Outline

1. One-step Profit Maximization

2. Second-Order Conditions

3. Introduction to Market Equilibrium

4. Aggregation

5. Market Equilibrium in the Short-Run

6. Comparative Statics of Equilibrium
1 One-step Profit Maximization

- Nicholson, Ch. 11, pp. 374-380 (Ch. 9, pp. 265–270, 9th)

- One-step procedure: maximize profits

- Perfect competition. Price $p$ is given
  - Firms are small relative to market
  - Firms do not affect market price $p_M$

  - Will firm produce at $p > p_M$?
  - Will firm produce at $p < p_M$?
  - $\Rightarrow p = p_M$
• Revenue: \( py = pf(L, K) \)

• Cost: \( wL + rK \)

• Profit \( pf(L, K) - wL - rK \)
• Agent optimization:

$$\max_{L,K} pf(L, K) - wL - rK$$

• First order conditions:

$$pf_L'(L, K) - w = 0$$

and

$$pf_K'(L, K) - r = 0$$

• Second order conditions? $$pf_{L,L}''(L, K) < 0$$ and

$$|H| = \begin{vmatrix} pf_{L,L}''(L, K) & pf_{L,K}''(L, K) \\ pf_{L,K}''(L, K) & pf_{K,K}''(L, K) \end{vmatrix} = p^2 \left[ f_{L,L}'' f_{K,K}'' - \left( f_{L,K}'' \right)^2 \right] > 0$$

• Need $$f_{L,K}''$$ not too large for maximum
• Comparative statics with respect to $p$, $w$, and $r$.

• What happens if $w$ increases?

$$\frac{\partial L^*}{\partial w} = -\frac{\begin{vmatrix} -1 & pf_{L,K}''(L,K) \\ 0 & pf_{K,K}''(L,K) \end{vmatrix}}{\begin{vmatrix} pf_{L,L}''(L,K) & pf_{L,K}''(L,K) \\ pf_{L,K}''(L,K) & pf_{K,K}''(L,K) \end{vmatrix}} < 0$$

and

$$\frac{\partial L^*}{\partial r} =$$

• Sign of $\partial L^*/\partial r$ depends on $f_{L,K}''$. 
2 Second Order Conditions in P-Max: Cobb-Douglas

• How do the second order conditions relate for:
  – Cost Minimization
  – Profit Maximization?

• Check for Cobb-Douglas production function
  \[ y = AK^\alpha L^\beta \]

• Cost Minimization. S.o.c.:
  \[ c''_y (y^*, w, r) > 0 \]

• As we showed, for CD prod. ftn.,
  \[ c''_y (y^*, w, r) = -\frac{1}{\alpha + \beta} \frac{1 - (\alpha + \beta)}{\alpha + \beta} B \frac{A^2}{A} \left( \frac{y}{A} \right) \frac{1 - 2(\alpha + \beta)}{\alpha + \beta} \]
  which is > 0 as long as \( \alpha + \beta < 1 \) (DRS)
• **Profit Maximization.** S.o.c.: $p f''_{L,L}(L, K) < 0$ and

$$|H| = p^2 \left[ f''_{L,L} f''_{K,K} - \left( f''_{L,K} \right)^2 \right] > 0$$

• As long as $\beta < 1$,

$$p f''_{L,L} = p \beta (\beta - 1) AK^\alpha L^{\beta - 2} < 0$$

• Then,

$$|H| = p^2 \left[ f''_{L,L} f''_{K,K} - \left( f''_{L,K} \right)^2 \right] =$$

$$= p^2 \left[ \beta (\beta - 1) AK^\alpha L^{\beta - 2} \star \right] =$$

$$= p^2 \left[ \alpha (\alpha - 1) AK^{\alpha - 2} L^{\beta - 1} \right] =$$

$$= p^2 A^2 K^{2\alpha - 2} L^{2\beta - 2} \alpha \beta [1 - \alpha - \beta]$$

• Therefore, $|H| > 0$ iff $\alpha + \beta < 1$ (DRS)

• The two conditions coincide
3 Introduction to Market Equilibrium

- Nicholson, (Ch. 10, pp. 279–295, 9th)

- Two ways to analyze firm behavior:
  - Two-Step Cost Minimization
  - One-Step Profit Maximization

- What did we learn?
  - Optimal demand for inputs $L^*, K^*$ (see above)
  - Optimal quantity produced $y^*$
• **Supply function.** $y = y^*(p, w, r)$

  – From profit maximization:

    $y = f(L^*(p, w, r), K^*(p, w, r))$

  – From cost minimization:

    $MC$ curve above $AC$

  – Supply function is increasing in $p$

• **Market Equilibrium.** Equate demand and supply.

