Outline

1. Oligopoly: Cournot

2. Oligopoly: Bertrand

3. Second-price Auction

4. Auctions: eBay Evidence
1 Oligopoly: Cournot

- Nicholson, Ch. 14, pp. 524-530 (*better* than Ch. 14, pp. 418–419, 421–422, 9th)

- Back to oligopoly maximization problem

- Assume 2 firms, cost $c_i(y_i) = cy_i$, $i = 1, 2$

- Firms choose simultaneously quantity $y_i$

- Firm $i$ maximizes:

  $$\max_{y_i} p \left( y_i + y_{-i} \right) y_i - cy_i.$$  

- First order condition with respect to $y_i$:

  $$p'_Y \left( y_i^* + y_{-i}^* \right) y_i^* + p - c = 0, \; i = 1, 2.$$
• Nash equilibrium:
  
  – $y_1$ optimal given $y_2$;
  
  – $y_2$ optimal given $y_1$.

• Solve equations:

$$p_Y' (y_1^* + y_2^*) y_1^* + p - c = 0 \text{ and }$$

$$p_Y' (y_2^* + y_1^*) y_2^* + p - c = 0.$$
2 Oligopoly: Bertrand

- Cournot oligopoly: firms choose quantities

- Bertrand oligopoly: firms first choose prices, and then produce quantity demanded by market

- Market demand function $Y(p)$

- 2 firms

- Profits:

$$
\pi_i(p_i, p_{-i}) = \begin{cases} 
(p_i - c) Y(p_i) & \text{if } p_i < p_{-i} \\
(p_i - c) Y(p_i) / 2 & \text{if } p_i = p_{-i} \\
0 & \text{if } p_i > p_{-i}
\end{cases}
$$
• First show that $p_1 = c = p_2$ is Nash Equilibrium

• Does any firm have a (strict) incentive to deviate?

• Check profits for Firm 1

• Symmetric argument for Firm 2
• Second, show that this equilibrium is unique.

• For each of the next 5 cases at least one firm has a profitable deviation

• Case 1. $p_1 > p_2 > c$

• Case 2. $p_1 = p_2 > c$

• Case 3. $p_1 > c \geq p_2$
• Case 4. $c > p_1 \geq p_2$

• Case 5. $p_1 = c > p_2$

• Only Case 6 remains: $p_1 = c = p_2$, which is Nash Equilibrium

• It is unique!
• Notice:

• To show that something is an equilibrium $\rightarrow$ Show that there is *no* profitable deviation

• To show that something is *not* an equilibrium $\rightarrow$ Show that there is *one* profitable deviation
• Surprising result of Bertrand Competition

• Marginal cost pricing

• Two firms are enough to guarantee perfect competition!

• Realistic? Price wars between PC makers
3 Second-price Auction

- Nicholson, Ch. 18, pp. 659–66 [Not in old book]
- Sealed-bid auction
- Highest bidder wins object
- Price paid is second highest price
- Two individuals: $I = 2$
- Strategy $s_i$ is bid $b_i$
- Each individual knows value $v_i$
• Payoff for individual $i$ is

$$u_i(b_i, b_{-i}) = \begin{cases} 
  v_i - b_i & \text{if } b_i > b_{-i} \\
  (v_i - b_{-i})/2 & \text{if } b_i = b_{-i} \\
  0 & \text{if } b_i < b_{-i}
\end{cases}$$

• Show: weakly dominant to set $b_i^* = v_i$

• To show:

$$u_i(v_i, b_{-i}) \geq u_i(b_i, b_{-i})$$

for all $b_i$, for all $b_{-i}$, and for $i = 1, 2$. 
1. Assume $b_{-i} > v_i$

- $u_i(v_i, b_{-i}) = 0 = u_i(b_i, b_{-i})$ for any $b_i < b_{-i}$
- $u_i(b_{-i}, b_{-i}) = (v_i - b_{-i}) / 2 < 0$
- $u_i(b_i, b_{-i}) = (v_i - b_{-i}) < 0$ for any $b_i > b_{-i}$

2. Assume now $b_{-i} = v_i$
3. Assume now $b_{-i} < v_i$.
4 Auctions: Evidence from eBay

• In second-price auction, optimal strategy is to bid one's own value

• Is this true?

