Outline

1. Who are we?

2. Prerequisites for the course

3. A test in maths

4. The economics of discrimination

5. Optimization with 1 variable

6. Multivariate optimization (Today or on Th)
1 Who are we?

Stefano DellaVigna

• Assistant Professor, Department of Economics

• Bocconi (Italy) undergraduate (Econ.), Harvard PhD (Econ.)

• Psychology and economics, applied microeconomics, behavioral finance, aging, media

• Evans 515, OH Th 12-2
Vikram Maheshri (3 Sections)

- Graduate Student, Department of Economics

- Rooms: To be announced
2 Prerequisites

• Mathematics
  – Good knowledge of multivariate calculus – Maths 1A or 1B and 53
  – Basic knowledge of probability theory and matrix algebra

• Economics
  – Knowledge of fundamentals – Ec1 or 2 or 3
  – High interest!
3 A Test in Maths

1. Can you differentiate the following functions with respect to $x$?

(a) $y = \exp(x)$

(b) $y = a + bx + cx^2$

(c) $y = \frac{\exp(x)}{b^x}$

2. Can you partially differentiate these functions with respect to $x$ and $w$?

(a) $y = axw + bx - c\frac{x}{w} + d\sqrt{xw}$

(b) $y = \exp(x/w)$

(c) $y = \int_0^1 (x + aw^2 + xs)ds$
3. Can you plot the following functions of one variable?

(a) \( y = \exp(x) \)

(b) \( y = -x^2 \)

(c) \( y = \exp(-x^2) \)

4. Are the following functions concave, convex or neither?

(a) \( y = x^3 \)

(b) \( y = -\exp(x) \)

(c) \( y = x^{5} y^{5} \) for \( x > 0, y > 0 \)
5. Consider an urn with 20 red and 40 black balls?

   (a) What is the probability of drawing a red ball?

   (b) What is the probability of drawing a black ball?

6. What is the determinant of the following matrices?

   (a) \( A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \)

   (b) \( A = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix} \)
4 The economics of discrimination

• Ok, I need maths. But where is the economics?

• Today: Simple model of discrimination (inspired by Becker, 1957)

• Workers:
  – $A$ and $B$. They produce 1 widget per day
  – Both have reservation wage $\bar{u}$

• Firm:
  – sells widgets at price $p > \bar{u}$ (assume $p$ given)
  – dislikes worker $B$
  – Maximizes profits ($p\times$ no of widgets – cost of labor) – disutility $d$ if employs $B$
• Wages and employment in this industry?

• Employment

  – Net surplus from employing $A$: $p - \bar{u}$
  
  – Net surplus from employing $B$: $p - \bar{u} - d$
  
  – If $\bar{u} < p < \bar{u} + d$, Firm employs $A$ but not $B$
  
  – If $\bar{u} + d < p$, Firm employs both

• What about wages?
• Case I. Firm monopolist/monopsonist and no union
  - Firm maximizes profits and gets all the net surplus
  - Wages of $A$ and $B$ equal $\bar{u}$

• Case II. Firm monopolist/monopsonist and union
  - Firm and worker get half of the net surplus each
  - Wage of $A$ equals $\bar{u} + .5 \ast (p - \bar{u})$
  - Wage of $B$ equals $\bar{u} + .5 \ast (p - \bar{u} - d)$

• Case III. Perfect competition among firms that discriminate ($d > 0$)
  - Prices are lowered to the cost of production
  - Wage of $A$ equals $p$ ($= \bar{u}$)
  - $B$ is not employed
• The magic of competition

• Case IIIb. Perfect competition + At least one firm does not discriminate ($d = 0$)
  
  – This firm offers wage $p$ to both workers
  
  – What happens to worker $B$?
  
  – She goes to the firm with $d = 0$!
  
  – In equilibrium now:
    
    * Wage of $A$ equals $p$
    
    * Wage of $B$ equals $p$ as well!

• Competition eliminates the pay and employment differential between men and women
• Is this true? Any evidence?

• S. Black and P. Strahan, AER 2001.
  
  – Local monopolies in banking industry until mid 70s
  
  – Mid 70s: deregulation
  
  – From local monopolies to perfect competition.
  
