Why Do People Work so Hard?

Paul Scanlon*

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Abstract

Conventional economic theory, coupled with standard preference parameters predict that labor supply should fall markedly over time. However, this contradicts the empirical reality that labor supply rises at the very onset of economic development, subsequently tends to fall, and then stabilizes. This paper addresses this pattern of labor supply over time, and explains it via the proliferation of new products coming on stream. This motivates consumers to continue to supply labor, so as avail of the new consumption opportunities. This induced labor supply, in turn, provides a market for new products, which itself is a driving incentive to innovate. On the balanced growth path, the two countervailing forces of a rising real wage and new consumption opportunities sustain a constant labor supply.

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*Department of Economics, UC Berkeley. Email: pscanlon@econ.berkeley.edu. This work is incomplete.
1 Introduction

A striking feature of time series data, indeed a stylized fact, is the relative stability of hours worked over time. Despite large differences in incomes across nations and time, hours worked per person betray remarkable stability. Put simply, the variation in hours worked bears little resemblance to large differences in income. Furthermore, a notable feature of time series data is that hours worked rise at the very onset of development, subsequently fall, and then remain fairly stable thereafter. This subsequent stability of hours work is, in fact, often listed along with the so-called “great ratios” as one of the necessary features of a balanced growth path; a necessary feature of any convincing model of long-run growth.

The relative constancy of hours work -the proverbial “40 hour work week”-, however, is not especially obvious. In standard micro theory, leisure is a normal good whose consumption should increase on a par with income levels. It’s hardly surprising, then, that various economists, most notably Keynes, predicted vastly shorter work weeks by this stage, coincident with rising income. Historical experience, however, has not borne out this prediction. In the US, for instance, hours worked per capita have remained roughly stable, and have even risen a little, in the past fifty years. This is despite a significant increase in labor real GDP per capita, and a marked fall in the price of basic consumption goods. In the Handbook of Macroeconomics, King and Rebelo remark: “the relative constancy of hours per capita is remarkable given the rise in real wages that accompanies economic growth. Over our sample period [since 1950s], the real wage....grew at 1.76% per year, but there is little evidence of a trend in hours worked per person”. Francis and Ramey (2004) go so far as to say that, over the twentieth century, “...there has been no long-run trend in hours worked per capita properly measured”. Indeed, according to estimates by Schor, labor hours today are approximately the same as they were eight centuries ago; and this is despite a vast increase in income levels over that period. It begs the question: if people are getting rich, why are they working as hard?

Cross sectional data betray another striking fact. Across developed nations, labor hours per capita have tended to plateau at different levels. The differences are especially pronounced among the US and Europe, as noted most prominently by Prescott. He notes that Americans now work 50 percent

\footnote{A point may soon be reached, much sooner perhaps than we are all aware of, when these needs are satisfied in the sense that we prefer to devote our further energies to the noneconomic”, from “Economic Possibilities For Our Grandchildren”.

\footnote{According to Mankiw(2006) real wages in the United States have increased by about 2% a year for the past fifty years, about the same amount as real GDP per worker.}
more than do the Germans, French, and Italians. Specifically, the average American person now works 25 hours a week; the French 18; and the Italians around 16. These are not trivial numbers. According to Blanchard, labor hours differentials now explain the bulk of the average 30% income differential among Europe and the US. Clearly, then, labor supply differentials are of first order importance, having large implications for welfare and growth.

Surprisingly, economic theory offers no coherent explanation for the aforesaid patterns of labor supply. While there is a vast literature on productivity and capital differentials across nations, the topic of labor hours, by contrast, has received little attention. This is surprising. Labor not only takes the bulk of national income, but such concepts as human capital (embodied in labor) and labor-augmenting technical change are now commonplace in economic theory. Also, it is ultimately labor that is the conduit to growth. All innovations and economic progress fundamentally stems from labor. In this sense, a theory of growth is seems incomplete without a theory of labor supply. This paper addresses these issues in labor supply, and seeks to actually explain its relative stability over time, together with cross sectional differentials.

Briefly, I endogenize labor supply as a function of new consumption opportunities. In the basic theory of labor/leisure choice, labor is supplied to afford new consumption opportunities. The marginal utility of consumption if therefore central to the labor supply decision. Based on widely accepted parameter values, a rising real wage should induce diminishing marginal utility, inducing a secular fall in labor supply. The question is, why has this not occurred? This paper maintains that new innovations are causing the marginal utility of the real wage to rise, offsetting the usual income effect, and sustaining constant labor supply. This is just analogous to the way expanding technology offsets diminishing returns to capital in the basic Solow model.

To give the main idea, consider a one-period framework, where the real wage is $\omega$ and the utility function (with the usual properties, $u' > 0, u'' < 0$) denoted $u(c) = u(\omega h)$. As $\omega$ rises, marginal utility of consumption falls which will tend to reduce labor supply over time. Now suppose we have $n$ goods with all goods separable, so utility is $nu(\frac{\omega h}{n})$. Now, if $n$ is rising too, diminishing marginal utility is countered, and a stable labor supply can be maintained. Moreover, the earnings accruing to that constant labor supply then act as a market for new innovations, which effectively keeps the entire dynamic on track. Regarding cross sectional differentials, real wages and variety differ markedly across nations, which can afford us an insight into disparate labor supplies. This analysis also has implications for the risk free rate puzzle. In this framework, people will still want to save, despite the apparent presence of diminishing marginal utility (as represented by coefficient of risk aversion).
Consider the fact that innovations counter diminishing marginal utility. Now, the prospect of a higher marginal utility in the future acts as an incentive to save, thus maintaining a low interest rate. In particular, the strong incentive to smooth consumption and borrow is attenuated.

