# Labor Supply and Risk Aversion: A Calibration Theorem

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#### Abstract

This paper shows that existing estimates of labor supply elasticities place a tight upper bound on risk aversion in an expected utility model. I derive a formula that relates the coefficient of relative risk aversion ( $\gamma$ ) to the ratio of the income elasticity of labor supply to the wage elasticity, holding fixed the degree of complementarity between consumption and leisure. The degree of complementarity can in turn be inferred from data on consumption choices when agents face unemployment risk. Calibration of the formula reveals that an upward sloping labor supply curve – as found in virtually all studies of labor supply – requires  $\gamma < 1.25$ . The bound on  $\gamma$  rises to at most 1.66 over the range of plausible values for the complementarity parameter. These results generalize to dynamic models with time non-separable or Kreps-Porteus preferences. Hence, conventional expected utility models cannot generate a high degree of risk aversion without sharply contradicting established facts about labor supply behavior.

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### 1 Introduction

Despite a series of well known critiques dating at least to Allais (1953), expected utility theory remains the workhorse framework used to analyze risky decisions. Recent applications of expected utility range from the analysis of optimal taxation and social insurance to principalagent problems and portfolio choice. In the expected utility model, risk aversion arises from the curvature of the underlying utility function, which is commonly measured by the coefficient of relative risk aversion ( $\gamma$ ). The conclusions drawn in any application of expected utility theory hinge on the level of risk aversion used to calibrate the model: the optimal rate of insurance is higher if  $\gamma$  is large, optimal contracts are less incentivized if  $\gamma$  is large, etc.

This paper derives a tight upper bound for  $\gamma$  using a well-established fact from the labor supply literature: Individuals work as much or more when their wages rise, i.e., the uncompensated wage elasticity is (weakly) positive. The intuition for the bounding result is roughly as follows.<sup>1</sup> If an individual chooses to work more when his wage rises, his marginal utility of consumption must not diminish very rapidly. If it did, he would choose to enjoy more leisure with his higher income rather than increasing labor supply to increase consumption. Since the rate at which the marginal utility of consumption diminishes determines risk aversion in expected utility models, it follows that an upper bound on risk aversion can be obtained using the fact that the wage elasticity is positive.

More precisely, the main theoretical result of the paper is that  $\gamma$  is directly related to the ratio of the income elasticity of labor supply to the price (substitution) elasticity of labor supply in any standard labor-leisure choice model, without any restrictions on preferences. To see why, recall that  $\gamma \propto \frac{u_{cc}}{u_c}$  where  $u_c$  denotes the first derivative of utility with respect to consumption, and  $u_{cc}$  denotes the second derivative. An agent's labor supply response

 $<sup>^{1}</sup>$ This intuition applies to the case where consumption and leisure are not complementary; the more general case is discussed in detail below.

to a wage increase is directly related to  $u_c$ , the marginal utility of consumption: The larger the magnitude of  $u_c$ , the greater the benefit of an additional dollar of income, and the more the agent will work when w goes up. The labor supply response to an increase in income is related to how much the marginal utility of consumption changes as income changes,  $u_{cc}$ . If  $u_{cc}$  is large, the marginal utility of consumption falls sharply as income rises, so the agent will reduce labor supply significantly when his income rises. It follows that there is a connection between  $\gamma$  and the ratio of income and price elasticities.

Formalizing this connection requires an additional step. Labor supply data cannot be used in isolation to identify cardinal properties of the utility function because data on certainty behavior only identify utility functions up to a monotonic transformation.<sup>2</sup> The cardinality of the utility function must be pinned down using information on risky decisions. To see how this can be done, observe that for a fixed degree of complementarity between consumption and leisure  $(u_{cl})$ , there is only one vN-M utility (up to affine transformations) that can be consistent with a given set of labor-leisure choices. Hence, given a value of  $u_{cl}$ , labor supply data can be used to infer risk aversion. I show that  $u_{cl}$  can in turn be estimated from data on consumption choices when employment is stochastic, e.g. when individuals anticipate being laid off with some probability.<sup>3</sup> Intuitively, the extent to which an agent chooses to smooth consumption across states in which labor supply differs (via an insurance policy or saving) reveals the degree of complementarity between consumption and leisure. Importantly, the estimates of risk aversion are very insensitive over the broad range of values of  $u_{cl}$  estimated in existing studies of complementarity, implying that labor supply data itself contains considerable information about the rate at which marginal utility diminishes .

<sup>&</sup>lt;sup>2</sup>In other words, identification of curvature requires a 1-1 map between observed choices and  $\gamma$ . In the general labor-leisure model, such a map does not exist because any monotonic transformation of utility generates the same labor supply choices.

<sup>&</sup>lt;sup>3</sup>It is critical to have data on choices under *uncertainty*; barring additional assumptions, a cardinal value for  $u_{cl}$  cannot be inferred from the usual certainty settings in which we typically think about estimating the degree of complementarity between consumption and leisure (e.g. timing-of-work, household production).

The formula relating the labor supply elasticities to  $\gamma$  results in a simple and surprisingly tight bound for risk aversion: If the uncompensated labor supply curve is upward sloping (as found by almost all studies of labor supply),  $\gamma < 1.25$ .<sup>4</sup> If  $\gamma$  were larger than 1.25, income effects of wage increases would be large relative to substitution effects, creating a downwardsloping labor supply curve. This bound rises to at most 1.66 for plausible perturbations in the degree of complementarity between consumption and leisure. To give a better sense of the range of values of  $\gamma$  that are consistent with evidence on labor supply behavior, I impute  $\gamma$  using twenty-nine sets of estimates of wage and income elasticities drawn from studies with different methodologies and data sources. In the benchmark case where  $u_{cl} = 0$ , the mean value of  $\gamma$  implied by the studies is 0.94, with a range of 0.15 to 1.45. Only one of the twenty-nine studies finds a slightly negative wage elasticity, yielding  $\gamma > 1.25$ . Labor supply data thus provide robust evidence that the marginal utility of wealth does not diminish very rapidly in practice.

These bounds generalize to a general dynamic life-cycle model of labor supply. The curvature of the value function over wealth – which determines risk preferences over wealth gambles – is bounded above by 1.25 by estimates of labor supply elasticities for dynamic models. In preference specifications where risk aversion and the elasticity of intertemporal substitution are free parameters (e.g. time non-separable or Kreps-Porteus preferences), the bounds applies only to risk aversion and the EIS remains unrestricted. Hence, the result of this paper pertains exclusively to risk aversion, distinguishing it from prior work on labor supply and the EIS in the "balanced growth" literature, which is discussed in greater detail below.

