Economics 101A
(Lecture 7)

Stefano DellaVigna

February 7, 2012
Outline

1. Utility maximization – Tricky Cases

2. Indirect Utility Function

3. Comparative Statics (Introduction)

4. Income Changes

5. Price Changes
1 Utility maximization – tricky cases

- First, re-solve CES utility function.

$$\max_{x_1, x_2} \left( \alpha x_1^\rho + \beta x_2^\rho \right)^{1/\rho}$$
$$s.t. \ p_1 x_1 + p_2 x_2 - M = 0$$

- Solution:

$$x_1^* = \frac{M}{p_1 \left( 1 + \left( \frac{\alpha}{\beta} \right)^{\frac{1}{\rho-1}} \left( \frac{p_2}{p_1} \right)^{\frac{\rho}{\rho-1}} \right)}$$
$$x_2^* = \frac{M}{p_2 \left( 1 + \left( \frac{\beta}{\alpha} \right)^{\frac{1}{\rho-1}} \left( \frac{p_1}{p_2} \right)^{\frac{\rho}{\rho-1}} \right)}$$
• Special case 1: $\rho \to 1^−$ (Perfect Substitutes)

- $\lim_{\rho \to 1^-} \frac{1}{\rho - 1} = \lim_{\rho \to 1^-} \frac{\rho}{\rho - 1} = -\infty$
  (here notice the convergence from the left)

- If $\frac{\alpha p_2}{\beta p_1} > 1$ (or $p_1/p_2 < \alpha/\beta$),

$$\left(\frac{\alpha}{\beta}\right)^{\frac{1}{\rho - 1}} \left(\frac{p_2}{p_1}\right)^{\frac{\rho}{\rho - 1}} \to 0$$

$$x^*_1 \to M/p_1$$

- If $\frac{\alpha p_2}{\beta p_1} < 1$ (or $p_1/p_2 > \alpha/\beta$),

$$\left(\frac{\alpha}{\beta}\right)^{\frac{1}{\rho - 1}} \left(\frac{p_2}{p_1}\right)^{\frac{\rho}{\rho - 1}} \to \infty$$

$$x^*_1 \to 0$$
• Solution for Perfect Substitutes Case is

\[ x_1^* = \begin{cases} 
0 & \text{if } p_1/p_2 > \alpha/\beta \\
M/p_1 & \text{if } p_1/p_2 < \alpha/\beta \\
\text{any } x_1 \in [0, M/p_1] & \text{if } p_1/p_2 = \alpha/\beta 
\end{cases} \]

\[ x_2^* = \begin{cases} 
M/p_2 & \text{if } p_1/p_2 > \alpha/\beta \\
0 & \text{if } p_1/p_2 < \alpha/\beta \\
x_2 \text{ such that B.C. holds} & \text{if } p_1/p_2 = \alpha/\beta 
\end{cases} \]

• Case $p_1/p_2 = \alpha/\beta$ has to be analyzed separately

• This is case in which budget line and indifference curves are parallel $\Rightarrow$ All points on budget line are tangent and hence optimal.
• Tricky Cases (ctd)

2. Solution does not satisfy $x_1^* > 0$ or $x_2^* > 0$. Example:

$$\max x_1 \ast (x_2 + 5)$$
$$s.t. \ p_1 x_1 + p_2 x_2 = M$$

• In this case consider corner conditions: what happens for $x_1^* = 0$? And $x_2^* = 0$?

- **Example 1:** Perfect Substitutes with $\frac{p_1}{p_2} = \frac{\alpha}{\beta}$

- **Example 2:** Non-convex preferences with two optima
2 Indirect utility function

• Nicholson, Ch. 4, pp. 124-127 (106–108, 9th)

• Define the indirect utility $v(p, M) \equiv u(x^*(p, M))$, with $p$ vector of prices and $x^*$ vector of optimal solutions.

• $v(p, M)$ is the utility at the optimum for prices $p$ and income $M$

• Some comparative statics: $\partial v(p, M)/\partial M = ?$

• Hint: Use Envelope Theorem on Lagrangean function
• What is the sign of $\lambda$?

• $\lambda = \frac{u'_{x_i}}{p} > 0$

• $\partial v(p, M)/\partial p_i =$?

• Properties:
  
  – Indirect utility is always increasing in income $M$
  
  – Indirect utility is always decreasing in the price $p_i$
3 Comparative Statics (introduction)

• Nicholson, Ch. 5, pp. 141-151 (121–131, 9th)

• Utility maximization yields $x_i^* = x_i^*(p_1, p_2, M)$

• Quantity consumed as a function of income and price

• What happens to quantity consumed $x_i^*$ as prices or income varies?
• Simple case: Equal increase in prices and income.

\[ M' = tM, \quad p_1' = tp_1, \quad p_2' = tp_2. \]

• Compare \( x^*(tM, tp_1, tp_2) \) and \( x^*(M, p_1, p_2) \).

• What happens?

• Write budget line: \( tp_1x_1 + tp_2x_2 = tM \)

• Demand is homogeneous of degree 0 in \( p \) and \( M \):

\[ x^*(tM, tp_1, tp_2) = t^0 x^*(M, p_1, p_2) = x^*(M, p_1, p_2). \]
• Consider Cobb-Douglas Case:

\[ x_1^* = \frac{\alpha}{\alpha + \beta} M/p_1, \quad x_2^* = \frac{\beta}{\alpha + \beta} M/p_2 \]

• What is \( \partial x_1^*/\partial M \)?

• What is \( \partial x_1^*/\partial p_1 \)?

• What is \( \partial x_1^*/\partial p_2 \)?

• General results?
4 Income changes

- Income increases from $M$ to $M' > M$.

- Budget line $(p_1x_1 + p_2x_2 = M)$ shifts out:
  \[
  x_2 = \frac{M'}{p_2} - x_1 \frac{p_1}{p_2}
  \]

- New optimum?
• Engel curve: $x_i^*(M)$: demand for good $i$ as function of income $M$ holding fixed prices $p_1, p_2$

• Does $x_i^*$ increase with $M$?
  
  – Yes. Good $i$ is normal

  – No. Good $i$ is inferior
5 Price changes

- Price of good $i$ decreases from $p_i$ to $p_i'$, $p_i' > p_i$

- For example, decrease in price of good 2, $p_2' < p_2$

- Budget line tilts:

$$x_2 = \frac{M}{p_2'} - x_1 \frac{p_1}{p_2'}$$

- New optimum?
Demand curve: \( x^*_i(p_i) \): demand for good \( i \) as function of own price holding fixed \( p_j \) and \( M \)

Odd convention of economists: plot price \( p_i \) on vertical axis and quantity \( x_i \) on horizontal axis. Better get used to it!
• Does $x_i^*$ decrease with $p_i$?

  – Yes. Most cases

  – No. Good $i$ is *Giffen*

  – Ex.: Potatoes in Ireland

  – Do not confuse with Veblen effect for luxury goods or informational asymmetries: these effects are real, but not included in current model
6 Next Class

- More comparative statics:
  - More on Price Effects
  - Slutsky Equation

- Then moving on to applications:
  - Labor Supply
  - Intertemporal choice
  - Economics of Altruism