Outline

1. Properties of Preferences II

2. From Preferences to Utility (and vice versa)

3. Common Utility Functions

4. Utility maximization
1 Properties of Preferences II

- Indifference relation $\sim$: $x \sim y$ if $x \succeq y$ and $y \succeq x$

- Strict preference: $x \succ y$ if $x \succeq y$ and not $y \succeq x$

- Exercise. If $\succeq$ is rational,
  - $\succ$ is transitive
  - $\sim$ is transitive
  - Reflexive property of $\succeq$. For all $x$, $x \succeq x$. 
• Other features of preferences

• Preference relation $\succeq$ is:

  – \textit{monotonic} if $x \geq y$ implies $x \succeq y$.

  – \textit{strictly monotonic} if $x \geq y$ and $x_j > y_j$ for some $j$ implies $x \succ y$.

  – \textit{convex} if for all $x, y,$ and $z$ in $X$ such that $x \succeq z$ and $y \succeq z$, then $tx + (1 - t)y \succeq z$ for all $t$ in $[0, 1]$.
2 From preferences to utility

- Nicholson, Ch. 3

- Economists like to use utility functions $u : X \to R$

- $u(x)$ is ‘liking’ of good $x$

- $u(a) > u(b)$ means: I prefer $a$ to $b$.

- **Def.** Utility function $u$ represents preferences $\succeq$ if, for all $x$ and $y$ in $X$, $x \succeq y$ if and only if $u(x) \geq u(y)$.

- **Theorem.** If preference relation $\succeq$ is rational and continuous, there exists a continuous utility function $u : X \to R$ that represents it.
• [Skip proof]

• Example:

\[(x_1, x_2) \succeq (y_1, y_2) \iff x_1 + x_2 \geq y_1 + y_2\]

• Draw:

• Utility function that represents it: \(u(x) = x_1 + x_2\)

• But... Utility function representing \(\succeq\) is not unique

• Take \(3u(x)\) or \(\exp(u(x))\)

• \(u(a) > u(b) \iff \exp(u(a)) > \exp(u(b))\)
• If $u(x)$ represents preferences $\succeq$ and $f$ is a strictly increasing function, then $f(u(x))$ represents $\succeq$ as well.

• If preferences are represented from a utility function, are they rational?
  
  – completeness
  
  – transitivity
• Indifference curves: \( u(x_1, x_2) = \bar{u} \)

• They are just implicit functions! \( u(x_1, x_2) - \bar{u} = 0 \)

\[
\frac{dx_2}{dx_1} = -\frac{U'_x}{U'_{x_2}} = MRS
\]

• Indifference curves for:
  
  – monotonic preferences;

  – strictly monotonic preferences;

  – convex preferences
3 Common utility functions

- Nicholson, Ch. 3, pp. 100-104 (82-86, 9th)

1. Cobb-Douglas preferences: \( u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha} \)
   - \( MRS = -\alpha x_1^{a-1} x_2^{1-\alpha} / (1-a)x_1^\alpha x_2^{-\alpha} = \frac{\alpha x_2}{1-\alpha x_1} \)

2. Perfect substitutes: \( u(x_1, x_2) = \alpha x_1 + \beta x_2 \)
   - \( MRS = -\alpha / \beta \)
3. Perfect complements: \( u(x_1, x_2) = \min(\alpha x_1, \beta x_2) \)

- \( MRS \) discontinuous at \( x_2 = \frac{\alpha}{\beta} x_1 \)

4. Constant Elasticity of Substitution: \( u(x_1, x_2) = (\alpha x_1^\rho + \beta x_2^\rho)^{1/\rho} \)

- \( MRS = -\frac{\alpha}{\beta} \left( \frac{x_1}{x_2} \right)^{\rho-1} \)

- if \( \rho = 1 \), then...

- if \( \rho = 0 \), then...

- if \( \rho \to -\infty \), then...
4 Utility Maximization

- Nicholson, Ch. 4, pp. 114–124 (94–105, 9th)

- $X = R^2_+$ (2 goods)

- Consumers: choose bundle $x = (x_1, x_2)$ in $X$ which yields highest utility.

- Constraint: income = $M$

- Price of good 1 = $p_1$, price of good 2 = $p_2$

- Bundle $x$ is feasible if $p_1 x_1 + p_2 x_2 \leq M$

- Consumer maximizes

$$\max_{x_1, x_2} u(x_1, x_2)$$

subject to:

- $p_1 x_1 + p_2 x_2 \leq M$
- $x_1 \geq 0$, $x_2 \geq 0$
• Maximization subject to inequality. How do we solve that?

• Trick: \( u \) strictly increasing in at least one dimension. \((\succeq \text{ strictly monotonic})\)

• Budget constraint always satisfied with equality

• Ignore temporarily \( x_1 \geq 0, x_2 \geq 0 \) and check afterwards that they are satisfied for \( x_1^* \) and \( x_2^* \).
• Problem becomes

\[
\max_{x_1, x_2} u(x_1, x_2) \\
\text{s.t. } p_1 x_1 + p_2 x_2 - M = 0
\]
5 Next Class

• Utility Maximization (ctd)

• Utility Maximization – tricky cases

• Indirect Utility Function