The fact that the curvature of utility over wealth is severely restricted by behavior observed in the labor market has two important implications. First, those who apply expected

<sup>&</sup>lt;sup>4</sup>Even if the uncompensated wage elasticity were somewhat negative, risk aversion remains tightly bounded; generating  $\gamma > 2$  requires an uncompensated wage elasticity more negative than any estimate in the labor supply literature to date (as reviewed in Pencavel (1986) and Blundell and MaCurdy (1999)).

utility theory to analyze problems such as taxation and portfolio choice must be content to calibrate their models with a low level of risk aversion, especially in applications where labor supply is endogenous. Second, the result implies that diminishing marginal utility itself is inadequate to explain the high estimates of risk aversion obtained in many studies of preferences over gambles. For instance, Barsky et. al. (1997) report estimates of  $\gamma > 4$  using responses to questions about large hypothetical gambles. Estimates of  $\gamma$  from portfolio choice and equity premiums exceed 10 (Mehra and Prescott 1985, Kocherlakota 1996), and estimates of  $\gamma$  using data on insurance deductibles are around 10 (Dreze 1987).<sup>5</sup> A departure from expected utility theory is needed to obtain such degrees of risk aversion without generating starkly counterfactual labor supply patterns.<sup>6</sup>

The remainder of the paper proceeds as follows. Section 2 places the result of this paper in the context of previous calibration results for expected utility theory and the balanced growth literature on intertemporal substitution. Section 3 derives formulas that connect risk aversion to labor supply behavior in standard static and dynamic models of labor supply. Section 4 gives the calibration argument and imputes  $\gamma$  using existing estimates of labor supply elasticities. The final section offers concluding remarks.

# 2 Related Literature

The idea that labor supply data place strong restrictions on risk aversion is related to but logically distinct from a well known result in the "balanced growth" literature, which shows that labor supply behavior restricts the elasticity of intertemporal substitution (EIS). In

<sup>&</sup>lt;sup>5</sup>There are, however, some studies that find very low levels of risk aversion as well. For instance, Szpiro (1986) estimates  $\gamma = 2$  using data on the fraction of insured assets and Metrick (1995) estimates  $\gamma = 0$  for Jeopardy players.

<sup>&</sup>lt;sup>6</sup>Even if expected utility is not the best positive model of risk preferences, having the bound on  $\gamma$  remains useful for two reasons. First, if expected utility is used as a normative model, knowing that  $\gamma$  is low can be helpful in determining optimal government policies (e.g. social insurance, taxation). Second, identifying  $\gamma$ can help in understanding which non-expected utility theory is closest to matching observed behavior.

a seminal article, King, Plosser, and Rebelo (1988) [KPR] show that balanced growth – which requires constant labor supply with increasing wages – places strong restrictions on the form of the flow utility function when utility is time-separable. Basu and Kimball (2002) develop the KPR point further by showing that reconciling low estimates of the EIS (as in Hall (1988) and Barsky et. al. (1997)) with balanced growth requires either strong complementarity between consumption and labor or time non-separable utility (e.g., habit formation).

While the EIS and  $\gamma$  are directly linked when utility is time-separable, they are two distinct characteristics of preferences when utility has a more general specification, as emphasized both theoretically and empirically by Hall (1988), Weil (1990), Epstein and Zin (1991), and others. This paper provides a bound on  $\gamma$ , without placing any restrictions on the EIS. This point is demonstrated formally in section 3.5 by considering two preference specifications commonly used to sever the link between  $\gamma$  and EIS in dynamic models: time non-separable utility and Kreps-Porteus preferences. In both of these cases, it is shown that effective risk aversion is identified by labor supply behavior, while the EIS remains unidentified. Hence, the calibration result of this paper is purely about preferences across states (risk) and not preferences across time periods (intertemporal substitution). Moreover, the solutions proposed by Basu and Kimball to reconcile a low EIS with labor supply behavior do not relax the risk aversion bound derived here.

The results of this paper are also related to those of Rabin (1999) and Kaplow (2003), who give other calibration results for risk preferences in an expected utility model. Rabin shows that expected utility cannot generate a reasonably high level of moderate-stakes risk aversion without creating unreasonably high large-stakes risk aversion.<sup>7</sup> Kaplow shows that estimates of the income elasticity of a value of a statistical life place an upper bound on risk

<sup>&</sup>lt;sup>7</sup>The difficulty of explaining moderate-stake risk aversion in conventional expected utility models has been documented by others as well (e.g., Segal and Spivak (1990), Epstein (1992)). More recently, Palacios-Huerta et. al. (2004), challenge the claim that small-stakes risk aversion is sufficiently high to be at odds with expected utility theory.

aversion in an expected utility setting. Each of these calibration arguments illuminates the restrictions inherent in expected utility theory in a different way. The labor supply bound tightly restricts risk aversion over moderate-stakes as well as large stakes, and is particularly relevant for the wide class of models that incorporate both uncertainty and labor supply choices. These models are often calibrated with risk aversion far above 1, violating the bound provided here.<sup>8</sup>

Finally, this paper relates to a large experimental literature documenting preferences over gambles inconsistent with expected utility (see e.g. Starmer (2000) for a review). The approach of this paper is very different: it identifies labor supply as a new source of information about the rate at which marginal utility diminishes, and shows that field evidence from this source robustly implies a very low  $\gamma$ . This result contributes to the existing literature on violations of expected utility in two ways. First, it restricts risk preferences over all gambles rather than just the small gambles that are feasible in experiments, significantly expanding the set of situations where the canonical expected utility model cannot pass muster as a "rough approximation." Second, it shows that the traditional notion of diminishing marginal utility can play at best a secondary role in explaining the high levels of risk aversion estimated in many studies, suggesting that an additional, quantitatively powerful source of risk aversion must be identified to understand risk preferences in these cases.

# 3 Labor Supply and Risk Aversion

This section derives estimators for  $\gamma$  in standard labor supply models, generalizing the model in steps to simplify the exposition. I begin by providing graphical intuition for the connection between labor supply and risk aversion in a static framework where the marginal

<sup>&</sup>lt;sup>8</sup>This issue is especially relevant for the applications of expected utility mentioned in the introductory paragraph. For example, see Gruber (1997) and Acemoglu and Shimer (1999) on optimal unemployment insurance, Haubrich (1994) on executive compensation, and Bodie and Samuelson (1992) on portfolio choice with endogenous labor supply.

disutility of labor is constant. I then derive an estimator for  $\gamma$  under the assumption that utility is additive in consumption and leisure  $(u_{cl} = 0)$ . The third subsection considers the case of arbitrary  $u_{cl}$ , and derives an estimator for  $\gamma$  in a static framework using both labor supply data and information on consumption choices when agents face unemployment risk. The fourth subsection considers a model where agents can make only extensive labor supply choices (i.e., hours cannot be chosen), and provides a corresponding formula for risk aversion based on elasticities of labor force participation. Finally, I consider a dynamic life cycle model with arbitrary time non-separable utility, where the restrictive assumption that consumption equals income inherent in the static analysis is dropped. In this model, the estimator derived for  $\gamma$  in the static model yields the curvature of the value function over wealth, which determines preferences over wealth gambles.

