Economics 101A
(Lecture 17)

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Outline

1. Elasticities

2. Response to Taxes

3. Producer Surplus

4. Consumer Surplus

5. Trade
1 Elasticities

- Nicholson, Ch.1, pp. 26-27 (pp. 27–28, 9th)

- How do we interpret magnitudes of $\partial p^*/\partial \alpha$?

- Result depends on units of measure.

- Can we write $\partial p^*/\partial \alpha$ in a unit-free way?

- Yes! Use elasticities.

- Elasticity of $x$ with respect to parameter $p$ is

$$\varepsilon_{x,p} = \frac{\partial x}{\partial p} \frac{p}{x}$$
• Interpretation: Percent response in $x$ to percent change in $p$:

$$\varepsilon_{x,p} = \frac{\partial x}{\partial p} x = \lim_{dp \to 0} \frac{x(p + dp) - x(p)p}{dp} x = \lim_{dp \to 0} \frac{dx}{dx} = \lim_{dp \to 0} \frac{dx}{x} \cdot \frac{dp}{dp/p}$$

where $dx \equiv x(p + dp) - x(p)$.

• Now, show

$$\varepsilon_{x,p} = \frac{\partial \ln x}{\partial \ln p}$$

• Notice: This makes sense only for $x > 0$ and $p > 0$
Proof. Consider function

\[ x = f(p) \]

Rewrite as

\[ \ln(x) = \ln f(p) = \ln f(e^{\ln(p)}) \]

Define \( \hat{x} = \ln(x) \) and \( \hat{p} = \ln(p) \)

This implies

\[ \hat{x} = \ln f(e^{\hat{p}}) \]

Get

\[
\frac{\partial \hat{x}}{\partial \hat{p}} = \frac{\partial \ln x}{\partial \ln p} = \frac{1}{f(e^{\hat{p}})} \frac{\partial f(e^{\hat{p}})}{\partial \hat{p}} e^{\hat{p}} = \frac{\partial x}{\partial p} \]
• Example with Cobb-Douglas utility function

\[ U(x, y) = x^\alpha y^{1-\alpha} \] implies solutions

\[ x^* = \alpha \frac{M}{p_x}, \quad y^* = (1 - \alpha) \frac{M}{p_y} \]

• Elasticity of demand with respect to own price \( \varepsilon_{x,p_x} \):

\[ \varepsilon_{x,p_x} = \frac{\partial x^*}{\partial p_x} \frac{p_x}{x^*} = -\frac{\alpha M}{(p_x)^2} \alpha \frac{M}{p_x} = -1 \]

• Elasticity of demand with respect to other price \( \varepsilon_{x,p_y} = 0 \)
• Go back to problem above:

\[
\frac{\partial p^*}{\partial \alpha} = -\left( \frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha} \right) \frac{\partial X^D}{\partial p} - \frac{\partial Y^S}{\partial p}
\]

• Use elasticities to rewrite response of \( p \) to change in \( \alpha \):

\[
\frac{\partial p^*}{\partial \alpha} \frac{\partial \alpha}{\partial p} = -\left( \frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha} \right) \frac{\alpha}{Y} \left( \frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p} \right) \frac{p}{Y}
\]

or (using fact that \( X^{D*} = Y^{s*} \))

\[
\varepsilon_{p,\alpha} = -\frac{\varepsilon_{S,\alpha} - \varepsilon_{D,\alpha}}{\varepsilon_{S,p} - \varepsilon_{D,p}}
\]

• We are likely to know elasticities from empirical studies
2 Response to taxes

- Nicholson, Ch. 12, pp. 423-426 (Ch. 11, pp. 322–323, 9th)

- Per-unit tax $t$

- Write price $p_i$ as price including tax

- Supply: $Y_i^S(p_i - t, w, r)$

- Demand: $X_i^D(p, M)$

\[ Y_i^S(p_i - t, w, r) - X_i^D(p, M) = 0 \]

- What is $dp^*/dt$?
• Comparative statics:

\[
\frac{\partial p^*}{\partial t} = - \frac{\partial Y^S}{\partial t} - \frac{\partial Y^S}{\partial p} \frac{\partial X^D}{\partial p} = - \frac{\partial Y^S}{\partial p} \frac{p}{X} = \frac{\varepsilon_{S,p}}{\varepsilon_{S,p} - \varepsilon_{D,p}}
\]