  – Wages?
    
    * Wages fall by 6.1 percent
  
  – Discrimination?
    
    * Wages fall by 12.5 percent for men
    
    * Wages fall by 2.9 percent for women
    
    * Employment of women as managers increases by 10 percent
• Summary: Competition is not great for workers (wages go down)

• BUT: Drives away the gender gap
• More evidence on discrimination

• Does black-white and male-female wage back derive from discrimination?

• Field experiment (Bertrand and Mullainathan, 2005)

• Send real CV with randomly picked names:
  – Male/Female
  – White/African American

<table>
<thead>
<tr>
<th>White Female</th>
<th>Perception</th>
<th>African American Female</th>
<th>Perception</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>$L(W)/L(B)$</td>
<td>Perception</td>
<td>$L(B)/L(W)$</td>
</tr>
<tr>
<td>Allison</td>
<td>$\infty$</td>
<td>0.926</td>
<td>209</td>
</tr>
<tr>
<td>Anne</td>
<td>$\infty$</td>
<td>0.962</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Carrie</td>
<td>$\infty$</td>
<td>0.923</td>
<td>116</td>
</tr>
<tr>
<td>Emily</td>
<td>$\infty$</td>
<td>0.925</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Jill</td>
<td>$\infty$</td>
<td>0.889</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Laurie</td>
<td>$\infty$</td>
<td>0.963</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Kristen</td>
<td>$\infty$</td>
<td>0.963</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Meredith</td>
<td>$\infty$</td>
<td>0.926</td>
<td>284</td>
</tr>
<tr>
<td>Sarah</td>
<td>$\infty$</td>
<td>0.852</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>
• Measure call-back rate from interview

- Results (Table 1):
  
  * Call-back rates 50 percent higher for Whites!
  
  * No effect for Male-Female call back rates

<table>
<thead>
<tr>
<th>Sample:</th>
<th>Callback Rate for White Names</th>
<th>Callback Rate for African American Names</th>
<th>Ratio</th>
<th>Difference (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All sent resumes</td>
<td>9.65% [2435]</td>
<td>6.45% [2435]</td>
<td>1.50</td>
<td>3.20% (0.0000)</td>
</tr>
<tr>
<td>Chicago</td>
<td>8.06% [1352]</td>
<td>5.40% [1352]</td>
<td>1.49</td>
<td>2.66% (0.0057)</td>
</tr>
<tr>
<td>Boston</td>
<td>11.63% [1083]</td>
<td>7.76% [1083]</td>
<td>1.50</td>
<td>4.05% (0.0023)</td>
</tr>
<tr>
<td>Females</td>
<td>9.89% [1860]</td>
<td>6.63% [1886]</td>
<td>1.49</td>
<td>3.26% (0.0003)</td>
</tr>
<tr>
<td>Females in administrative jobs</td>
<td>10.46% [1358]</td>
<td>6.55% [1359]</td>
<td>1.60</td>
<td>3.91% (0.0003)</td>
</tr>
<tr>
<td>Females in sales jobs</td>
<td>8.37% [502]</td>
<td>6.83% [527]</td>
<td>1.22</td>
<td>1.54% (0.3523)</td>
</tr>
<tr>
<td>Males</td>
<td>8.87% [575]</td>
<td>5.83% [540]</td>
<td>1.52</td>
<td>3.04% (0.0513)</td>
</tr>
</tbody>
</table>

*Notes:
1. The table reports, for the entire sample and different subsamples of sent resumes, the callback rates for applicants with a White sounding name (column 1) and an African American sounding name (column 2), as well as the ratio (column 3) and difference (column 4) of these callback rates. In brackets in each cell is the number of resumes sent in that cell.
2. Column 4 also reports the p-value for a test of proportion testing the null hypothesis that the callback rates are equal across racial groups.
• Strong evidence of discrimination against African Americans

• Example of Applied Microeconomics

• Not (really) covered in this class: See Ec131 (Public), 142 (Applied Metrics) and (partly) 151 (Labor)+154 ( Discrimination)

• Also: URAP – Get involved in a professor’s research

• If curious: read Steven Levitt and Stephen Dubner, *Freakonomics*

• At end of class, two more examples:
  
  – Deductibles and Choice of Home Insurance (Sydnor, 2007)
  
  – Effect of Fox News on Voting (DellaVigna and Kaplan, 2007)
5 Optimization with 1 variable

• Nicholson (9th), Ch.2, pp. 22-26

• Example. Function \( y = -x^2 \)

• What is the maximum?