#### 3.1 Graphical Intuition

Consider an agent who has a vN-M utility function u(c, l) over consumption and labor. This agent's expected utility from a gamble  $(\tilde{c}, \tilde{l})$  is obtained by taking an expectation over the vN-M utility function:

$$U(\tilde{c},\tilde{l}) = Eu(\tilde{c},\tilde{l})$$

The goal of this paper is to identify the curvature of the underlying cardinal utility over outcomes, u(c, l).<sup>9</sup>

This section illustrates the connection between risk aversion and labor supply graphically in the special case where the marginal disutility of labor is constant. Let c denote consumption, l labor supply, and w the wage. Abusing notation slightly, let u(c) denote utility over consumption; assume that  $u_c > 0$  and  $u_{cc} < 0$ . Let  $\psi$  denote the marginal disutility of labor. I first derive an expression for compensated labor supply,  $l^c(w)$ , by solving an expenditure

<sup>&</sup>lt;sup>9</sup>I focus on the curvature of utility over consumption  $\gamma = \frac{u_{cc}}{u_c}c$  here, but show below that deriving the curvature of utility over wealth (when *l* is endogenous) is straightforward once the value of  $\gamma$  is known.

minimization problem:

$$\min c + w(1-l)$$
 s.t.  $u(c) - \psi l \ge \overline{v}$ 

At an interior optimum,  $l^{c}(w)$  satisfies the first order condition

$$u_c(u^{-1}(\overline{v}+\psi l^c)) = \frac{\psi}{w}$$

This condition is intuitive: the individual chooses labor supply by equating the marginal utility of an extra dollar of consumption with the marginal disutility of working to earn that dollar. These choices are depicted by the intersections of the  $u_c$  and  $\frac{\psi}{w}$  curves in Figure 1.

Now consider the effect of an increase in w from  $w_0$  to  $w_1$  on compensated labor supply. As shown in Figure 1, a change in w shifts the flat marginal disutility of labor curve downward. If the utility function is highly curved (case A), the marginal utility of consumption  $(u_c)$  falls quickly as labor supply and income rise. Consequently, the increase in w leads to a small increase in  $l_A^c$ . When the utility function is not very curved (case B), marginal utility declines slowly as a function of wealth and the same  $\Delta w$  leads to a larger increase in  $l_B^c$ . Figure 1 therefore illustrates that the compensated wage (price) elasticity of labor supply,  $\varepsilon_{l,w}^c$  is inversely related to the curvature of utility over consumption.

The intuition for this relationship is as follows. Following a compensated wage increase, agents increase their labor supply up to the point where the marginal utility of an additional dollar is offset by the marginal disutility of the additional work necessary to earn that dollar. If utility is very curved, this condition is met by a small increase in labor supply. If utility is not very curved, the agent needs to increase l much more before his marginal utility of money falls sufficiently to equal the new  $\frac{\psi}{w_1}$ .

The preceding argument relies on the assumption that disutility of labor,  $\psi$ , does not vary with l. When it does, the curvature of  $\psi(l)$  is confounded with the curvature of u and  $\varepsilon_{l,w}^c$  is no longer sufficient to recover  $\gamma$ . In this case, the elasticity of labor supply with respect to unearned income,  $\varepsilon_{l,y}$  is needed to separate the two curvature parameters. Abstractly, the

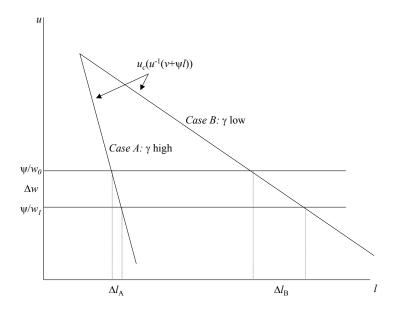


Figure 1: Recovering  $\gamma$  from Labor Supply

income and compensated wage elasticities are both functions of the two curvatures. One can therefore back out  $\gamma$  and the curvature of  $\psi$  by solving a system of two equations and two unknowns, conditional on the degree of complementarity between consumption and leisure.

The next section derives the relationship between  $\gamma$  and labor supply elasticities formally in the more general case where the marginal disutility of labor varies with l.

### 3.2 Base Case: Additive Utility

It is convenient to redefine u(c, l) as the agent's utility over both consumption and labor. Assume  $u_c > 0, u_l < 0, u_{cc} < 0, u_{ll} < 0$ . In addition, assume utility is additive:  $u_{cl} = 0.10$ 

Consider an agent with wage w and unearned income y. Unearned income in this model should not be thought of as only asset wealth. It also includes the income of the other earner

<sup>&</sup>lt;sup>10</sup>Note that this restriction is stronger than assuming that the utility function permits an additively separable representation. For example, Cobb-Douglas utility is "additively separable" but does *not* satisfy the additivity restriction (however, the log of a Cobb-Douglas utility does).

in dual-earner households and Hausman's (1985) definition of "virtual income" to correct for the non-linearity of the budget set created by the progressive tax system.<sup>11</sup> This agent chooses (Marshallian) labor supply l by solving

$$\max_{l} u(y+wl,l)$$

At an interior optimum, l satisfies the first order condition

$$wu_c(y+wl,l) = -u_l(y+wl,l)$$
(1)

Consider the effects of increasing w and y on l:

$$\begin{array}{lll} \displaystyle \frac{\partial l}{\partial y} & = & -\frac{w u_{cc}}{w^2 u_{cc} + u_{ll}} \\ \displaystyle \frac{\partial l}{\partial w} & = & -\frac{u_c + w l u_{cc}}{w^2 u_{cc} + u_{ll}} \end{array}$$

Using the Slutsky decomposition for compensated labor supply  $\left(\frac{\partial l^c}{\partial w}\right)$ 

$$\frac{\partial l^c}{\partial w} = \frac{\partial l}{\partial w} - l \frac{\partial l}{\partial y} \tag{2}$$

it follows that

$$\frac{\partial l/\partial y}{\partial l^c/\partial w} = \frac{w u_{cc}(y+wl,l)}{u_c(y+wl,l)}$$
(3)

The definition of the coefficient of relative risk aversion at consumption c is

$$\gamma(c) \equiv -\varepsilon_{u_c,c} = \frac{\partial u_c(c)}{\partial c} \frac{c}{u_c(c)} = -\frac{u_{cc}(c)}{u_c(c)}c$$
(4)

<sup>&</sup>lt;sup>11</sup>Non-linear budget sets and dual earners could be explicitly introduced in the model, but these effects can be handled more easily by changing the definition of y appropriately in the simple single-earner model. The empirical implementation in section 3 defines y appropriately to take these effects into account as in the modern labor supply literature.

which implies, using (3), that

$$\gamma(y+wl) = -\frac{y+wl}{w} \frac{\partial l/\partial y}{\partial l^c/\partial w} = -(1+\frac{wl}{y}) \frac{\varepsilon_{l,y}}{\varepsilon_{l^c,w}}(y,w)$$
(5)

where  $\varepsilon_{l,y}$  denotes the income elasticity,  $\varepsilon_{l^c,w}$  the compensated wage (price) elasticity, and  $\frac{wl}{y}$  the ratio of earned to unearned income.

As in the graphical example, the coefficient of relative risk aversion is inversely related to the price elasticity. In addition, when disutility of labor is not constant,  $\gamma$  is directly related to the magnitude of the income elasticity. To see why the ratio of these elasticities determines risk aversion, recall that  $\gamma \propto \frac{u_{cc}}{u_c}$  where  $u_c$  denotes the first derivative of utility with respect to consumption, and  $u_{cc}$  denotes the second derivative. An agent's labor supply response to a wage increase is directly related to  $u_c$ : the larger the magnitude of  $u_c$ , the greater the benefit of an additional dollar of income, and the more the agent will work when w goes up. The labor supply response to an increase in income is related to how much the marginal utility of consumption changes as income changes,  $u_{cc}$ . A large income effect implies that the agent is willing to increase effort significantly in order to recoup lost income, which means that the marginal utility of consumption rises quickly as income falls, i.e. the magnitude of  $u_{cc}$  is large. The connection between  $\gamma$  and the ratio of income and price elasticities follows from these two observations.

