Economics 101A
(Lecture 8)

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Outline

1. Indirect Utility Function

2. Comparative Statics (Introduction)

3. Income Changes

4. Price Changes

5. Expenditure minimization
1 Indirect utility function

• Nicholson, Ch. 4, pp. 124-127 (106–108, 9th)

• Define the indirect utility $v(p, M) \equiv u(x^*(p, M))$, with $p$ vector of prices and $x^*$ vector of optimal solutions.

• $v(p, M)$ is the utility at the optimumum for prices $p$ and income $M$

• Some comparative statics: $\partial v(p, M)/\partial M =$?

• Hint: Use Envelope Theorem on Lagrangean function
• What is the sign of $\lambda$?

• $\lambda = \frac{u'_x}{p}/p > 0$

• $\frac{\partial v(p, M)}{\partial p_i} =$?

• Properties:
  
  – Indirect utility is always increasing in income $M$

  – Indirect utility is always decreasing in the price $p_i$
2 Comparative Statics (introduction)

• Nicholson, Ch. 5, pp. 141-151 (121–131, 9th)

• Utility maximization yields \( x_i^* = x_i^*(p_1, p_2, M) \)

• Quantity consumed as a function of income and price

• What happens to quantity consumed \( x_i^* \) as prices or income varies?
• Simple case: Equal increase in prices and income.

• $M' = tM$, $p'_1 = tp_1$, $p'_2 = tp_2$.

• Compare $x^*(tM, tp_1, tp_2)$ and $x^*(M, p_1, p_2)$.

• What happens?

• Write budget line: $tp_1 x_1 + tp_2 x_2 = tM$

• Demand is homogeneous of degree 0 in $p$ and $M$:

$$x^*(tM, tp_1, tp_2) = t^0 x^*(M, p_1, p_2) = x^*(M, p_1, p_2).$$
• Consider Cobb-Douglas Case:

\[ x_1^* = \frac{\alpha}{\alpha + \beta} \frac{M}{p_1}, x_2^* = \frac{\beta}{\alpha + \beta} \frac{M}{p_2} \]

• What is \( \partial x^*/\partial M \)?

• What is \( \partial x^*/\partial p_x \)?

• What is \( \partial x^*/\partial p_y \)?

• General results?
3 Income changes

- Income increases from $M$ to to $M' > M$.

- Budget line $(p_1x_1 + p_2x_2 = M)$ shifts out:

$$x_2 = \frac{M'}{p_2} - x_1 \frac{p_1}{p_2}$$

- New optimum?
• Engel curve: $x_i^*(M)$: demand for good $i$ as function of income $M$ holding fixed prices $p_1, p_2$

• Does $x_i^*$ increase with $M$?
  
  – Yes. Good $i$ is normal

  – No. Good $i$ is inferior
4 Price changes

• Price of good $i$ increases from $p_i$ to to $p'_i > p_i$

• For example, decrease in price of good $2$, $p'_2 < p_2$

• Budget line tilts:

$$x_2 = \frac{M}{p'_2} - x_1 \frac{p_1}{p'_2}$$

• New optimum?
• Demand curve: $x_i^*(p_i)$: demand for good $i$ as function of own price holding fixed $p_j$ and $M$

• Odd convention of economists: plot price $p_i$ on vertical axis and quantity $x_i$ on horizontal axis. Better get used to it!
• Does $x_i^*$ decrease with $p_i$?
  
  – Yes. Most cases

– No. Good $i$ is *Giffen*

– Ex.: Potatoes in Ireland

– Do not confuse with Veblen effect for luxury goods or informational asymmetries: these effects are real, but not included in current model
5 Expenditure minimization

- Nicholson, Ch. 4, pp. (109–113, 9th)

- Solve problem EMIN (minimize expenditure):

  \[
  \min p_1 x_1 + p_2 x_2 \\
  \text{s.t. } u(x_1, x_2) \geq \bar{u}
  \]

- Choose bundle that attains utility \(\bar{u}\) with minimal expenditure

- Ex.: You are choosing combination CDs/restaurant to make a friend happy

- If utility \(u\) strictly increasing in \(x_i\), can maximize s.t. equality

- Denote by \(h_i(p_1, p_2, \bar{u})\) solution to EMIN problem

- \(h_i(p_1, p_2, \bar{u})\) is Hicksian or compensated demand
• Graphically:

  – Fix indifference curve at level $\bar{u}$
  
  – Consider budget sets with different $M$
  
  – Pick budget set which is tangent to indifference curve

• Optimum coincides with optimum of Utility Maximization!

• Formally:

$$h_i(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p_1, p_2, \bar{u}))$$
• Expenditure function is expenditure at optimum

• \( e(p_1, p_2, \bar{u}) = p_1 h_1(p_1, p_2, \bar{u}) + p_2 h_2(p_1, p_2, \bar{u}) \)

• \( h_i(p_i) \) is *Hicksian or compensated demand* function

• Is \( h_i \) always decreasing in \( p_i \)? Yes!

• Graphical proof: moving along a convex indifference curve

• (For non-convex indifferent curves, still true)
• Using first order conditions:

\[ L(x_1, x_2, \lambda) = p_1 x_1 + p_2 x_2 - \lambda (u(x_1, x_2) - \bar{u}) \]

\[ \frac{\partial L}{\partial x_i} = p_i - \lambda u'_i(x_1, x_2) = 0 \]

• Write as ratios:

\[ \frac{u'_1(x_1, x_2)}{u'_2(x_1, x_2)} = \frac{p_1}{p_2} \]

• \( MRS \) = ratio of prices as in utility maximization!

• However: different constraint \( \rightarrow \lambda \) is different
• Example 1: Cobb-Douglas utility

\[
\begin{align*}
\min & \quad p_1 x_1 + p_2 x_2 \\
\text{s.t.} & \quad x_1^\alpha x_2^{1-\alpha} \geq \bar{u}
\end{align*}
\]

• Lagrangean =

• F.o.c.:

• Solution: \( h_1^* = \), \( h_2^* = \)

• \( \partial h_i^*/\partial p_i < 0, \partial h_i^*/\partial p_j > 0, j \neq i \)
6 Next Lectures

• Slutsky Equation

• Complements and Substitutes

• Then moving on to applications:
  – Labor Supply
  – Intertemporal choice
  – Economics of Altruism