Outline

1. Application 2: Intertemporal choice II
2. Application 3: Altruism and charitable donations
3. Introduction to probability
4. Expected Utility
1 Intertemporal choice II

- Nicholson Ch. 16, pp. 573-581 (477–484, 9th) [*Labor Supply*]

- Nicholson Ch. 17, pp. 597-601 (502–506, 9th)

- Comparative statics with respect to income $M_0$

- Rewrite ratio of f.o.c.s as

  \[ U'(c_0) - \frac{1 + r}{1 + \delta} U'(c_1) = 0 \]

- Substitute $c_1$ in using $c_1 = M_1 + (M_0 - c_0)(1 + r)$ to get

  \[ U'(c_0) - \frac{1 + r}{1 + \delta} U'(M_1 + (M_0 - c_0)(1 + r)) = 0 \]
• Apply implicit function theorem:

\[
\frac{\partial c_0^*(r, M)}{\partial M_0} = -\frac{-\frac{1+r}{1+\delta}U''(c_1)(1 + r)}{U'''(c_0) - \frac{1+r}{1+\delta}U''(c_1) * (- (1 + r))}
\]
• Denominator is always negative

• Numerator is positive

• $\partial c_0^* (r, M) / \partial M_0 > 0$ — consumption at time 0 is a normal good.

• Can also show $\partial c_0^* (r, M) / \partial M_1 > 0$
• Comparative statics with respect to interest rate $r$

• Apply implicit function theorem:

$$\frac{\partial c_0^*(r, M)}{\partial r} = -\frac{\frac{1}{1+\delta}U'(c_1)}{U''(c_0) - \frac{1+r}{1+\delta}U''(c_1) * (- (1 + r))} - \frac{\frac{1+r}{1+\delta}U''(c_1) * (M_0 - c_0)}{U''(c_0) - \frac{1+r}{1+\delta}U''(c_1) * (- (1 + r))}$$

• Denominator is always negative

• Numerator: First term negative (substitution eff.)

• Numerator: Second term (income effect:)

  – positive if $M_0 > c_0$

  – negative if $M_0 < c_0$. 
2 Altruism and Charitable Donations

• Maximize utility = satisfy self-interest?
• No, not necessarily

• 2-person economy:
  – Mark has income $M_M$ and consumes $c_M$
  – Wendy has income $M_W$ and consumes $c_W$

• One good: $c$, with price $p = 1$
• Utility function: \( u(c) \), with \( u' > 0, u'' < 0 \)

• Wendy is altruistic: she maximizes \( u(c_W) + \alpha u(c_M) \) with \( \alpha > 0 \)

• Mark simply maximizes \( u(c_M) \)

• Wendy can give a donation of income \( D \) to Mark.
• Wendy computes the utility of Mark as a function of the donation $D$

• Mark maximizes

$$\max_{c_M} u(c_M)$$

$$s.t. \ c_M \leq M_M + D$$

• Solution: $c_M^* = M_M + D$

• Wendy maximizes

$$\max_{c_M,D} u(c_W) + \alpha u(M_M + D)$$

$$s.t. \ c_W \leq M_W - D$$
• Rewrite as:

$$\max_D u(M_W - D) + \alpha u (M_M + D)$$

• First order condition:

$$-u'(M_W - D^*) + \alpha u' (M_M + D^*) = 0$$

• Second order conditions:

$$u''(M_W - D^*) + \alpha u'' (M_M + D^*) < 0$$
- Assume $\alpha = 1$.
  - Solution?
  - $u'(M_W - D) = u'(M_M + D^*)$
  - $M_W - D^* = M_M + D^*$ or $D^* = (M_W - M_M) / 2$
  - Transfer money so as to equate incomes!
  - Careful: $D < 0$ (negative donation!) if $M_M > M_W$

- Corrected maximization:
  $\max_D u(M_W - D) + \alpha u(M_M + D)$
  $s.t. D \geq 0$

- Solution ($\alpha = 1$):
  \[
  D^* = \begin{cases} 
  (M_W - M_M) / 2 & \text{if } M_W - M_M > 0 \\
  0 & \text{otherwise}
  \end{cases}
  \]
• Assume interior solution. \((D^* > 0)\)

• Comparative statics 1 (altruism):

\[
\frac{\partial D^*}{\partial \alpha} = -\frac{u'(M_M + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} > 0
\]

• Comparative statics 2 (income of donor):

\[
\frac{\partial D^*}{\partial M_W} = -\frac{-u''(M_W + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} > 0
\]

• Comparative statics 3 (income of recipient):

\[
\frac{\partial D^*}{\partial M_M} = -\frac{\alpha u''(M_M + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} < 0
\]
• A quick look at the evidence

• From Andreoni (2002)
3 Introduction to Probability

- So far deterministic world:
  - income given, known $M$
  - interest rate known $r$

- But some variables are unknown at time of decision:
  - future income $M_1$?
  - future interest rate $r_1$?