The reader may be puzzled that we can identify a unique value for  $\gamma$  by observing only labor supply. Since non-linear monotonic transformations of u(c, l) do not affect the choice of l, are there not infinitely many values of  $\gamma$  that could be associated with observed labor supply behavior? While this is true in general, the key is to observe that any non-linear transformation of u will change the value of  $u_{cl}$ . However, (5) was derived under the assumption that u(c, l) is additive, i.e.  $u_{cl} = 0$ .

It should be reiterated that  $\gamma$  is the curvature of utility over consumption.<sup>12</sup> This

<sup>&</sup>lt;sup>12</sup>This is the parameter estimated by most studies of choice under uncertainty, insofar as labor supply

coincides with the curvature of utility over wealth – the parameter that determines risk preferences in an expected utility model – when labor supply is fixed. When labor supply is variable, the curvature of utility over wealth is *lower* than  $\gamma$ . To see this, define indirect utility over unearned income, which is equivalent to wealth in the static model, as

$$v(y) = u(y + wl(y), l(y))$$

It is shown in the appendix that the curvature of utility over unearned income is

$$-\frac{v_{yy}}{v_y}y = \gamma \frac{y + wl\varepsilon_{l,y}}{y + wl} < \gamma$$

Hence, our bound on  $\gamma$  effectively serves as a bound on curvature over wealth as well when l is endogenous. Intuitively, when there are more margins over which an agent can adjust expenditure, the curvature of utility over wealth falls because the marginal dollar can be allocated more efficiently.

It is worth noting that frictions which prevent agents from reoptimizing fully in response to perturbations in w and y will not affect our inferences about  $\gamma$ . For example, Status-quo biases or institutional constraints that make small adjustments in labor supply difficult do not affect the estimate. This is because the method is biased only by factors that affect the price and income elasticities differently.<sup>13</sup>

The preceding derivation hinges on the assumption that utility is additive  $(u_{cl} = 0)$ . The next section relaxes this assumption.

is omitted from these analyses. More importantly, it is the relevant parameter for many analyses of risky behavior where shocks to labor supply are exogenous (e.g. optimal unemployment insurance).

<sup>&</sup>lt;sup>13</sup>To see this formally, note that an adjustment cost or status quo bias can be modeled as a cost  $k(l, l_0)$  of changing labor supply to l from  $l_0$ . Since we make no assumptions about the way in which labor supply l enters u(c, l), (5) still obtains. The reason is that curvature is identified from the ratio of income and price effects, and k > 0 attenuates both effects.

#### 3.3 Complementarity Between Labor and Consumption

When  $u_{cl} \neq 0$ , (3) becomes

$$\frac{\partial l/\partial y}{\partial l^c/\partial w} = w u_{cc}/u_c + u_{cl}/u_c \tag{6}$$

which, after some rearrangement, implies that

$$\gamma(y+wl) = (1+\frac{wl}{y})\frac{-\varepsilon_{l,y}}{\varepsilon_{l^c,w}} + (1+\frac{y}{wl})\varepsilon_{u_c,l}$$
(7)

where  $\varepsilon_{u_c,l}$  denotes the elasticity of the marginal utility of consumption with respect to labor supply. Note that labor supply data is sufficient to identify  $\gamma$  given *any* value of  $\varepsilon_{u_c,l}$  because no non-linear transformation of u will leave  $\varepsilon_{u_c,l}$  unchanged; in other words,  $\varepsilon_{u_c,l}$  pins down the cardinal normalization of the Bernoulli utility function.

It remains to estimate  $\varepsilon_{u_c,l}$ . Since it is a cardinal concept, this parameter must be estimated from choices under uncertainty. The most obvious method of estimating  $\varepsilon_{u_c,l}$ , which has been implemented in a set of empirical studies, is to exploit data on the consumption choices of individuals who anticipate exogenous shocks to labor supply. For example, if an agent plans to keep consumption fairly constant across states in which he is employed and unemployed,  $\varepsilon_{u_c,l}$  must be small.

To derive the relationship between consumption choices when employment is stochastic and  $\varepsilon_{u_c,l}$  formally, consider a world with two states. Agents supply  $l_1$  units of labor in state 1 and  $l_2$  units of labor in state 2. Suppose that the agent can trade consumption fairly between the two states by purchasing state-contingent commodities (e.g. using an insurance policy). He chooses consumption in the two states by maximizing expected utility

$$\max_{c_1, c_2} pu(c_1, l_1) + (1 - p)u(c_2, l_2)$$
  
s.t.  $pc_1 + (1 - p)c_2 = pwl_1 + (1 - p)wl_2 \equiv \overline{W}$ 

The agent's first-order condition for consumption is obtained by equating marginal utilities

across the two states:

$$u_c(c_1, l_1) = u_c(\overline{W} - c_1, l_2)$$

Now, suppose we observe data from the following experiment. Assume that the agent starts out supplying a constant  $l_1 = l_2 = \overline{l}$  units of labor in each state. Suppose there is a balanced-budget change in labor supply, increasing state 1 labor supply by  $\delta_1$  units while decreasing state 2 labor supply by  $\delta_2$  units to keep expected earnings fixed at  $\overline{W}$ . We can think of this as a decision to increase work effort in state 1 to compensate for (partial) unemployment in state 2.

Differentiating the first order condition with respect to  $l_1$  while holdings earnings fixed at  $\overline{W}$  yields the following identity:

$$\varepsilon_{u_c,l} = \gamma \varepsilon_{c_1,l_1}$$

Here,  $\varepsilon_{c_1,l_1}$  denotes the elasticity of consumption with respect to labor supply in state 1 (while labor supply in state 2 changes so that total income remains constant). Plugging this expression into (7) and solving gives an estimator for risk aversion in terms of  $\varepsilon_{c_1,l_1}$ :

$$\gamma = \left(1 + \frac{wl}{y}\right) \frac{-\varepsilon_{l,y}}{\varepsilon_{l^c,w}} / \left(1 - \left(1 + \frac{y}{wl}\right)\varepsilon_{c_1,l_1}\right) \tag{8}$$

This formula reduces to (5) when utility is additive in labor and consumption ( $\varepsilon_{c_1,l_1} = 0$ ). When consumption and labor are complements,  $\varepsilon_{c_1,l_1} > 0$ , and the true  $\gamma$  is higher relative to the estimate obtained when additive utility over c and l is assumed.<sup>14</sup> Hence, deriving an upper bound on  $\gamma$  requires an upper bound on the value of  $\varepsilon_{c_1,l_1}$ .

To obtain a bound on  $\varepsilon_{c_1,l_1}$ , it is helpful to derive a connection between  $\varepsilon_{c_1,l_1}$  and the consumption drop during (full) unemployment that has been estimated in consumption studies.

<sup>&</sup>lt;sup>14</sup>The sign of  $\varepsilon_{c_1,l_1}$  is theoretically ambiguous. If consumption requires time, as in Becker (1965), leisure and consumption are complements ( $\varepsilon_{c_1,l_1} < 0$ ). On the other hand, work-related expenses can make labor and consumption complementary ( $\varepsilon_{c_1,l_1} > 0$ ). I show below that regardless of the sign of  $\varepsilon_{c_1,l_1}$ , the estimates of  $\gamma$  are not very sensitive to its magnitude.

