</td>
<td>$12.50</td>
<td>4</td>
<td>1d 00h 14m</td>
</tr>
<tr>
<td>Rich Dad's Cashflow Quadrant by Robert T. ...</td>
<td>$9.00</td>
<td>9</td>
<td>1d 00h 43m</td>
</tr>
<tr>
<td>Real Estate Investment Cashflow Software $$$!</td>
<td>$10.49</td>
<td>2</td>
<td>1d 04h 36m</td>
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<tr>
<td>CASHFLOW® 101 202 Robert Kiyosaki Best Pak $</td>
<td>$207.96</td>
<td></td>
<td>1d 06h 47m</td>
</tr>
<tr>
<td>TRY IT TODAY, WITH ABSOLUTELY NO RISK,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CASHFLOW® 101 Robert Kiyosaki Plus Bonuses!</td>
<td>$129.95</td>
<td></td>
<td>1d 08h 02m</td>
</tr>
<tr>
<td>Your satisfaction is GUARANTEED, 100% $ back</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MIINT Cashflow 101 *Robert Kiyosaki Game NR!</td>
<td>$140.00</td>
<td>13</td>
<td>1d 08h 04m</td>
</tr>
<tr>
<td>It's easy to be rich. Brand New. Still sealed</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>cashflow Hard Money Funding 101 real estate</td>
<td>$14.99</td>
<td></td>
<td>1d 09h 28m</td>
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<tr>
<td>BRANDNEW RICHDAD CASHFLOW FOR KIDS E-GAME</td>
<td>$20.00</td>
<td>1</td>
<td>1d 13h 54m</td>
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<td>CASHFLOW® 101 Robert Kiyosaki Plus Bonuses!</td>
<td>$129.95</td>
<td></td>
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<tr>
<td>CASHFLOW® 101 202 Robert Kiyosaki Best Pak $</td>
<td>$207.96</td>
<td></td>
<td>1d 15h 47m</td>
</tr>
<tr>
<td>TRY IT TODAY, WITH ABSOLUTELY NO RISK,</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Listing Example – Magnified

Pricing:

- [Buy Now] $129.95
- $140.00

- CASHFLOW® 101 202 Robert Kiyosaki Best Pak
- $207.96

Pricing:

- $129.95

- CASHFLOW® 101 Robert Kiyosaki Plus Bonuses!
- Your satisfaction is GUARANTEED, 100% $ back

- MINT Cashflow 101 *Robert Kiyosaki Game NR!
- It's easy to be rich, Brand New. Still sealed
# Bidding history of an item

**Item title:** CASHFLOW 101 Board Game Rich Dad Poor Dad

**Time left:** Auction has ended.

Only actual bids (not automatic bids generated up to a bidder's maximum) are shown. Automatic bids may be placed days or hours before a listing ends. Learn more about bidding.

<table>
<thead>
<tr>
<th>User ID</th>
<th>Bid Amount</th>
<th>Date of bid</th>
</tr>
</thead>
<tbody>
<tr>
<td>beezebugs</td>
<td>US $152.50</td>
<td>Aug-11-04 09:51:21 PDT</td>
</tr>
<tr>
<td>mndm-hdl</td>
<td>US $150.00</td>
<td>Aug-11-04 09:39:53 PDT</td>
</tr>
<tr>
<td>beezebugs</td>
<td>US $140.00</td>
<td>Aug-08-04 12:08:05 PDT</td>
</tr>
<tr>
<td>sj_orbit</td>
<td>US $130.01</td>
<td>Aug-08-04 25:49:02 PDT</td>
</tr>
<tr>
<td>succeedbroker</td>
<td>US $110.00</td>
<td>Aug-08-04 19:56:26 PDT</td>
</tr>
<tr>
<td>succeedbroker</td>
<td>US $105.00</td>
<td>Aug-06-04 17:19:21 PDT</td>
</tr>
<tr>
<td>succeedbroker</td>
<td>US $102.50</td>
<td>Aug-06-04 17:11:31 PDT</td>
</tr>
<tr>
<td>succeedbroker</td>
<td>US $100.00</td>
<td>Aug-06-04 15:41:40 PDT</td>
</tr>
<tr>
<td>002la</td>
<td>US $95.00</td>
<td>Aug-06-04 17:10:21 PDT</td>
</tr>
<tr>
<td>002la</td>
<td>US $95.00</td>
<td>Aug-06-04 17:10:21 PDT</td>
</tr>
<tr>
<td>12-gauge</td>
<td>US $88.00</td>
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</tr>
<tr>
<td>lmdeyque</td>
<td>US $58.00</td>
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<tr>
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<td>lmdeyque</td>
<td>US $40.00</td>
<td>Aug-05-04 10:45:08 PDT</td>
</tr>
<tr>
<td>bearhnbulls22</td>
<td>US $31.00</td>
<td>Aug-05-04 06:49:19 PDT</td>
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<tr>
<td>75ion</td>
<td>US $30.00</td>
<td>Aug-04-04 19:45:54 PDT</td>
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<tr>
<td>bearhnbulls22</td>
<td>US $29.00</td>
<td>Aug-04-04 06:49:26 PDT</td>
</tr>
<tr>
<td>bearhnbulls22</td>
<td>US $25.00</td>
<td>Aug-04-04 06:48:01 PDT</td>
</tr>
</tbody>
</table>

If you and another bidder placed the same bid amount, the earlier bid takes priority.
Hypotheses

Given the information on the listing website:

• (H1) An auction should never end at a price above the concurrently available purchase price.

• (H2) Mentioning of higher outside prices should not affect bidding behavior.
Figure 1. Starting Price (*startprice*)

- 45% below $20; mean=$46; SD=43.88
- only 6 auctions with first bid (not price) above buy-it-now
Figure 2. Final Price (finalprice)

41% are above “buy-it-now” (mean $132; SD 16.83)
Figure 4. Total Price (incl. shipping cost)

51% are above “buy-it-now” plus its shipping cost (mean=$144.20; SD=15.00)
The Other Lesson?

Some unsolicited eBay advice.

• Can make money by selling “Cashflow 101” to those who aspire to become financially smart, and overpay for the board game!

• Sellers : add exaggerated retail price, pay 20 cents extra (now 40 cents) for 10 day listing!

• Buyers : check out the “buy-it-now” price before you bid!
5 Next lecture

- Dynamic Games
- Stackelberg duopoly