In section 1, I briefly discuss some background information. In section 2, I present a partial equilibrium model, which presents the basic insights. Section 3 presents a general equilibrium model which yields additional insights, particularly relevant for long run growth. In section 4, I present some applications of the model. Section 5 presents some empirical evidence. Section 6 concludes.

2 Background Information

2.1 Labor Supply and Macro Modelling

In most macroeconomic models, labor is simply exogenous. The standard practice is to simply assume a constant labor supply and invoke a utility function - typically Cobb Douglas- that produces stable labor hours. This function is convenient since the income and substitution effects of rising wage rates offset, mathematically yielding constant labor supply. However, there is no explanation as such; this formulation is, rather, neat way of ensuring a balanced growth path with stable hours. Moreover, this formulation cannot explain the patterns of labor supply as an economy develops, and is little help in explaining cross-country differentials. It dictates that labor hours are always constant.

Arguably, this is a reasonable assumption for neoclassical models, which emphasize capital accumulation. But when one considers the centrality of labor supply to the new growth theory, the assumed exogeneity of labor is surprising. In these models, it is labor supply that underlies growth; a higher labor supply engenders greater markets and specialization, thereby enhancing productivity and growth. Nonetheless, in these models, most notably that of P. Romer, labor supply, the factor underpinning growth, is exogenous. Likewise for the endogenous growth model of Lucas. He attributes differential growth rates to differences in human capital accumulation, or, really, the quality of the labor supply.

\[3^{3}\text{In the 700 page text, “Endogenous Growth Theory” by Aghion and Howitt, there is no reference to the issue of endogenizing labor supply; throughout the text, labor supply is simply exogenous.}\]
2.2 Some Theory

It turns out that the coefficient of relative risk aversion, $\sigma$, is central to any analysis of labor supply.\textsuperscript{4} Namely, it reveals how quickly the marginal utility of consumption declines, as consumption increases. And work effort is ultimately supplied so as to provide for consumption. The marginal utility of consumption is thus key; how the consumer values further units of consumption is fundamental to any analysis of labor supply. If marginal utility of consumption declines sharply as consumption rises (that is, $\sigma$ is high), then, \textit{ceteris paribus}, labor supply will fall markedly as the real wage rises. Why supply more labor for such lower utility gains? More formally, if $\sigma$ exceeds unity, labor supply will fall in the face of a rising real wage. The reason being, with a rising real wage, a given amount of work yields ever more consumption, but those subsequent units of consumption are valued less and less. If $\sigma > 1$, the income effect of a rising real wage will offset the substitution effect, with labor supply falling to zero over time. And conversely. Only for the knife-edge case of logarithmic utility (i.e. $\sigma = 1$) will the two effects just offset.

A important question for examining labor supply trends over time, then, is the magnitude of $\sigma$. In most growth models, this is assumed to be one -Cobb Douglas utility- so as to permit a balanced growth path. In this way, the countervailing income and substitution effects of a rising real wage just offset. Primarily, though, this utility formulation is chosen so as to yield a constant labor supply.

What does the empirical evidence say?

An abundance of empirical evidence indicates that $\sigma$ actually does, in fact, exceed unity. The most striking evidence is from empirical testing of consumption capital asset pricing model, where estimates of sigma exceed 40, and lately (from 1979 to 2003) even as high as 169 as noted by D.Romer\textsuperscript{5} While this is extreme (reasonable estimates, according to Prescott, don’t exceed ten) other information corroborates a relatively high value of sigma. Barro, for instance\textsuperscript{6}, claims that this is the only way to reconcile data on savings rates and income levels, which, he claims rise upon economic development. On empirical estimates, he remarks \textit{“the usual view in the finance literature is that sigma is between 2 and 5”}. Recently, Weitzman\textsuperscript{7} claims \textit{“there seems to be some rough agreement within the economics profession as a whole that an array of evidence from a variety of sources suggests that it is somewhere}

\textsuperscript{4}Strictly speaking, I should refer to the \textit{elasticity of intertemporal substitution.}, since I don’t deal with the concept of risk per se. However, with conventional power utility, one is the inverse of the other (abstracting from the Epstein/Zin separation.) Hereafter, for convenience, I’ll continue to refer to “risk aversion”.

\textsuperscript{5}\textit{Advanced Macroeconomics}, third edition.