Normalize the agent's labor supply to 1 when working and 0 when unemployed. Since we are using a discrete change in labor supply to infer an elasticity at a specific (c, l) pair, we must choose a functional form for consumption in terms of labor supply. For simplicity, I assume a linear form:

$$c = a + bl$$

In this case,  $\varepsilon_{c_1,l_1}(l_1 = 1) = \frac{b}{a+b}$ , which is precisely the percentage drop in consumption from the employed to unemployed state. Note that estimates of the consumption drop during unemployment give an *upper* bound for the true  $\varepsilon_{c_1,l_1}$  if agents cannot smooth consumption across states to their desired level because of the inadequacy of available insurance policies, lack of savings, or credit constraints (i.e., incomplete markets). Put differently, unemployed agents are always free to consume less than they actually do when unemployed; it is only consuming more that may be impossible because of liquidity constraints. Hence, their actual consumption choices should, if anything, overstate the true degree of complementarity between c and l.

Most estimates of the consumption drop are quite small. In a recent study of unemployment in the US, Gruber (1997) estimates  $\frac{b}{a+b} = 0.068$  using data on food consumption from the PSID. Gruber (1998) obtains estimates of similar magnitudes when examining data for a broad set of consumption goods from the CEX.<sup>15</sup> Browning, Hansen, and Heckman [BHH] (1999) survey a number of other studies that estimate the consumption drop associated with an exogenous labor supply change. Several studies estimate that c and l are actually substitutes ( $\varepsilon_{c_1,l_1} < 0$ ). The largest estimate of complementarity cited by BHH is a 30% drop in consumption during unemployment. In view of this evidence, a plausible upper bound for  $\varepsilon_{c_1,l_1}$  is around 20% and an extreme value appears to be 30%. We will see in section 4 below that even this extreme value of  $\varepsilon_{c_1,l_1}$  has little effect on our bound for  $\gamma$ . The bound

<sup>&</sup>lt;sup>15</sup>Similarly, Browning and Crossley (2001) use data from the Canadian Out of Employment Panel to show that the consumption drop is not statistically distinguishable from zero for households that have positive liquid assets before their unemployment spell.

on risk aversion implied by labor supply behavior is thus insensitive to assumptions about the degree of complementarity between consumption and labor, which is fortunate given the lack of consensus in the literature about this parameter.

#### 3.4 Extensive Labor Supply Decisions

Until this point, we have made the assumption that agents are able to freely choose the number of hours they work. However, spikes in the hours distribution at 20 hours and 40 hours question the validity of this assumption. Many individuals appear to face the narrower choice of either working for a fixed number of hours or not working at all.<sup>16</sup> In this section, I consider a model where hours choices are restricted and show that an estimator for risk aversion can be derived using labor force participation data in this case.

To model extensive labor supply decisions, let us return to the static framework and assume that the agent makes a binary decision to work and supply 1 unit of labor or not work. As above, let y denote uncarned income and w the income earned by working. Returning temporarily to additive utility over consumption and leisure, redefine u(c) as the utility from consumption. Let  $\psi$  denote disutility of supplying 1 unit of labor. The agent chooses labor supply by solving

$$\max_{l \in \{0,1\}} u(y+wl) - \psi l$$

He works if his disutility of labor is less than the utility of an additional w units of consumption, i.e. if

$$\psi < \widehat{\psi}(y, w) \equiv u(y + w) - u(y)$$

<sup>&</sup>lt;sup>16</sup>The purpose of this section is to show that inferences about curvature can be made even when changes in labor supply are lumpy. The nature of the lumpiness itself (e.g. whether there are restrictions on hours worked in a week or weeks worked in a year) is not important.

Let us model the heterogeneity of disutility of labor in the economy by a smooth density  $f(\psi)$ . Then the fraction of workers who participate in the labor force is

$$\theta(y,w) = \int_0^{\widehat{\psi}(y,w)} f(\psi) d\psi \tag{9}$$

It follows that

$$-\frac{\partial\theta/\partial y}{\partial\theta/\partial w} = \frac{u_c(y) - u_c(y+w)}{u_c(y+w)}$$
(10)

This expression shows that the percent change in marginal utility of wealth from y to y + w is equal to the ratio of the income and wage effects on labor supply. In the intensive labor supply model, we could compute  $\gamma(c)$  at any level c without making any functional form assumptions because we could observe how marginal utility changes for small changes in income. In a world with extensive labor supply decisions, we observe only the change in marginal utility between y and y + w. Consequently, we need to make a functional form assumption for u(c) to translate the change in marginal utilities into a coefficient of relative risk aversion. I assume CRRA utility:<sup>17</sup>

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

Under this assumption, (10) implies

$$-\frac{\partial\theta/\partial y}{\partial\theta/\partial w} = \frac{y^{-\gamma} - (y+w)^{-\gamma}}{(y+w)^{-\gamma}}$$

Solving for  $\gamma$  yields

$$\gamma = \frac{\log[1 - \frac{\varepsilon_{\theta, y}}{\varepsilon_{\theta, w}} \frac{w}{y}]}{\log[1 + \frac{w}{y}]} \tag{11}$$

Finally, a model of unemployment analogous to that above can be used to derive an

<sup>&</sup>lt;sup>17</sup>If  $\gamma(c)$  actually varies with c, this method yields the best constant- $\gamma$  fit of the data, which can be loosely interpreted as the average  $\gamma(c)$  in the region  $c \in [y, y + w]$ .

estimator for  $\gamma$  when utility is not additive:

$$\gamma = \frac{\log[1 - \frac{\varepsilon_{\theta, y}}{\varepsilon_{\theta, w}} \frac{w}{y}]}{\log[(1 - \frac{\Delta c}{c})(1 + \frac{w}{y})]}$$
(12)

where  $\frac{\Delta c}{c}$  denotes the consumption drop associated with unemployment.

This concludes the analysis of risk aversion and labor supply in the static labor-leisure choice model. The next subsection extends the preceding results to a dynamic setting.

### 3.5 Dynamics: Distinguishing EIS and $\gamma$

The fact that we have derived an estimator for risk aversion in a static model suggests that the estimator fundamentally restricts preferences over states (risk aversion) rather than preferences over time (intertemporal substitution). Establishing this point formally requires analysis of a dynamic model where the EIS and  $\gamma$  are independent parameters. This subsection considers a standard dynamic life-cycle model and shows that (5) estimates "effective risk aversion" – the curvature of the value function over wealth – leaving the EIS unidentified.

Consider a T period life-cycle model, and let consumption in each period be denoted by  $c_t$ and labor supply by  $l_t$ . Note that the restrictive assumption that consumption equals income implicit in the static analysis is dropped in this model. Allow utility to be completely nonseparable over time, leaving parameters governing intertemporal substitution unrestricted.<sup>18</sup> Let  $w\theta_t$  denote the wage in period t and y uncarned income (period 0 wealth). In the standard terminology of MaCurdy (1981), a change in  $\theta_t$  is a transitory parametric wage change, while changes in w are *permanent* wage changes, i.e. shifts in the *entire profile* of wages. We will be interested in the latter effect: the relationship between l and w.