• eBay has proxy system: If you have highest bid, you pay bid of second-highest bidder

• eBay is essentially a second-price auction

• Two deviations:
  1. People bid multiple times – they should not in this theory
  2. People may overbid
An example: 
eBay Bidding for a Board Game

• Bidding environment with clear boundary for rational willingness to pay (“buy-it-now price”).
• Empirical environment unaffected by common-value arguments (presumably bidding for private use; in addition “buy-it-now” price).
• Still non-negligible amount ($100-$200).

→ Is there evidence of overbidding?
→ If so, can we detect determinants of overbidding?
The Object
The Data

• Cashflow 101: board game with the purpose of finance/accounting education.

• Retail price: $195 plus shipping cost ($10.75) from manufacturer (www.richdad.com).

• Two ways to purchase Cashflow 101 on eBay
  – Auction (quasi-second price proxy bidding)
  – Buy-it-now

• Hand-collected data of all auctions and Buy-it-now transactions of Cashflow 101 on eBay from 2/19/2004 to 9/6/2004.
Sample

• Listings
  – 206 by individuals (187 auctions only, 19 auctions with buy-it-now option)
  – 493 by two retailers (only buy-it-now)

• Remove non-US$, terminated, unsold items and items without simultaneous professional buy-it-now listing. → 169 auctions

• Buy-it-now offers of the two retailers
  – Continuously present for all but six days. (Often individual buy-it-now offers present as well; they are often lower.)
  – 100% and 99.9% positive feedback scores.
  – Same prices $129.95 until 07/31/2004; $139.95 since 08/01/2004.
  – Shipping cost $9.95; other retailer $10.95.
  – New items (with bonus tapes/video).
# Listing Example (02/12/2004)

<table>
<thead>
<tr>
<th>Item Description</th>
<th>Price</th>
<th>Quantity</th>
<th>Delivery Time</th>
</tr>
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<tbody>
<tr>
<td>Rich Dad's Cashflow Quadrant, Rich dad ...</td>
<td>$12.50</td>
<td>4</td>
<td>1d 00h 14m</td>
</tr>
<tr>
<td>Rich Dad's Cashflow Quadrant by Robert T. ...</td>
<td>$9.00</td>
<td>9</td>
<td>1d 00h 43m</td>
</tr>
<tr>
<td>Real Estate Investment Cashflow Software $$$!</td>
<td>$10.49</td>
<td>2</td>
<td>1d 04h 36m</td>
</tr>
<tr>
<td>CASHFLOW® 101 202 Robert Kiyosaki Best Pak $</td>
<td>$207.96</td>
<td></td>
<td>1d 06h 47m</td>
</tr>
<tr>
<td>TRY IT TODAY, WITH ABSOLUTELY NO RISK,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CASHFLOW® 101 Robert Kiyosaki Plus Bonuses!</td>
<td>$129.95</td>
<td></td>
<td>1d 08h 02m</td>
</tr>
<tr>
<td>Your satisfaction is GUARANTEED, 100% $ back</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MIINT Cashflow 101 *Robert Kiyosaki Game NR!</td>
<td>$140.00</td>
<td>13</td>
<td>1d 08h 04m</td>
</tr>
<tr>
<td>It's easy to be rich. Brand New. Still sealed</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cashflow Hard Money Funding 101 real estate</td>
<td>$14.99</td>
<td></td>
<td>1d 09h 28m</td>
</tr>
<tr>
<td>BRANDNEW RICHDAD CASHFLOW FOR KIDS E-GAME</td>
<td>$20.00</td>
<td>1</td>
<td>1d 13h 54m</td>
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<tr>
<td>CASHFLOW® 101 Robert Kiyosaki Plus Bonuses!</td>
<td>$129.95</td>
<td></td>
<td>1d 14h 17m</td>
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<td>Your satisfaction is GUARANTEED, 100% $ back</td>
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<tr>
<td>CASHFLOW® 101 202 Robert Kiyosaki Best Pak $</td>
<td>$207.96</td>
<td></td>
<td>1d 15h 47m</td>
</tr>
<tr>
<td>TRY IT TODAY, WITH ABSOLUTELY NO RISK,</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
Listing Example – Magnified

CASHFLOW® 101 202 Robert Kiyosaki Best Pak $  $207.96
TRY IT TODAY, WITH ABSOLUTELY NO RISK.