\textsuperscript{6}“Rare Events and the Equity Premium”, 2005

\textsuperscript{7}“Risk, Uncertainty and Asset Pricing Antipuzzles”, 2006
between about one and about three. Robert Lucas, in his classic paper on consumption variability and the business cycle, notes that “a value of unity means logarithmic preferences; people appear more risk averse than this”.\(^8\) John Campbell (2001) notes that “direct evidence on the elasticity of intertemporal substitution suggests that it is fairly low, certainly well below one [i.e. \(\sigma > 1\)].” Similarly, others such as Robert Hall (1988) note that consumption growth is very insensitive to changes in interest rates over time, indicating \(\sigma\) is very low; his estimates of the elasticity of intertemporal substitution \([\frac{1}{\sigma}]\) are not statistically different from zero.\(^9\) In D. Romer’s text, he summarizes much empirical evidence and writes that it “suggests that the intertemporal elasticity of substitution is low”. Blanchard and Fisher claim “the bulk of the empirical evidence suggests a relatively low value of the elasticity of substitution. In short, then, a substantial body of evidence suggests that \(\sigma > 1\). Yet, in most macroeconomic models encapsulating labor supply, \(\sigma\) is simply assumed to be one; such a “knife edge” and apparently unrealistic assumption is an undesirable feature of such models. But its use is understandable. Moving away from \(\sigma = 1\) yields growth models with secularly falling or rising labor supply, and so no balanced growth paths - an even more undesirable feature.\(^10\)

Some authors concede this point, and invoke another utility function, the so-called King, Plosser and Rebelo utility function. This function can indeed can reconcile higher values of risk aversion and stable labor supply. However, underpinning this solution is another strong assumption. Consumption and leisure are nonseparable and are complements; that is, the marginal utility of consumption is increasing in work hours. An example of this would be where, after a hard day’s work, driving home in an fancy car yields more utility. According to this view, a constant labor supply is sustained since people work so as to increase the marginal utility of consumption.\(^11\) While this has merit, it’s equally plausible, though, that the marginal utility of consumption is decreasing in work hours. Surely, long hours of labor mean less time to enjoy consumption goods - the case of a cruise or vacation being obvious examples. Indeed, this prompts Barro and Xala I Martin, in their text of growth,\(^12\) to remark that this case is “introspectively more plausible”. Also, in Becker’s theory of the household, a notable feature of his model is that consumption is time-consuming; that consumption and leisure

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\(^8\) Models of Business Cycles.

\(^9\) See Basu and Kimball (2000)

\(^10\) With standard CRRA preferences, we have \(U(C) = \frac{C^{1-\sigma}}{1-\sigma}\). If we combine a value of \(\sigma \approx 4\), say, with the first order condition \(U'(C) \frac{W}{P} = V'(L)\) and impose that \(L\) is constant, we get \(\frac{1}{\sigma} \frac{W}{P}\) was constant. But per capita consumption roughly doubled since 1960, but wages did not increase sixteen-fold. Something else must have changed on the left hand side of the labor optimality condition.

\(^11\) People tire themselves out working so as to derive extra benefit from consumption after a long day at work.

\(^12\) Economic Growth, 2004.
are complements. In any case, there has been little empirical evidence done to lend support to this complementarity assumption. John Cochrane, in his text on asset pricing, is skeptical of this approach, remarking that the issue “a priori not clear”. According to Cochrane, this nonseparability has also been used to attempt to rectify asset pricing puzzles, but empirically has been unsuccessful. On the whole, to predicate a theory of labor choice on the assumption of labor/consumption complementarity seems contrived. Like Cobb Douglas, this formulation seems more of a mathematical fix than a compelling portrayal of human behavior.\textsuperscript{13}

2.3 Consumption

The concept of diminishing marginal utility to a given consumption good pervades economic theory. Traditionally, though, growth theory has paid little attention to the consumption variety and imperfect substitutability among goods.\textsuperscript{14} In standard growth models, there is only a single consumption good or, more specifically, all consumption goods are perfectly substitutable; and so exhibiting infinite elasticity of substitution. In steady state, consumption of a given good increases over time at the rate of growth of technology.

Given standard preference values, labor supply would fall sharply in such a one good world. Indeed, basic introspection leads one to conclude that work effort would fall sharply in this one-good world. In a one-good world, would people continue to supply work effort as more and more of that same good become plentiful? Or, to put it another way, if the product space today was identical to that of a century years ago, but today’s real wages still prevailed, would the average person still work the same hours? This indeed seems unlikely, but conventional theory answers in the affirmative; labor hours worked are independent of the product space.

In the standard neoclassical labor/leisure model, the marginal utility of the real wage is a crucial determinant labor supply. Product variety, then, is central to any explanation of labor supply patterns. New products and better qualities of existing products forever come on stream, and an entire industry — advertising — exists to inform us of such goods. And these products no doubt affect the marginal utility of consumption; a second car, say, hardly provides as much utility as a new boat. A central feature of

\textsuperscript{13} There is another model by Rogerson which posits another utility function where productivity of household work rises over time, thereby yielding more market work. In his framework leisure time incorporates work in the home and leisure. In equilibrium, the trend in home productivity growth is equal to the trend in market productivity growth, which maintains constant market labor supply (but more leisure). However, this is another knife edge restriction, and this approach has rarely been invoked in standard models.

\textsuperscript{14} One exception is Grossman and Helpman. However, their motivating concern is very different to mine.
this paper is that the marginal utility of consumption is dependent on the product or characteristic space available (which of course varies across time and space). And that product space is forever expanding, thereby countering diminishing marginal utility of consumption. In my framework, it is this countervailing force that acts to continually raise the utility value of marginal work hours. This bolsters the substitution effect of a rising wage, thereby preventing labor hours from falling. I present a \textit{theory} of why income and substitution effects offset in the long run.