For simplicity, I first consider the case where utility is separable in c and l. Complementarity is addressed below. Exploiting separability, let  $u(c_1, ..., c_T)$  denote utility over

<sup>&</sup>lt;sup>18</sup>This preference structure nests commonly analyzed cases such as habit formation.

consumption and  $\psi(l_1, ..., l_T)$  denote disutility over labor.

The agent solves

$$\max_{c_t, l_t} u(c_1, ..., c_T) - \psi(l_1, ..., l_T)$$
  
s.t.  $c_1 + ... + c_T = y + w(\theta_1 l_1 + ... + \theta_T l_T)$ 

This problem can be re-written as a two-stage maximization problem as follows:

$$\max_{c,L} v(c) - \Psi(L) \text{ s.t. } c = y + wL$$
where  $v(c) = \max_{c_t} u(c_1, ..., c_T) | c_1 + ... + c_T = c$ 
and
$$\Psi(L) = \min_{l_t} \psi(l_1, ..., l_T) | \theta_1 l_1 + ... + \theta_T l_T = L$$
(13)

Here, c is the composite commodity that represents total consumption, L is the composite commodity that represents total labor, v is indirect utility over the composite consumption good, and  $\Psi$  is indirect disutility over composite labor. Re-writing the problem in this way is essentially an application of the composite commodity theorem. Note that since the problem can be re-written in terms of indirect utilities even if the consumption path is uncertain, introducing shocks into this model will not affect the results below.

The agent's utility maximization problem over c and L in (13) has precisely the same form as the static labor-leisure utility maximization problem given in section 3.2. The derivation there thus yields an expression for the curvature of v:

$$\gamma_v \equiv \frac{-v_{cc}}{v_c}c = -(1 + \frac{wL}{y})\frac{\varepsilon_{L,y}}{\varepsilon_{L^c,w}}(y,w)$$
(14)

This formula is the same as that in (5), with two exceptions. First, the risk aversion parameter  $\gamma_v$  is the curvature over indirect utility over the *composite* consumption good c,

rather than consumption in any single period.<sup>19</sup> This is intuitive: In a dynamic model, the curvature of v is "effective risk aversion," the parameter that matters for preferences over gambles that are immediately resolved, i.e. wealth gambles (assuming L is fixed).<sup>20</sup> The second difference is that the elasticities in the formula are for lifetime labor supply, L, rather than labor in any single period. This is because single-period labor supply in the static model is analogous to total labor supply in a life-cycle model. Consequently, inferring risk aversion in the general dynamic model requires knowledge of labor supply responses over the lifetime to permanent shifts in wage profiles, i.e. changes in w.<sup>21</sup> Such estimates are available from the dynamic labor supply literature (e.g. MaCurdy (1981)), as discussed in section 4 below.

Complementarity between consumption and labor can be addressed at a mathematical level as in the static analysis, using estimates of the drop in consumption when agents face exogenous changes in L across states. An important conceptual difference in the dynamic setting is that it is necessary to know how c covaries with L, where c and L are composite commodities that represent *total* consumption and labor over the lifetime. In practice, most estimates of complementarity are from transitory fluctuations in l rather than longterm shocks to L.<sup>22</sup> But evidence on the consumption response to longer-term shocks such as disability or illness suggests that these responses are only slightly larger than the responses to short-term shocks (Cochrane 1991). Moreover, a large body of evidence suggests that the expenditure drop at retirement is no larger than 20% (see e.g., Aguiar and Hurst (2004)). If

<sup>&</sup>lt;sup>19</sup>The  $\gamma_v$  parameter coincides with the curvature of the flow (felicity) utility function,  $\gamma$ , when preferences are time-separable.

<sup>&</sup>lt;sup>20</sup>Risk aversion over wealth gambles is lower than  $\gamma_v$  if L is endogenous: see section 3.2 and the appendix. <sup>21</sup>However, if utility is time-separable, our main bounding result goes through even if we only know that the labor supply curve in a single period is upward sloping in w. This is because when utility is time-separable,  $\frac{\partial l_t}{\partial w} > 0$  for any t implies  $\frac{\partial l_t}{\partial w} > 0 \ \forall t$ , so  $\frac{\partial L}{\partial w} > 0$ . <sup>22</sup>Put differently, in a dynamic setting, if consumption in period t is perimitted to be complementary with

<sup>&</sup>lt;sup>22</sup>Put differently, in a dynamic setting, if consumption in period t is perimitted to be complementary with labor in another period t', then we must bound not only complementarity between c and l in a given period (as in the static model), but also between c and l across periods. Insofar as within-period cross-partials are larger in magnitude than cross-period cross-partials, evidence on complementarity from transitory shocks continues to suffice.

one interprets this drop as an upper bound on complementarity between c and L, it would be consistent with our bound on  $\varepsilon_{c_1,l_1}$  in the static case. Hence, despite limited direct evidence on what happens to c when there is a shock to L, it is difficult to imagine that complementarity between consumption and labor in the long run is significantly larger than the bounds placed on this parameter in section 3.3.

An important corollary of the preceding analysis is that the intertemporal elasticity of substitution remains unrestricted while effective risk aversion is pinned down by labor supply behavior when utility is time non-separable. It may be helpful to emphasize this point by considering Kreps-Porteus utility, another commonly used method of severing the link between EIS and risk aversion.<sup>23</sup> Note that the Kreps-Porteus recursive utility specification is simply a special case of the general time non-separable class of utility functions analyzed above when agents are solving a labor-leisure choice problem without uncertainty. The comparative statics of labor supply in the Kreps-Porteus model necessarily have the same form as those in a general time non-separable model. An argument formally identical to that given above for time non-separable utility therefore establishes that the estimator in (5) yields the curvature of the value function over wealth for Kreps-Porteus utility. Hence, our estimator identifies risk aversion over immediately-resolved (wealth) gambles for agents with Kreps-Porteus utility as well.

The distinction between the EIS and  $\gamma$  clarifies the connection between this paper and the work by King, Plosser and Rebelo (1988) and Basu and Kimball (2002) on labor supply and intertemporal substitution discussed in section 2. Consider the case where utility is additive over c and l. Here, the Basu and Kimball result is that *time* separability is inconsistent with  $\varepsilon_{l,w} > 0$  and low EIS. In contrast, this paper shows that *state* separability (i.e., expected utility theory) is inconsistent with  $\varepsilon_{l,w} > 0$  and high  $\gamma$ . The mathematics underlying these two results is very similar, except that "time periods" have been relabeled as "states" in this

<sup>&</sup>lt;sup>23</sup>The Kreps-Porteus specification is an expected-utility model for immediately resolved risks, and therefore falls within the general framework analyzed in this paper.

paper. Conceptually, however, the results are quite different for two reasons. First, they address two aspects of preferences – intertemporal substitution and risk aversion – which are empirically and intuitively distinct from each other, as noted above. The result here places no restrictions on the EIS; conversely, the results in KPR and Basu and Kimball place no restrictions on risk aversion because they do not analyze a model with uncertainty. Second, the solutions proposed by Basu and Kimball to resolve their "EIS puzzle" would not break the link between risk aversion and labor supply behavior demonstrated here.

This concludes the theoretical portion of the paper. The next section implements the formulas derived above using existing estimates of labor supply elasticities.