• **Aggregation?**

• **Industry supply function!**
4 Aggregation

4.1 Producers aggregation

- $J$ companies, $j = 1, \ldots, J$, producing good $i$

- Company $j$ has supply function

$$y_i^j = y_i^{j*}(p_i, w, r)$$

- Industry supply function:

$$Y_i(p_i, w, r) = \sum_{j=1}^{J} y_i^{j*}(p_i, w, r)$$

- Graphically,
4.2 Consumer aggregation

- Nicholson, (Ch. 10, pp. 279–282)

- One-consumer economy

- Utility function $u(x_1, \ldots, x_n)$

- Prices $p_1, \ldots, p_n$

- Maximization $\Rightarrow$

\[
\begin{align*}
x_1^* &= x_1^*(p_1, \ldots, p_n, M), \\
&\vdots \\
x_n^* &= x_n^*(p_1, \ldots, p_n, M).
\end{align*}
\]
• Focus on good \( i \). Fix prices \( p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n \) and \( M \).

• Single-consumer demand function:

\[
x_i^* = x_i^* \left( p_i \mid p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n, M \right)
\]

• What is sign of \( \partial x_i^* / \partial p_i \)?

• Negative if good \( i \) is normal

• Negative or positive if good \( i \) is inferior
• Aggregation: \( J \) consumers, \( j = 1, \ldots, J \)

• Demand for good \( i \) by consumer \( j \):
\[
x^{j*}_i = x^{j*}_i (p_1, \ldots, p_n, M^j)
\]

• Market demand \( X_i \):
\[
X_i \left( p_1, \ldots, p_n, M^1, \ldots, M^J \right) = \sum_{j=1}^{J} x^{j*}_i \left( p_1, \ldots, p_n, M^j \right)
\]

• Graphically,
• Notice: market demand function depends on distribution of income $M^J$

• Market demand function $X_i$:
  
  – Consumption of good $i$ as function of prices $p$

  – Consumption of good $i$ as function of income distribution $M^j$
5 Market Equilibrium in the Short-Run

- Nicholson, (Ch. 14, pp. 368–382, 9th)

- What is equilibrium price $p_i$?

- Magic of the Market...

- Equilibrium: No excess supply, No excess demand

- Prices $p^*$ equates demand and supply of good $i$:

$$Y^* = Y^S_i (p^*_i, w, r) = X^D_i (p^*_1, ..., p^*_n, M^1, ..., M^J)$$
• Graphically,

• Notice: in short-run firms can make positive profits
• Comparative statics exercises with endogenous price $p_i$:
  
  - increase in wage $w$ or interest rate $r$:

  - change in income distribution
6 Comparative statics of equilibrium

• Nicholson, Ch. 12, pp. 403-406 (Ch. 10, pp. 293-295, 9th)

• Supply and Demand function of parameter $\alpha$:

  $- Y^S_i (p_i, w, r, \alpha)$

  $- X^D_i (p, M, \alpha)$

• How does $\alpha$ affect $p^*$ and $Y^*$?

• Comparative statics with respect to $\alpha$

• Equilibrium:

  $$Y^S_i (p_i, w, r, \alpha) = X^D_i (p, M, \alpha)$$
Can write equilibrium as implicit function:

\[ Y_i^S(p_i, w, r, \alpha) - X_i^D(p, M, \alpha) = 0 \]

What is \( \Delta p^*/\Delta \alpha \)?

Implicit function theorem:

\[
\frac{\partial p^*}{\partial \alpha} = - \frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha} \frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}
\]

What is sign of denominator?

Sign of \( \partial p^*/\partial \alpha \) is negative of sign of numerator
• Examples:

1. *Fad.* Good becomes more fashionable: \( \frac{\partial X^D}{\partial \alpha} > 0 \iff \frac{\partial p^*}{\partial \alpha} > 0 \)

2. *Recession in Europe.* Negative demand shock for US firms: \( \frac{\partial X^D}{\partial \alpha} < 0 \iff \frac{\partial p^*}{\partial \alpha} < 0 \)

3. *Oil shock.* Import prices increase: \( \frac{\partial Y^S}{\partial \alpha} < 0 \iff \frac{\partial p^*}{\partial \alpha} > 0 \)

4. *Computerization.* Improvement in technology. \( \frac{\partial Y^S}{\partial \alpha} > 0 \iff \frac{\partial p^*}{\partial \alpha} < 0 \)
7 Next Lecture

- Market Equilibrium
- Comparative Statics of Equilibrium
- Elasticities
- Taxes and Subsidies