3 Model

3.1 Description

In the model, there is a representative household, and I normalize the number of households to one. The household chooses between consumption and leisure. More generally, “leisure” incorporates any nonmarket activity - the black market, home production, conventional leisure, and so on. There is a variety of consumption goods, and the product space increases over time. The elasticity of substitution between goods is less than one; namely, the goods are significantly different from each other. These sectors basically represent relatively new goods. They could also represent new characteristics (such as a small tv on a cell phone, say) in the Lancaster sense. In this way, my framework doesn’t explicitly distinguish between vertical and horizontal product differentiation.

Note that there are infinitely many \textit{potential} brands (all of which the consumer has a latent demand for), with \( n(t) \) existing in period \( t \), entering utility in an additively separable way. The marginal utility of any good is independent of other goods. For the purposes of growth over the long run, this assumption is realistic; over time we continually see the arrival of goods that are imperfect substitutes for previous products.\textsuperscript{15}

3.1.1 Partial Equilibrium

Consumers have utility:

\[
\int_0^\infty \left( \int_0^\infty (v(c_{jt}) - v(0)) \, dj - \beta \frac{t^{1+\theta}}{1+\theta} \exp(-\rho t) \right) \, dt
\]

which it maximizes subject to:

\textsuperscript{15}In fact, this is more realistic that conventional new growth models, where all goods are, by construction, easily substitutable. The monopolistic competition model truly only applies to slightly different varieties of a given good - like, say, different brands of cereal.
\[ \dot{b}_t = r_t b_t + W_t t - C_t \]

where:

\[ v(c_{jt}) = \frac{(c_{jt} + \epsilon)^{1-\sigma}}{1-\sigma} \quad c_{jt} \gg \epsilon \]

So, utility reduces to:

\[ \int_0^\infty \left( \int_0^{n_t} (v(c_{jt}) - v(0)) \, dj - \beta \frac{t^{1+\theta}}{1+\theta} \right) \exp(-\rho t) \, dt \]

where:

\[ \int_0^{n_t} p_j c_{jt} \, di = C_t \]

At any point, \( t \), there are \( n_t \) brands available so utility reduces to:

\[ \int_0^\infty \left( \int_0^{n(t)} (v(c_{jt}) - v(0)) \, di - \beta \frac{t^{1+\theta}}{1+\theta} \right) \exp(-\rho t) \, dt \]

### 3.2 The Solution

#### 3.2.1 The Intratemporal Problem

The model is completely symmetric so, by concavity, the consumer will consume all available goods.\(^{16}\)

The price of each good is fixed at \( p \), which I normalize to one. So the consumer can purchase quantity \( \frac{C_t}{n_t} \) of each, where \( C_t \) is consumption in period \( t \) (from the intertemporal problem).

To make things tractable, I set the intra and intertemporal elasticities of substitution equal. For my purposes, this is convenient. The consumer will smooth consumption over products within a given period.

\[ \int_0^\infty \left( \int_0^{n_t} (v(c_{jt}) - v(0)) \, dj - \beta \frac{t^{1+\theta}}{1+\theta} \right) \exp(-\rho t) \, dt \]

\[ \int_0^{n_t} (v(c_{jt}) - v(0)) \, dj = \frac{n_t \left( \frac{C_t}{p_t n_t} + \epsilon \right)^{1-\sigma}}{1-\sigma} + \frac{n_t}{(\sigma - 1)\epsilon^{\sigma-1}} \]

Letting \( p = 1 \), we have:

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\(^{16}\)At any point in time, \( t \), goods that are not available to purchase (but which there is latent demand for), are assumed to have infinite prices; and so \( p_{jt} > \frac{v'(c_{jt})}{\lambda_t} \) \( \forall j \in (n_t, \infty) \), where \( \lambda_t \) is the marginal utility of wealth in period \( t \).
\[
\frac{n(C_t^* + \epsilon)^{1-\sigma}}{1-\sigma}
\]

And given that \( \epsilon \approx 0 \), or \( \frac{C_t^*}{\epsilon} \gg \epsilon \) \forall t \) I henceforth take the approximation:

\[
\frac{n_t(C_t^* + \epsilon)^{1-\sigma}}{1-\sigma} \approx \frac{n_t(C_t^*)^{1-\sigma}}{1-\sigma}
\]

The utility function for the intertemporal problem now reduces to:

\[
V(C, n, l) = \frac{n^\sigma C_t^{1-\sigma}}{1-\sigma} + \frac{n_t}{e^{\sigma-1}} - \beta \frac{l_t^{1+\theta}}{1+\theta}
\]

Consumers then maximize:

\[
\int_0^\infty \left( \frac{n^\sigma C_t^{1-\sigma}}{1-\sigma} + \frac{n_t}{e^{\sigma-1}} - \beta \frac{l_t^{1+\theta}}{1+\theta} \right) \exp(-\rho t) \, dt
\]

subject to:

\[
b_t = r_t b_t + W_t l_t - C_t
\]

\[
0 \leq l_t \leq 1
\]

\[
b_0 > 0
\]

\[
\lim_{t \to \infty} \lambda_t b_t \exp(-\rho t) = 0
\]