## 4 Empirical Implementation

#### 4.1 A Calibration Argument

A large body of evidence points to a basic fact about labor supply: Individuals work more when they get paid more to do so. An rich and diverse set of empirical studies of labor supply, which are described in detail below, uniformly find that uncompensated labor supply curves are weakly upward sloping ( $\varepsilon_{l,w} \geq 0$ ).<sup>24</sup> The fact that  $\varepsilon_{l,w} \geq 0$  has strong implications for risk aversion. To see this, first rewrite the Slutsky equation given in (2) in terms of elasticities:

$$\varepsilon_{l^c,w} = \varepsilon_{l,w} - \frac{lw}{y}\varepsilon_{l,y}$$

Given  $\varepsilon_{l,w} \ge 0$ , this formula yields a lower bound on  $\varepsilon_{l^c,w}$  of  $-\frac{lw}{y}\varepsilon_{l,y}$ . If  $u_{cl} = 0$ , (5) can be applied to obtain a bound on  $\gamma$ :

$$\gamma < 1 + \frac{y}{wl}$$

<sup>&</sup>lt;sup>24</sup>This claim is further substantiated by a recent survey of 134 labor and public economists at 40 leading research institutions by Fuchs, Krueger, and Poterba (1998). They found that the vast majority of these experts believe that the best estimate of the uncompensated wage elasticity is weakly positive.

The ratio of unearned income to earned income varies across the population, but in the aggregate it equals the ratio of capital income to labor income, which is  $\frac{1}{2}$  in the U.S. This places an upper bound of  $\gamma = 1.5$  for a representative agent whose utility is an income-weighted average of individual utilities. Since capital income is highly concentrated, if we are interested in the curvature of an equally-weighted average of utilities, the relevant value of  $\frac{y}{wl}$  is much lower; a reasonable estimate is  $\frac{1}{4}$ .<sup>25</sup> In this case,  $\varepsilon_{l,w} > 0$  implies  $\gamma < 1.25$  for the representative agent when utility is additive in c and l. This bound also applies to the curvature of the value function in dynamic models based on preceding arguments.

When additivity is relaxed, the upper bound becomes

$$\gamma < (1 + \frac{y}{wl}) / (1 - (1 + \frac{y}{wl})\varepsilon_{c_1, l_1})$$
(15)

As shown in section 3.3, a fairly loose upper bound for  $\varepsilon_{c_1,l_1}$  is 0.2, which implies that  $\gamma < 1.25/(1-0.25) = 1.66$ . Even in the extreme-case scenario of a 30% drop in consumption associated with unemployment – a value that Browning, Hansen, and Heckman (1999) argue is an upward-biased estimate of the true degree of complementarity between c and l – the estimate of  $\gamma$  rises to only 2. Hence, with relatively modest assumptions on complementarity between c and l, the fact that the uncompensated labor supply curve is upward sloping requires that risk aversion must be quite low in an expected utility model.

#### 4.2 Imputing $\gamma$ from Labor Supply Elasticities

This subsection gives a more complete picture of the values of  $\gamma$  implied by studies of labor supply that estimate income and price elasticities. The traditional labor supply literature, summarized by Pencavel (1986) and Blundell and MaCurdy (1999), defines "labor supply" as hours worked or work participation. These studies estimate labor supply responses to

<sup>&</sup>lt;sup>25</sup>Tabulations by the US Census Bureau (1999, Table E) adjusted for the progressivity of the income tax indicate that  $\frac{y}{wl} \approx \frac{1}{4}$  for the median family in the U.S., which has an income of approximately \$40,000.

permanent changes in wages and unearned income using variation such as tax changes, crosssectional differences, or lottery winnings.<sup>26</sup> This literature can be broken into two strands. The "static" or "reduced-form" literature estimates wage and income elasticities that yield the relevant parameters for a static labor-leisure choice model. These studies employ crosssectional variation or changes in tax rates as a means of creating permanent, unanticipated variations in budget sets. The credibility of static estimates has been questioned on the grounds that cross-sectional variation introduces significant omitted variable bias and individuals are forward-looking in practice, which calls for formal modelling of the dynamics involved. The "life cycle" or "structural" literature, pioneered by MaCurdy (1981) and others, addresses this problem by explicitly modelling dynamic labor supply and consumption choices and estimating labor supply responses to permanent shifts in wage profiles and unearned income. I consider studies from both strands of the literature and show that the implications for risk aversion are generally quite similar.

A more recent literature on labor supply, starting with Feldstein (1995), emphasizes that hours worked is only one component of labor supply and that other margins such as effort or training might adjust as well. When a multi-dimensional definition of labor supply is incorporated into the models analyzed above, (5) still obtains except that the elasticity ratio  $\frac{\varepsilon_{Ly}}{\varepsilon_{L^c,u^-}}$  is replaced by  $\frac{\varepsilon_{LI,y}}{\varepsilon_{LI^c,1-\tau}}$ , where LI is labor income and  $1 - \tau$  the net-of-tax rate. This result follows the lines of Feldstein (1999), who shows that the elasticity of taxable labor income with respect to the net-of-tax rate captures all margins on which taxable income can be adjusted. The reason is that the relative prices of each mechanism of adjustment remain fixed when tax rates change. Hence, recent estimates of the response of earned income to changes in unearned income and tax rates can also be used to make inferences about  $\gamma$ .

Table 1 presents a set of income and wage elasticities for three definitions of labor supply:

<sup>&</sup>lt;sup>26</sup>These studies estimate labor supply elasticities for a group of individuals, which might raise concerns about aggregation if there is unobserved heterogeneity in risk preferences across agents. However, it can be proved that the estimator for  $\gamma$  derived in the representative agent model remains meaningful in this case; in particular, it yields a weighted average of risk aversion in the group.

(A) Hours worked, (B) Participation, and (C) Earned income.<sup>27</sup> To get a sense of the plausible range for  $\gamma$  that is consistent with labor supply behavior, I include elasticity estimates for a wide range of groups, such as prime age males, married women, retired individuals, and low income families. The inclusion of a diverse set of studies yields a substantial amount of variation in the elasticities, ranging from 0.035 to 1.0 for the compensated wage elasticity and -0.3 to -0.008 for the income elasticity. In general, elasticity estimates for groups who are not as attached to the labor force (married women and older individuals) tend to be higher than the elasticity estimates for groups with greater labor force attachment (prime age males).

Column (6) of Table 1 reports estimates of  $\gamma$  at the average value of  $\frac{y}{w}$  and l in each study under the additive utility assumption. Note that the mean values of  $\frac{y}{wl}$  vary widely across the studies – e.g., married women's unearned income equals at least their husband's income, which is generally larger than their own earned income. These variations are taken into account in implementing (5). In addition, following Hausman (1985), y is defined as "virtual" unearned income to account for the progressivity of the tax system.<sup>28</sup>

Strikingly, of the 29 sets of estimates used to construct this table, only one study implies a value of  $\gamma$  above our calibration bound of 1.25 under the  $u_{cl} = 0$  assumption.<sup>29</sup> The overall (unweighted) mean estimate of  $\gamma$  across the twenty-nine studies is  $\gamma = 0.94$ , implying that a 10% increase in consumption reduces the marginal utility of consumption by 9.4%. This similarity of the estimates of  $\gamma$  despite the use of different methodologies, definitions of labor supply, and groups of the population may be surprising at first. But it is quite intuitive in view of the calibration argument given above – nearly all the studies of labor supply find

<sup>&</sup>lt;sup>27</sup>In part (C) of the table, it is important to distinguish between *taxable* labor income and total labor income. In the calculation of  $\gamma$ , we are interested in changes in *total* earned income, irrespective of the form of compensation. The measure of income used in the studies reported in the table is adjusted gross income (AGI), and may not capture forms of compensation that are not reported on tax returns such as office perks.