• How about price received by suppliers \( p^* - t \)?

\[
\frac{\partial (p^* - t)}{\partial t} = \frac{\partial Y^S}{\partial p} - 1 = \frac{\varepsilon_{D,p}}{\varepsilon_{S,p} - \varepsilon_{D,p}}
\]
• *Inflexible Supply.* (Capacity is fixed) Supply curve vertical \((\varepsilon_{S,p} = 0)\)

• Producers bear burden of tax

• *Flexible Supply.* (Constant Returns to Scale) Supply curve horizontal \((\varepsilon_{S,p} \to \infty)\)

• Consumers bear burden of tax
• **Inflexible demand.** Demand curve vertical ($\varepsilon_{D,p} = 0$)?

• Consumers bear burden

• General lesson: Least elastic side bears larger part of burden

• What happens with a subsidy ($t < 0$)?

• What happens to quantity sold?

• Use demand curve:

$$\frac{\partial X^{D*}}{\partial t} = \frac{\partial X^{D*}}{\partial p^*} \frac{\partial p^*}{\partial t}$$

and use expression for $\partial p^*/\partial t$ above
3 Welfare: Producer Surplus

- Nicholson, Ch. 11, pp. 371-374 (Ch. 9, pp. 261–263, 9th)

- Producer Surplus is easier to define:
  \[ \pi (p, y_0) = p y_0 - c(y_0). \]

- Can give two graphical interpretations:

- **Interpretation 1.** Rewrite as
  \[ \pi (p, y_0) = y_0 \left[ p - \frac{c(y_0)}{y_0} \right]. \]

- Profit equals rectangle of quantity times (p - Av. Cost)
• **Interpretation 2.** Remember:

\[ f(x) = f(0) + \int_0^x f'_x(s) \, ds. \]

• Rewrite profit as

\[
\left[ p \ast 0 + p \int_0^{y_0} 1 \, dy \right] - \left[ c(0) + \int_0^{y_0} c'_y(y) \, dy \right] = \\
= \int_0^{y_0} \left( p - c'_y(y) \right) \, dy - c(0).
\]

• Producer surplus is area between price and marginal cost (minus fixed cost)
4 Welfare: Consumer Surplus

- Nicholson, Ch. 5, pp. 165-169 (Ch. 5, pp. 145–149, 9th)

- Welfare effect of price change from $p_0$ to $p_1$

- Proposed measure:
  \[ e(p_0, u) - e(p_1, u) \]

- Can rewrite expression above as
  \[
  e(p_0, u) - e(p_1, u) = \left( e(0, u) + \int_0^{p_0} \frac{\partial e(p, u)}{\partial p} dp \right) - \\
  \left( e(0, u) + \int_0^{p_1} \frac{\partial e(p, u)}{\partial p} dp \right)
  \]

  \[ = \int_{p_1}^{p_0} \frac{\partial e(p, u)}{\partial p} dp \]

- What is $\frac{\partial e(p, u)}{\partial p}$?
• Remember envelope theorem...

• Result:

\[ \frac{\partial e(p, u)}{\partial p} = h(p, u) \]

• Welfare measure is integral of area to the side of Hick-sian compensated demand

• Graphically,
Example of welfare effects: Imposition of Tax

Welfare before tax

Welfare after tax
5 Trade

• Nicholson, Ch. 12, pp. 427-429 (Ch. 11, pp. 326–327, 9th)

• Assume that domestic industry opens to trade

• Is this a good or a bad thing?

• Consider graphically

• Equilibrium with no trade at quantity $X_D^*$ and price $p_D^*$
• Trade: Goods available at lower price $p_T^*$

• (Otherwise, openness to trade irrelevant)

• Shift in price to $p_T^* < p_D^*$ and in quantity to $X_T^* > X_D^*$

• Label domestic production and imports
• What happens to profits of domestic firms?

• What happens to consumer surplus?

• More total surplus, but firms lost some profits and some employment → Difficult trade-off
6  Next Lecture

- Trade

- Market Equilibrium in the Long-Run

- Then: Market Power

- Monopoly

- Price Discrimination