The current valued Hamiltonian is:

\[
H = \frac{n^\sigma C_t^{1-\sigma}}{1-\sigma} + \frac{n_t}{e^{\sigma-1}} - \beta \frac{l_t^{1+\theta}}{1+\theta} + \lambda_t (r_t b_t + W_t l_t - C_t)
\]

\[
\frac{\partial H}{\partial C_t} = 0 \Rightarrow \frac{n_t^\sigma}{C_t^\sigma} = \lambda_t
\]

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17Now, taking a Taylor expansion of the function around \( \frac{C_t}{\epsilon} \), we get:

\[
\frac{n(C_t^* + \epsilon)^{1-\sigma}}{1-\sigma} = \frac{n(C_t^*)^{1-\sigma}}{1-\sigma} + \frac{n\epsilon}{(C_t^*)^\sigma} - \frac{n\epsilon^2}{(C_t^*)^{\sigma+1}} + O(3)
\]

Note that other terms go to zero in equilibrium.

18To have a bounded integral we need \( \rho > \left(\frac{\sigma-1}{\sigma}\right)g \). Plus, note that \( g = r - \rho \), so this means \( r - g > \frac{\sigma-1}{\sigma}g \) or \( r > g(2 - \frac{1}{\sigma}) \).
\[
\frac{\partial H}{\partial l_t} = 0 \Rightarrow \beta_t^0 = \lambda W_t
\]

\[
\frac{\dot{\lambda}_t}{\lambda_t} = \rho - r_t
\]

The Keynes-Ramsey rule reduces to:

\[
\frac{\dot{C}_t}{C_t} = \frac{r_t - \rho}{\sigma} + \frac{\dot{n}_t}{n_t}
\]

For a fixed labor supply, we have (from diff leisure/lab condition):

\[
\frac{\dot{W}_t}{W_t} = r_t - \rho
\]

### 3.2.2 Balanced Growth Path

On the \( bgp \) we have: \( \frac{\dot{C}_t}{C_t} = \frac{\dot{W}_t}{W_t} \)

So, in this setup, consumption growth is independent of risk aversion. Note that variety growth will also increase saving growth, since it raises the marginal utility of future wealth.

In steady state, we have:

\[
\frac{\dot{n}_t}{n_t} = \frac{\sigma - 1}{\sigma} \frac{\dot{W}_t}{W_t}
\]

This is an important result. To maintain fixed labor supply the real wage and the number of products the consumer is able to purchase must be growing proportionally.\(^{20}\) This already presents us with an explanation for why labor supply is approximately constant across time and space. As real wages increase, the products we can purchase increase accordingly. This counters diminishing marginal utility, and maintains a constant labor supply. The income effect of a rising wage is being countered by new products bolstering marginal utility all the time.

Note that, when \( \sigma = 1 \), the above equation stipulates that we can sustain a constant labor supply without variety growth. This is of course just the standard Cobb Douglas (or log utility) model. When \( \sigma < 1 \), variety growth must be negative. As mentioned, empirical evidence suggests that \( \sigma > 1 \) is the

\(^{19}\)Note that \( \frac{\dot{C}_t}{C_t} = \frac{\dot{W}_t}{W_t} = r_t - \rho \) imposes a constant labor supply. Otherwise consumption growth would be greater or less than wage growth. In equilibrium, the agent will consume his wage and save asset income.

\(^{20}\)Note that this analysis deals with permanent changes in the real wage and uncompensated elasticities of supply. As such, I’m not concerned with transient changes in wages or the intertemporal allocation of labor.
empirically relevant case, which is my focus here. Noteworthy is the fact that now have a reasonable formulation of utility that is consistent with constant labor supply, irrespective of how high $\sigma$ actually is. Also, note that in the conventional model we have $C_t = \int_0^n c_{jt} dj$. That is, all consumption goods are perfect substitutes, and are effectively transformed into a single good so we end up with $n = 1$. This of course would lead to falling labor supply given $\sigma > 1$. In this conventional approach, higher wages merely lead to “more of the same” consumption goods, thus inducing a sharp fall in labor supply with a rising wage. In my approach, rising variety prevents this occurring, thus sustaining a stable labor supply in the presence of a rising wage.

Importantly, a rise in the real wage without proportional variety growth will cause labor supply to fall. Higher variety growth without a change in the real wage will increase labor supply. Put another way, one can conceive of labor supply over time as a “tug-of-war” between the real wage and variety growth.

### 3.2.3 Risk free rate puzzle

Consider again the Keynes Ramsey rule when we’re off the balanced growth path:

$$\frac{\dot{C}_t}{C_t} = \frac{r_t - \rho}{\sigma} + \frac{n_t}{n_t}$$

This formulation of utility can resolve the risk free rate puzzle. Recall that the risk free rate puzzle was why consumers saved, given consumption growth over time. It really hinged on the concept of diminishing marginal utility and the empirical evidence indicating high risk aversion (or low intertemporal elasticity of substitution) and consumption growth of about 2% a year. If there were lots of goods in the future, and the consumer wants a smooth profile of consumption, why not just borrow and have more today? This should lead to a higher risk free rate of interest.