<sup>&</sup>lt;sup>28</sup>Since the earned income estimates combine different studies, they are evaluated at  $\frac{y}{wl} = \frac{1}{4}$ , which reflects the median value of unearned to earned income in the US (see above).

<sup>&</sup>lt;sup>29</sup>The estimates reported for the Blundell and MaCurdy (1999) are an average of the estimates of 20 studies surveyed in their paper, all of which imply  $\gamma < 1.25$ .

 $\varepsilon_{l,w} \ge 0$ , which we know places a tight upper bound on  $\gamma$ .<sup>30</sup>

Column (7) of Table 1 reports estimates of  $\gamma$  that account for the degree of complementarity implied by the upper-bound on the unemployment consumption drop computed in section 3.3 ( $\frac{\Delta c}{c} = 20\%$ ). This adjustment increases the average estimate of  $\gamma$  to 1.33. In view of the calibration arguments, it should not be surprising that taking complementarity between labor and consumption into account does not raise the estimates of  $\gamma$  significantly.

# 5 Conclusion

Empirical studies of labor supply uniformly find that the uncompensated wage elasticity of labor supply is positive: Individuals work more (or at least not much less) when their wages rise. This paper has shown that this fact reveals considerable information about the rate at which the marginal utility of wealth diminishes, which determines risk aversion in expected utility models. To be consistent with an upward-sloping labor supply curve, the coefficient of relative risk aversion must be less than 1.25 in the case where consumption and leisure are not complementary. This bound rises to at most 1.66 over the range of plausible perturbations of complementarity between consumption and labor. In a dynamic life-cycle model with a fully general time non-separable utility, effective risk aversion – the parameter that determines risk preferences over wealth gambles – is bounded by the same values.

The most important message of this paper is that those who work in the canonical onegood expected utility framework must be content with a relatively low level of risk aversion in view of well-established facts about labor supply behavior. This means that either true risk preferences cannot explained by a standard expected utility model or that risk aversion is truly low. It should be noted, however, that many economists think that risk aversion is high and that uncompensated wage elasticities are positive. For instance, in

<sup>&</sup>lt;sup>30</sup>Pencavel (1986), Blundell and MaCurdy (1999), and Gruber and Saez (2000) summarize more than sixty other studies with an array of methodologies, nearly all of which find  $\varepsilon_{l,w} \ge 0$  and therefore imply  $\gamma < 1.25$  as well.

recommending parameters for neoclassical macro models on the basis of micro-econometric studies, Browning, Hansen, and Heckman (1999) state that (1) the uncompensated wage elasticity of labor supply is (weakly) positive, (2) the elasticity of consumption with respect to labor, holding wealth fixed, is at most 0.3 (i.e., the degree of complementarity between consumption and labor is not extremely large), and (3) the best estimate of  $\gamma$  is around 4. These three recommendations are not mutually consistent with the canonical expected utility theory of risk preferences.

How can high risk aversion be reconciled with an upward-sloping labor supply curve? What is needed is a dimension to risk aversion beyond diminishing marginal utility. Several existing non-expected utility theories that alter probability weights on outcomes have such a dimension. Examples include Chew and MacCrimmon's (1979) weighted expected utility, Quiggin's (1982) rank-dependent utility, and Tversky and Kahneman's (1992) cumulative prospect theory. Abstractly, these theories change an agent's utility valuation of a lottery  $x = (p_1, x_1; ...; p_n, x_n)$  from  $\sum p_i u(x_i)$  to  $\sum g(p_i)u(x_i)$  where g is a weighting function that, appropriately parametrized, can introduce an additional source of risk aversion into the model.<sup>31</sup> Testing the performance of such theories under the constraint that u has a low degree of curvature could be a fruitful course for further research.

 $<sup>^{31}</sup>$ See Starmer (2000) and references therein for a detailed review of these and other theories of risk preferences.

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### Appendix: Curvature of utility over unearned income

The Envelope theorem implies that

$$v_y(y) = u_c(c(y), l(y))$$

and it follows that

$$\gamma^y = -\frac{v_{yy}}{v_y}y = -\frac{u_{cc}}{u_c}\frac{\partial c}{\partial y}y$$

Recognizing that  $\partial c/\partial y = 1 + w \partial l/\partial y$ , we obtain

$$\gamma^y = \gamma \varepsilon_{c,y}$$

where  $\varepsilon_{c,y}$  denotes the income elasticity of consumption. Finally, observe that

$$\varepsilon_{c,y} = \frac{y + wl\varepsilon_{l,y}}{y + wl}$$

where  $\varepsilon_{l,y}$  is the income elasticity of labor supply. Since  $\varepsilon_{c,y} < 1$ , it follows that  $\gamma^y < \gamma$ .

Study (1)	Sample (2)	Identification (3)	Income <u>Elasticity</u> (4)	Compensated <u>Wage Elasticity<sup>a</sup></u> (5)	γ <u>Additive</u> (6)	γ <u>Δc/c=0.2</u> (7)
A. Hours						
MaCurdy (1983)	Married Men	Panel	-0.020	0.130	0.77	1.03
Blundell and MaCurdy (1999) <sup>b</sup>	Men	Various	-0.120	0.567	1.06	1.41
MaCurdy et al (1990)	Married Men	Cross Section	-0.010	0.035	1.47	1.96
Eissa and Hoynes (1998)	Married Men, Inc < 30K	EITC Expansions	-0.030	0.192	0.88	1.17
	Married Women, Inc < 30K	EITC Expansions	-0.040	0.088	0.64	2.12
Friedberg (2000)	Older Men (63-71)	Soc. Sec. Earnings Test	-0.297	0.545	0.93	1.80
Average B. Participation <sup>c</sup>					1.03	1.45
			0.000	0.000		0.50
Eissa and Hoynes (1998)	Married Men, Inc < 30K	EITC Expansions	-0.008	0.033	0.44	0.50
A. 10 70 70	Married Women, Inc < 30K	EITC Expansions	-0.038	0.288	0.15	0.45
Average					0.29	0.48
C. Earned Income <sup>d</sup>						
Imbens et al (2001)	Lottery Players in MA	Lottery Winnings	-0.110			
Feldstein (1995)	Married, Inc > 30K	TRA 1986		1.040	0.53	0.71
Auten and Carroll (1997)	Single and Married, Inc>15K	TRA 1986		0.660	0.83	1.11
Average					0.45	0.61
Overall Average					0.94	1.33

 TABLE 1

 Labor Supply Elasticities and Implied Coefficients of Relative Risk Aversion

<sup>a</sup>In part (C), this column gives the elasticity of earned income with respect to the net-of-tax rate

<sup>b</sup>This row uses an unweighted average of the 20 elasticities reported in Blundell and MaCurdy (1999) and assumes y/wl=1/4

<sup>c</sup>Participation elasticities assume CRRA utility

<sup>d</sup>Since studies on earned income do not estimate income elasticties, I use the Imbens et. al. estimate in each case