Economics 101A  
(Lecture 13)

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Outline

1. Insurance II

2. Mid-Term Feedback

3. Investment in Risky Asset

4. Measures of Risk Aversion

5. Intertemporal Consumption and Time Consistency
1 Mid-Term Feedback

- Thanks for the feedback!
2 Insurance II

• Nicholson, Ch. 7, pp. 216–221 (18, pp. 545–551, 9th)

• Individual maximization:

\[
\max_{\alpha} (1 - p) u (w - q\alpha) + pu (w - q\alpha - L + \alpha)
\]

\[
s.t. \alpha \geq 0
\]

• Assume \( \alpha^* \geq 0 \), check later

• First order conditions:

\[
0 = -q (1 - p) u' (w - q\alpha) + (1 - q) pu' (w - q\alpha - L + \alpha)
\]

or

\[
\frac{u' (w - q\alpha)}{u' (w - q\alpha - L + \alpha)} = \frac{1 - q}{q} \frac{p}{1 - p}.
\]
• What if $q > p$ (insurance needs to cover operating costs)?

• $\alpha^* < L$ : Insurance will be only partial (if at all)

• Exercise: Check second order conditions!
3 Investment in Risk Asset

- Nicholson, Ch. 7, pp. 214–215 (not in 9th Ed.)

- Individual has:
  - wealth $w$
  - utility function $u$, with $u' > 0$

- Two possible investments:
  - Asset B (bond) yields return 1 for each dollar
  - Asset S (stock) yields uncertain return $(1 + r)$:
    * $r = r_+ > 0$ with probability $p$
    * $r = r_- < 0$ with probability $1 - p$
    * $Er = pr_+ + (1 - p)r_- > 0$

- Share of wealth invested in stock $S = \alpha$
• Individual maximization:

\[
\max_{\alpha} (1 - p) u(w [(1 - \alpha) + \alpha (1 + r_-)]) +
+ pu(w [(1 - \alpha) + \alpha (1 + r_+)])
\]

\[s.t. 0 \leq \alpha \leq 1\]

• Case of risk neutrality: \( u(x) = a + bx, b > 0 \)

• Assume \( a = 0 \) (no loss of generality)

• Maximization becomes

\[
\max_{\alpha} b (1 - p) (w [1 + \alpha r_-]) + bp (w [1 + \alpha r_+])
\]

or

\[
\max_{\alpha} bw + \alpha bw [(1 - p) r_- + pr_+]
\]

• Sign of term in square brackets? Positive!

• Set \( \alpha^* = 1 \)
• Case of risk aversion: $u'' < 0$

• Assume $0 \leq \alpha^* \leq 1$, check later

• First order conditions:

$$0 = (1 - p)(wr_-)u'(w[1 + \alpha r_-]) +$$
$$+ p(wr_+)u'(w[1 + \alpha r_+])$$

• Can $\alpha^* = 0$ be solution?

• Solution is $\alpha^* > 0$ (positive investment in stock)

• Exercise: Check s.o.c.
4 Measures of Risk Aversion

• Nicholson, Ch. 7, pp. 209-213 (Ch. 18, pp. 541–545, 9th)

• How risk averse is an individual?

• Two measures:

  – Absolute Risk Aversion $r_A$:

    \[ r_A = -\frac{u''(x)}{u'(x)} \]

  – Relative Risk Aversion $r_R$:

    \[ r_R = -\frac{u''(x)}{u'(x)} x \]

• Examples in the Problem Set
5 Time consistency

• Intertemporal choice

• Three periods, $t = 0$, $t = 1$, and $t = 2$

• At each period $i$, agents:
  
  – have income $M_i' = M_i + \text{savings/debts from previous period}$

  – choose consumption $c_i$;

  – can save/borrow $M_i' - c_i$

  – no borrowing in last period: at $t = 2$ $M_2' = c_2$
Utility function at $t = 0$

$$u(c_0, c_1, c_2) = U(c_0) + \frac{1}{1 + \delta} EU(c_1) + \frac{1}{(1 + \delta)^2} EU(c_2)$$

Utility function at $t = 1$

$$u(c_1, c_2) = U(c_1) + \frac{1}{1 + \delta} EU(c_2)$$

Utility function at $t = 2$

$$u(c_2) = U(c_2)$$

$U' > 0, \ U'' < 0$
• Question: Do preferences of agent in period 0 agree with preferences of agent in period 1?

• Period 1.

• Budget constraint at $t = 1$:

$$c_1 + \frac{1}{1 + r}c_2 \leq M'_1 + \frac{1}{1 + r}M_2$$

• Maximization problem:

$$\max U(c_1) + \frac{1}{1 + \delta}EU(c_2)$$

$$s.t. \quad c_1 + \frac{1}{1 + r}c_2 \leq M'_1 + \frac{1}{1 + r}M_2$$

• First order conditions:

• Ratio of f.o.c.s:

$$\frac{U'(c_1)}{EU'(c_2)} = \frac{1 + r}{1 + \delta}$$
• Back to period 0.

• Agent at time 0 can commit to consumption at time 1 as function of uncertain income $M_1$.

• Anticipated budget constraint at $t = 1$:
\[ c_1 + \frac{1}{1 + r}c_2 \leq M_1' + \frac{1}{1 + r}M_2 \]

• Maximization problem:
\[ \max U(c_0) + \frac{1}{1 + \delta}U(c_1) + \frac{1}{(1 + \delta)^2}EU(c_2) \]
\[ s.t. c_1 + \frac{1}{1 + r}c_2 \leq M_1' + \frac{1}{1 + r}M_2 \]

• First order conditions:

• Ratio of f.o.c.s:
\[ \frac{U''(c_1)}{EU'(c_2)} = \frac{1 + r}{1 + \delta} \]
• The two conditions coincide!

• **Time consistency.** Plans for future coincide with future actions.

• To see why, rewrite utility function \( u(c_0, c_1, c_2) \):

\[
U(c_0) + \frac{1}{1 + \delta} U(c_1) + \frac{1}{(1 + \delta)^2} EU(c_2)
\]

\[
= U(c_0) + \frac{1}{1 + \delta} \left[ U(c_1) + \frac{1}{1 + \delta} EU(c_2) \right]
\]

• Expression in brackets coincides with utility at \( t = 1 \)

• Is time consistency right?
  
  – addictive products (alcohol, drugs);
  
  – good actions (exercising, helping friends);
  
  – immediate gratification (shopping, credit card borrowing)
6 Next Lecture

- An Example: Health club Attendance

- Cost Minimization

- Solve an Example

- Cases in which s.o.c. are not satisfied

- Start Profit Maximization