My formulation of utility offers an explanation of why the consumer would not want to dissave on a grand scale. In accord with the permanent income hypothesis, the consumer wishes to smooth marginal utility over time. With variety growth, the consumer has an incentive to save and increase consumption growth, regardless of sigma. Greater prospective variety makes the consumer more efficient at creating utility in the future. This, then, attenuates the desire to dissave.

Most resolutions involve some kind of time non separable utility -like habit persistence. However, the foregoing analysis yields a simple resolution within the more conventional time separable framework.

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21The Epstein/Zin resolution is a notable exception; they offer a resolution with a utility formulation separating risk aversion and intertemporal elasticity of substitution.
3.3 Labor supply

Getting back to labor dynamics, let’s solve now for the actual labor supply. To get a closed form solution for labor, I restrict the analysis to linear disutility of labor. Ignoring time scripts, we then have:

\[ W_t n_t^\sigma = C_t^\sigma l_t^\theta \]

\[ \dot{A}_t = r_t A_t + W_t l_t - C_t \]

\[ C_t = r_t A_t + W_t l_t - \dot{A}_t \]

\[ W_t n_t^\sigma = (r_t A_t + W_t l_t - \dot{A}_t)^\sigma l_t^\theta \]

On the balanced growth path, steady state labor supply is defined implicitly by the equation:

\[ W_t n_t^\sigma = (r_t A_t + W_t l^* - \dot{A}_t)^\sigma (l_t^*)^\theta \]

Now, to get a closed for solution generalize \( \frac{l^{1+\theta}}{1+\theta} \) to \( l \) (i.e \( \theta = 0 \), linear disutility). Solving for \( l \) (again, ignoring time scripts) then gives:

\[ l = \frac{n}{W^{1-\frac{1}{\theta}}} - \frac{r A}{W} + \frac{\dot{A}}{W} \]

Then noting that, on the balanced growth path, \( \dot{A} = Ag \), and \( \frac{\dot{W}}{W} = \frac{\dot{C}}{C} = r - \rho \)

\[ l = \frac{n}{W^{1-\frac{1}{\theta}}} - \rho \frac{A}{W} \in [0,1] \]

but \( \frac{\dot{A}}{W} \) is constant in steady state.\(^{22}\)

A vast body of evidence suggests, however, that consumption tracks labor income very well over the lifecycle, particularly over the prime earning years.\(^{23}\) In this case, we simple have \( C = Wl \) and our equation for labor supply is simply \( l = \frac{n}{W^{1-\frac{1}{\theta}}} \).

\(^{22}\)With power utility, consumption and wealth grow at same rate on the balanced growth path (consumption is a constant fraction of wealth.) The wage and consumption grow at same rate too.

\(^{23}\)Mankiw and Campbell show, for instance, that about half of agents live paycheck to paycheck.
3.4 An Exogenous Rise in Variety

Suppose an economy is in steady state and the variety of goods starts to increase, while the wage remains constant. This could be an outgrowth, say, of globalization or product market deregulation. How does this play out? To see, I use a simple two period model.

Without loss of generality, set disutility of labor linear and \( r = \rho \). Suppose we have \( n_2 > n_1 > \pi \) whereas, ordinarily, we have \( n_1 = n_2 = \pi \).

3.4.1 Case 1: No Change in Variety

Ordinarily, we have:

\[
1 = Wu'(C_1) = W \frac{\pi^{\sigma-1}}{C^{\sigma}}
\]

\[
u'(C_1) = u'(C_2)
\]

The labor market condition is:\(^24\)

\[
V'(l_1) = Wu'(C_1) \Rightarrow l_1 = l_2
\]

3.4.2 Case 2: Higher Variety

These optimality conditions above also hold for the situation where variety is higher in both periods and grows. But the levels of consumption and labor change. Examining the labor conditions for each period more closely indicates that:

\[
1 = Wu'(C_2) = W \frac{n_1^\sigma}{(WI + S)^\sigma}
\]

\[
1 = Wu'(C_1) = W \frac{n_2^\sigma}{(WI - S)^\sigma}
\]

where \( S \) denotes savings. The usual labor market condition from case 1 is:

\[
1 = Wu'(C_1) = W \frac{\pi^\sigma}{(WI)^\sigma}
\]

But with increase in variety, we have (combining conditions):

\[
1 = Wu'(C_1) = W \frac{\pi^\sigma}{(WI)^\sigma} = W \frac{n_2^\sigma}{(WI - S)^\sigma}
\]

So given that \( n_2 > \pi \), we necessarily have \( l > l_1 \). That is, relative to before, labor supply will now be higher in both periods.

\(^{24}\)Where I make disutility of labor explicit.
4 General Equilibrium

Turning now to general equilibrium, I incorporate the foregoing concepts into an endogenous growth model.

4.1 Utility Function

4.1.1 The economic environment

A imperfectly competitive equilibrium in this economy is an allocation $(C_t, b_t, Y_t, I_t, E_t, l_t^d, l_t^s, η, A_t, n_t, π_t, x_{ijt})_{t=0}^{∞}$ and a price system $(W_t, r_t, η, (p_{it})_{t=0}^{∞}, η)_{t=0}^{∞}$.