- Generalize framework to allow for uncertainty
  - Events that are truly unpredictable (weather)
  - Event that are very hard to predict (future income)
• Probability is the language of uncertainty

• Example:
  
  – Income $M_1$ at $t = 1$ depends on state of the economy
  – Recession ($M_1 = 20$), Slow growth ($M_2 = 25$), Boom ($M_3 = 30$)
  – Three probabilities: $p_1$, $p_2$, $p_3$
  – $p_1 = P(M_1) = P(\text{recession})$

• Properties:
  
  – $0 \leq p_i \leq 1$
  – $p_1 + p_2 + p_3 = 1$
• Mean income: \( EM = \sum_{i=1}^{3} p_i M_i \)

• If \((p_1, p_2, p_3) = (1/3, 1/3, 1/3)\),

\[
EM = \frac{1}{3}20 + \frac{1}{3}25 + \frac{1}{3}30 = \frac{75}{3} = 25
\]

• Variance of income: \( V(M) = \sum_{i=1}^{3} p_i (M_i - EM)^2 \)

• If \((p_1, p_2, p_3) = (1/3, 1/3, 1/3)\),

\[
V(M) = \frac{1}{3}(20 - 25)^2 + \frac{1}{3}(25 - 25)^2 + \frac{1}{3}(30 - 25)^2
\]
\[
= \frac{1}{3}5^2 + \frac{1}{3}5^2 = 2/3 * 25
\]

• Mean and variance if \((p_1, p_2, p_3) = (1/4, 1/2, 1/4)\)?
4 Expected Utility

- Nicholson, Ch. 7, pp. 202-209 (Ch. 18, pp. 533–541, 9th)

- Consumer at time 0 asks: what is utility in time 1?

- At $t = 1$ consumer maximizes

$$\max U(c^1)$$

$$s.t. \ c^1_i \leq M^1_i + (1 + r)(M^0 - c^0)$$

with $i = 1, 2, 3$.

- What is utility at optimum at $t = 1$ if $U' > 0$?

- Assume for now $M^0 - c^0 = 0$

- Utility $U \left( M^1_i \right)$

- This is uncertain, depends on which $i$ is realized!
• How do we evaluate future uncertain utility?

• **Expected utility**

\[
EU = \sum_{i=1}^{3} p_i U \left( M_i^1 \right)
\]

• In example:

\[
EU = \frac{1}{3} U(20) + \frac{1}{3} U(25) + \frac{1}{3} U(30)
\]

• Compare with \( U(EC) = U(25) \).

• Agents prefer riskless outcome \( EM \) to uncertain outcome \( M \) if

\[
\frac{1}{3} U(20) + \frac{1}{3} U(25) + \frac{1}{3} U(30) < U(25) \text{ or } \frac{1}{2} U(20) + \frac{1}{2} U(30) < \frac{2}{3} U(25) \text{ or } \frac{1}{2} U(20) + \frac{1}{2} U(30) < U(25)
\]
• Picture
• Depends on sign of $U''$, on concavity/convexity

• Three cases:

  - $U''(x) = 0$ for all $x$. (linearity of $U$)
    * $U(x) = a + bx$
    * $1/2U(20) + 1/2U(30) = U(25)$

  - $U''(x) < 0$ for all $x$. (concavity of $U$)
    * $1/2U(20) + 1/2U(30) < U(25)$

  - $U''(x) > 0$ for all $x$. (convexity of $U$)
    * $1/2U(20) + 1/2U(30) > U(25)$
• If $U''(x) = 0$ (linearity), consumer is indifferent to uncertainty

• If $U''(x) < 0$ (concavity), consumer dislikes uncertainty

• If $U''(x) > 0$ (convexity), consumer likes uncertainty

• Do consumers like uncertainty?

• Do you like uncertainty?
• **Theorem. (Jensen’s inequality)** If a function \( f(x) \) is concave, the following inequality holds:

\[
f(Ex) \geq Ef(x)
\]

where \( E \) indicates expectation. If \( f \) is strictly concave, we obtain

\[
f(Ex) > Ef(x)
\]

• Apply to utility function \( U \).

• Individuals dislike uncertainty:

\[
U(Ex) \geq EU(x)
\]

• Jensen’s inequality then implies \( U \) concave \((U'' \leq 0)\)

• Relate to diminishing marginal utility of income
5 Next Lectures

- Risk aversion

- Applications:
  - Insurance
  - Portfolio choice
  - Consumption choice II