• Maximum is at 0

• General method?
• Sure! Use derivatives

• Derivative is slope of the function at a point:
\[
\frac{\partial f(x)}{\partial x} = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
\]

• **Necessary condition for maximum** $x^*$ is
\[
\frac{\partial f(x^*)}{\partial x} = 0 \quad (1)
\]

• Try with $y = -x^2$.

\[
\frac{\partial f(x)}{\partial x} = 0 \quad \Rightarrow \quad x^* =
\]
• Does this guarantee a maximum? No!

• Consider the function $y = x^3$

• $\frac{\partial f(x)}{\partial x} = 0 \implies x^* =$

• Plot $y = x^3$. 
**Sufficient condition for a (local) maximum:**

\[
\frac{\partial f(x^*)}{\partial x} = 0 \quad \text{and} \quad \left. \frac{\partial^2 f(x)}{\partial^2 x} \right|_{x^*} < 0 \quad (2)
\]

**Proof:** At a maximum, \( f(x^* + h) - f(x^*) < 0 \) for all \( h \).

**Taylor Rule:**

\[
f(x^* + h) - f(x^*) = \frac{\partial f(x^*)}{\partial x} h + \frac{1}{2} \frac{\partial^2 f(x^*)}{\partial^2 x} h^2 + \text{higher order terms.}
\]

**Notice:** \( \frac{\partial f(x^*)}{\partial x} = 0 \).

\[
f(x^* + h) - f(x^*) < 0 \quad \text{for all} \quad h \implies \frac{\partial^2 f(x^*)}{\partial^2 x} h^2 < 0 \implies \frac{\partial^2 f(x^*)}{\partial^2 x} < 0
\]

**Careful:** Maximum may not exist: \( y = \exp(x) \)
• Tricky examples:

  – *Minimum.* \( y = x^2 \)

  – *No maximum.* \( y = \exp(x) \) for \( x \in (-\infty, +\infty) \)

  – *Corner solution.* \( y = x \) for \( x \in [0, 1] \)
6 Multivariate optimization

- Nicholson, Ch.2, pp. 26–32

- Function from $\mathbb{R}^n$ to $\mathbb{R}$: $y = f(x_1, x_2, \ldots, x_n)$

- Partial derivative with respect to $x_i$:

  \[
  \frac{\partial f(x_1, \ldots, x_n)}{\partial x_i} = \lim_{h \to 0} \frac{f(x_1, \ldots, x_i + h, \ldots x_n) - f(x_1, \ldots, x_i, \ldots x_n)}{h}
  \]

  - Slope along dimension $i$

- Total differential:

  \[
  df = \frac{\partial f(x)}{\partial x_1} dx_1 + \frac{\partial f(x)}{\partial x_2} dx_2 + \ldots + \frac{\partial f(x)}{\partial x_n} dx_n
  \]
• One important economic example

• Example 1: Partial derivatives of $y = f(L, K) = L^{0.5}K^{0.5}$

  • $f'_L =$
    (marginal productivity of labor)

  • $f'_K =$
    (marginal productivity of capital)

  • $f''_{L,K} =$
Maximization over an open set (like $\mathbb{R}$)

- **Necessary condition for maximum** $x^*$ is
  \[
  \frac{\partial f(x^*)}{\partial x_i} = 0 \quad \forall i
  \]  
  (3)

  or in vectorial form

  \[
  \nabla f(x) = 0
  \]

- These are commonly referred to as first order conditions (f.o.c.)
7 Next Class

- Multivariate Maximization (ctd.)
- Comparative Statics
- Implicit Function Theorem
- Envelope Theorem

- Going toward:
  - Preferences
  - Utility Maximization (where we get to apply maximization techniques the first time)