- There is a single infinitely lived representative agent. The consumer takes the path of $(W_t, r_t, p_{it}, A_t, n_t)_{t=0}^{∞}$ for all $t$, and the household production technology for new consumption ideas as given. $A_t$ is a measure of the number of inputs produced by firms at time $t$ out of potentially infinitely many.

The consumer chooses $(C_t, c_{jt}, x_{it}^C, x_{ijt}, l_t^s, b_t)_{t=0}^{∞}$ to maximize:

$$\int_0^{∞} \left( \int_0^{∞} (v(c_{jt}) - v(0)) dj - β l_t^{1+θ} \frac{1}{1+θ} \right) \exp(-ρt) dt$$

subject to:

$$\dot{b}_t = r_t b_t + W_t l_t - C_t$$

where

$$v(c_{jt}) = \frac{(c_{jt} + ε)^{1-σ}}{1-σ} \quad c_{jt} \gg ε$$

and

The path of $n_t$, the number of consumption goods the household can produce at time $t$, is given by the household technology:

$$n_t = φ A_t^Γ \quad φ, γ > 0$$

where $A_t$ is the measure of nonrival knowledge available to consumers to produce consumption “recipes”, indexed $j$. $φ$ parameterizes how easy it is to exploit available knowledge, $A_t$, to produce new consumption goods or the efficiency with which the household can create consumption goods.

So, utility reduces to:
\[
\int_0^\infty \left( \int_0^{n_t} (v(c_{jt}) - v(0)) \, dj - \beta \frac{t^1 + \theta}{1 + \theta} \right) \exp(-\rho t) \, dt
\]

where:

\[
\int_0^{A_t} p_{it} x_{it}^C \, di = C_t
\]

where \(x_{it}^C\) denotes inputs purchased from firm \(i \in [0, A_t]\).

The production (or subutility) function for composite commodity, \(c_{jt} \forall j = 1 \ldots n\), is:

\[
c_{jt} = \left( \int_0^{A_t} x_{jdt}^C \, di \right)^{\frac{1}{\alpha}}
\]

The consumer problem gives us \((C_t, c_{jt}, x_{it}^C, x_{ijt}, l_{it}^s, b_{it})_{t=0}^\infty\).

\(x_{it}^C\) denotes total demand by consumer for input \(i \in [0, A_t]\):

\[
x_{it}^C = \int_0^{n_t} x_{ijt}^d \, dj
\]

• Input firms (indexed by \(i\)) are monopolistically competitive, and take \((W_t, A_t, E_t, l_{it}^s, p_{it})_{t=0}^\infty\) as given. The firm sells inputs to consumers and the R & D sector. Each firm issues bonds to pay a fixed start up cost of \(\eta\), which is a patent purchased from the R & D sector. The only factor of production is labor, and the labor market is competitive. The production function for firm \(i\) is \(y_{it} = A_t l_{it}^d\). Worker efficiency, \(E_t\), is given by \(E_t = A_t\). Then, the marginal cost of the firm, \(\frac{W_t}{A_t}\), is normalized to one (i.e the numeraire).

The representative firm production function is \(y_{it} = A_t l_{it}^d \forall i \in [0, A_t]\) and profit, \(\pi_{it}\) is:

\[
\pi_{it} = p_{it} x_{it}^d (p_{it}) - W_t \frac{x_{it}^d}{A_t}
\]

so \(l_{it}^d = \frac{x_{it}^d}{A_t}\).

Each firm \(i\) chooses \((p_{it})_{t=0}^\infty\) to solve:

\[
\max_{p_{it}} \pi_{it} = x_{it}^d (p_{it}) (p_{it} - \frac{W_t}{A_t})
\]

where the total demand \(x_{it}\) is given from the consumer and R & D problem, that is, \(x_{it}^d = x_{it}^C + x_{it}^R\).

This problem gives an expression for \((p_{it}, x_{it}, \pi_{it})_{t=0}^\infty\) for all \(i\) and \(t\).

\[25\]This is standard in new growth models, but I omit the explicit modelling here.
• The research sector is competitive (free entry), and of the lab equipment form. It issues debt, \( I_t \), on the capital market each period, and purchases inputs from existing firms, then selling blueprints to new start-ups. It takes \( (A_t, p_{it})_{t=0}^{\infty} \) as given and chooses \( (x_{it})_{t=0}^{\infty} \) to maximize:

\[
\dot{A} \quad s.t \quad \int_0^{A_t} p_{it} x_{it} \, di = I_t
\]

where \( I_t \) denotes R & D outlays, and:

\[
\dot{A} = \frac{\alpha A_t^{1-\frac{1}{n}}}{\eta} \left( \int_0^{A_t} x_{it}^{\alpha} \, di \right)^{\frac{1}{\alpha}}
\]

The conveys the notion that one unit of expenditure on R & D creates \( \frac{1}{\eta} \) blueprints. So the cost of production of a single blueprint is \( \eta \) and free entry and zero profits in R & D means a patent sells for \( \eta \). Regarding the blueprint production function, there are two forces. More \( A \) makes it both easier and harder to come up with new ideas, the respective “standing and shoulders” and “stepping on toes” effects. Both effects are assumed to just offset, and it always takes \( \eta \) of expenditure to produce one blueprint.

