Outline

1. Dynamic Games II

2. Oligopoly: Stackelberg

3. General Equilibrium: Introduction

4. Edgeworth Box: Pure Exchange

5. Barter
1 Dynamic Games II

• Can use this to study finitely repeated games

• Suppose we play the prisoner’s dilemma game ten times.

\[
\begin{array}{c|ccc}
1 & 2 & D & ND \\
\hline
D & -4, -4 & -1, -5 \\
ND & -5, -1 & -2, -2 \\
\end{array}
\]

• What is the subgame perfect equilibrium?
• The result differs if infinite repetition with a probability of terminating

• Can have cooperation

• Strategy of repeated game:
  – Cooperate (ND) as long as opponent always cooperate
  – Defect (D) forever after first defection

• Theory of repeated games: Econ. 104
2 Oligopoly: Stackelberg

• Nicholson, Ch. 15, pp. 543-545 (better than Ch. 14, pp. 423-424, 9th)

• Setting as in problem set

• 2 Firms

• Cost: $c(y) = cy$, with $c > 0$

• Demand: $p(Y) = a - bY$, with $a > c > 0$ and $b > 0$

• Difference: Firm 1 makes the quantity decision first

• Use subgame perfect equilibrium
Solution:

Solve first for Firm 2 decision as function of Firm 1 decision:

$$\max_{y_2} (a - by_2 - by_1^*) y_2 - cy_2$$

F.o.c.: $a - 2by_2^* - by_1^* - c = 0$

Firm 2 best response function:

$$y_2^* = \frac{a - c}{2b} - \frac{y_1^*}{2}.$$
• Firm 1 takes this response into account in the maximization:

$$\max_{y_1} (a - by_1 - by_2^* (y_1)) y_1 - cy_1$$

or

$$\max_{y_1} \left( a - by_1 - b \left( \frac{a - c}{2b} - \frac{y_1}{2} \right) \right) y_1 - cy_1$$

• F.o.c.:

$$a - 2by_1 - \frac{(a - c)}{2} + by_1 - c = 0$$

or

$$y_1^* = \frac{a - c}{2b}$$

and

$$y_2^* = \frac{a - c}{2b} - \frac{y_1^*}{2} = \frac{a - c}{2b} - \frac{a - c}{4b} = \frac{a - c}{4b}.$$
• Total production:

\[ Y_D^* = y_1^* + y_2^* = 3 \frac{a - c}{4b} \]

• Price equals

\[ p^* = a - b \left( \frac{3a - c}{4b} \right) = \frac{1}{4}a + \frac{3}{4}c \]

• Compare to monopoly:

\[ y_M^* = \frac{a - c}{2b} \]

and

\[ p_M^* = \frac{a + c}{2} \].

• Compare to Cournot:

\[ Y_D^* = y_1^* + y_2^* = 2 \frac{a - c}{3b} \]

and

\[ p_D^* = \frac{1}{3}a + \frac{2}{3}c. \]
• Compare with Cournot outcome

• Firm 2 best response function:

\[ y_2^* = \frac{a - c}{2b} - \frac{y_1^*}{2} \]

• Firm 1 best response function:

\[ y_1^* = \frac{a - c}{2b} - \frac{y_2^*}{2} \]

• Intersection gives Cournot
• Stackelberg: Equilibrium is point on Best Response of Firm 2 that maximizes profits of Firm 1

• Plot iso-profit curve of Firm 1:

\[ \bar{\Pi}_1 = (a - c) y_1 - b y_1 y_2 - b y_1^2 \]

• Solve for \( y_2 \) along iso-profit:

\[ y_2 = \frac{a - c}{b} - y_1 - \frac{\bar{\Pi}_1}{b y_1} \]

• Iso-profit curve is flat for

\[ \frac{dy_2}{dy_1} = -1 + \frac{\bar{\Pi}}{b (y_1)^2} = 0 \]

or

\[ y_1 = \]
Figure
3 General Equilibrium: Introduction

• So far, we looked at consumers
  – Demand for goods
  – Choice of leisure and work
  – Choice of risky activities

• We also looked at producers:
  – Production in perfectly competitive firm
  – Production in monopoly
  – Production in oligopoly
• We also combined consumers and producers:
  – Supply
  – Demand
  – Market equilibrium

• Partial equilibrium: one good at a time

• General equilibrium: Demand and supply for all goods!
  – supply of young worker↑ ⟷ wage of experienced workers?
  – minimum wage↑ ⟷ effect on higher earners?
  – steel tariff↑ ⟷ effect on car price
4 Edgeworth Box: Pure Exchange

- Nicholson, Ch. 13, pp. 441-444, 476-478 (Ch. 12, pp. 335–338, 369–370, 9th)

- 2 consumers in economy: $i = 1, 2$

- 2 goods, $x_1, x_2$

- Endowment of consumer $i$, good $j$: $\omega^i_j$

- Total endowment: $(\omega_1, \omega_2) = (\omega_1^1 + \omega_1^2, \omega_2^1 + \omega_2^2)$

- No production here. With production (as in book), $(\omega_1, \omega_2)$ are optimally produced
• Edgeworth box

• Draw preferences of agent 1

• Draw preferences of agent 2
• Consumption of consumer $i$, good $j$: $x_j^i$

• Feasible consumption:

$$x_i^1 + x_i^2 \leq \omega_i \text{ for all } i$$

• If preferences monotonic, $x_i^1 + x_i^2 = \omega_i$ for all $i$

• Can map consumption levels into box
5 Barter

- Consumers can trade goods 1 and 2

- Allocation \( ((x_1^1, x_2^1), (x_1^2, x_2^2)) \) can be outcome of barter if:

- **Individual rationality.**

  \[
  u_i(x_1^i, x_2^i) \geq u_i(\omega_1, \omega_2) \text{ for all } i
  \]

- **Pareto Efficiency.** There is no allocation \( ((\hat{x}_1^1, \hat{x}_2^1), (\hat{x}_1^2, \hat{x}_2^2)) \) such that

  \[
  u_i(\hat{x}_1^i, \hat{x}_2^i) \geq u_i(x_1^i, x_2^i) \text{ for all } i
  \]

  with strict inequality for at least one agent.
• Barter outcomes in Edgeworth box

• Endowments \((\omega_1, \omega_2)\)

• Area that satisfies individual rationality condition

• Points that satisfy pareto efficiency

• **Pareto set.** Set of points where indifference curves are tangent
- **Contract curve.** Subset of Pareto set inside the individually rational area.

- Contract curve = Set of barter equilibria

- Multiple equilibria. Depends on bargaining power.

- Bargaining is time- and information-intensive procedure

- What if there are prices instead?
6 Next lecture

- Walrasian Equilibrium