CASHFLOW® 101 Robert Kiyosaki Plus Bonuses! $129.95
Your satisfaction is GUARANTEED, 100% $ back

MINT Cashflow 101 Robert Kiyosaki Game NR! $140.00
It's easy to be rich. Brand New. Still sealed

Pricing:
[Buy Now] $129.95

Pricing:
$140.00
**Bidding history of an item**

Item title: CASHFLOW 101 Board Game Rich Dad Poor Dad

Time left: Auction has ended.

Only actual bids (not automatic bids generated up to a bidder's maximum) are shown. Automatic bids may be placed days or hours before a listing ends. Learn more about [bidding](#).

<table>
<thead>
<tr>
<th>User ID</th>
<th>Bid Amount</th>
<th>Date of bid</th>
</tr>
</thead>
<tbody>
<tr>
<td>beezebugs (21)</td>
<td>US $152.50</td>
<td>Aug-11-04 09:51:21 PDT</td>
</tr>
<tr>
<td>mlady-b (21)</td>
<td>US $150.00</td>
<td>Aug-11-04 09:53:53 PDT</td>
</tr>
<tr>
<td>beezebugs (21)</td>
<td>US $140.00</td>
<td>Aug-08-04 12:06:05 PDT</td>
</tr>
<tr>
<td>sj_orbit (66)</td>
<td>US $130.01</td>
<td>Aug-08-04 25:49:02 PDT</td>
</tr>
<tr>
<td>successbroker (991)</td>
<td>US $110.00</td>
<td>Aug-08-04 19:56:26 PDT</td>
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<tr>
<td>successbroker (991)</td>
<td>US $105.00</td>
<td>Aug-06-04 17:19:21 PDT</td>
</tr>
<tr>
<td>002la (1)</td>
<td>US $102.50</td>
<td>Aug-06-04 17:11:31 PDT</td>
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<tr>
<td>successbroker (991)</td>
<td>US $100.00</td>
<td>Aug-06-04 15:41:40 PDT</td>
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<td>002la (1)</td>
<td>US $90.00</td>
<td>Aug-05-04 17:10:46 PDT</td>
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<tr>
<td>002la (1)</td>
<td>US $85.00</td>
<td>Aug-05-04 17:10:21 PDT</td>
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<td>12-gauge (29)</td>
<td>US $85.00</td>
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<td>imdyque (110)</td>
<td>US $58.00</td>
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<td>US $45.00</td>
<td>Aug-05-04 10:45:41 PDT</td>
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<td>US $40.00</td>
<td>Aug-05-04 10:45:08 PDT</td>
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<td>bearinb Bunns (3)</td>
<td>US $31.00</td>
<td>Aug-04-04 08:49:19 PDT</td>
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<td>75ton (1)</td>
<td>US $30.00</td>
<td>Aug-04-04 19:45:54 PDT</td>
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<td>bearinb Bunns (3)</td>
<td>US $25.00</td>
<td>Aug-04-04 08:43:26 PDT</td>
</tr>
</tbody>
</table>

If you and another bidder placed the same bid amount, the earlier bid takes priority.
Hypotheses

Given the information on the listing website:

• (H1) An auction should never end at a price above the concurrently available purchase price.

• (H2) Mentioning of higher outside prices should not affect bidding behavior.
Figure 1. Starting Price (startprice)

- 45% below $20; mean=$46; SD=43.88
- only 6 auctions with first bid (not price) above buy-it-now
Figure 2. Final Price (finalprice)

41% are above “buy-it-now” (mean $132; SD 16.83)
Figure 4. Total Price (incl. shipping cost)

⇒ 51% are above “buy-it-now” plus its shipping cost (mean=$144.20; SD=15.00)
5 Next lecture

- Dynamic Games

- Stackelberg duopoly