• The value of a blueprint is \( V_t = \frac{\pi_t}{r_t} \). Arbitrage then ensures that this is equated to the cost of a blueprint, \( \eta \). It then follows that \( \frac{\pi_t}{r_t} = \eta \), which gives the interest rate \( r_t \).

• \( Y_t = \int_0^{A_t} A_t l_{it} \, di \).

• Labor constraint: \( \int_0^{A_t} l_{it} \, di = l_t^s \).

• Capital market equilibrium: \( b_t = \int_0^t I_t \, dt = \eta A_t \)

• Resource constraint: \( Y_t = C_t + \eta \dot{A}_t \).

4.2 Solving the model

4.3 Intratemporal Problem

Conveniently, we can solve the consumer problem in two stages, the intra and intertemporal problems. By symmetry and concavity, the agent will consume an equal amount, \( \frac{C_j}{n_t} \), of each good \( j \in [0, n_t] \).

For each composite commodity, \( c_{jt} \), the consumer then solves:

\[
\max_{x_{itj}} \left( \int_0^{A_t} x_{ijt}^{\alpha} \, di \right)^{\frac{1}{\alpha}} + \lambda_t (\frac{C_t}{n_t} - \int_0^{A_t} p_{it} x_{ijt} \, di)
\]
the demand for input \( i \in [0, A_t] \) for each commodity \( c_{jt} \) where \( j \in [0, n_t] \) is given by:

\[
    x_{ijt} = \frac{p_{it}^{-\alpha}}{\left( \int_0^{A_t} p_{it}^{-\alpha} \, di \right)^{n_t}} C_t
\]

By symmetry, then, total consumer demand for input \( i \), \( x^C_{it} \), is:

\[
    x^C_{it} = \int_0^{n_t} x_{ijt} \, dj = \frac{p_{it}^{-\alpha}}{\left( \int_0^{A_t} p_{it}^{-\alpha} \, di \right)^{n_t}} C_t
\]

### 4.4 Consumer Problem

In symmetric equilibrium we have \( c_{jt} = \frac{C_t}{\beta^{\frac{1}{\sigma}} A_t^{\frac{1}{\alpha}}} \). The indirect utility function (for goods expenditure) in period \( t \) is then (noting our approximation \( c_{jt} \gg \epsilon \)):

\[
    U(C_t) = n \left( \frac{C_t}{\beta^{\frac{1}{\sigma}} A_t^{\frac{1}{\alpha}}} \right)^{1-\sigma} + \frac{n_t}{(\sigma-1)\epsilon^{\sigma-1}}
\]

So total utility is:

\[
    U(C_t) = \sigma^{-1} n_t^\sigma \left( A_t^{\frac{1}{\alpha}-1} C_t \right)^{1-\sigma} + \frac{n_t}{(\sigma-1)\epsilon^{\sigma-1}}
\]

which reduces to:

\[
    u'(C_t) = \frac{\sigma^{-1} n_t^\sigma}{A_t^{(\sigma-1)(\frac{1}{\alpha}-1)} C_t^\sigma}
\]

Having solved intratemporal problem, now let’s turn to the intertemporal problem, and solve for the balanced growth path. The consumer allocates funds (from dividends and wages) between consumption and assets/saving. There is a single representative consumer who maximizes:

Consumers then maximize:

\[
    \int_0^\infty \left( \frac{\sigma^{-1} n_t^\sigma (A_t^{\frac{1}{\alpha}-1} C_t)^{1-\sigma}}{1-\sigma} + \frac{n_t}{(\sigma-1)\epsilon^{\sigma-1}} - \beta l_t^{1+\theta} \right) \exp(-dt) \, dt
\]

subject to:

\[
    \dot{b}_t = rb_t + W_l l_t - C_t
\]

\[
    0 \leq l_t \leq 1
\]
\[ b_0 > 0 \]

The TVC:

\[ \lim_{t \to \infty} \lambda_t b_t \exp(-\rho t) = 0 \]

The solution is:

\[
H = \frac{p_t^{\sigma-1} n_t^\sigma (A_t^{1-\frac{1}{\alpha}} C_t)^{1-\sigma}}{1 - \sigma} + \frac{n_t}{(\sigma - 1)e^{\sigma - 1}} - \beta \frac{I_t^{1+\theta}}{1 + \theta} + \lambda_t (r_t b_t + W_t l_t - C_t)
\]

\[
\frac{\partial H}{\partial C} = 0 \Rightarrow \frac{p_t^{1-\sigma} n_t^\sigma}{A_t^{(\sigma-1)(\frac{1}{\alpha}-1)} C_t^\sigma} = \lambda_t
\]

\[
\frac{\partial H}{\partial l} = 0 \Rightarrow \beta l_t^\theta = \lambda_t W_t
\]

\[
\dot{\lambda}_t = \rho - r_t
\]

In steady state:

\[
\frac{\dot{C}_t}{C_t} = \frac{\dot{W}_t}{W_t} = \frac{\dot{A}_t}{A_t}
\]

Differentiating the labor condition gives, imposing constant \( l \) gives:

\[
\frac{\dot{n}_t}{n_t} = \frac{\sigma - 1}{\sigma \alpha \cdot A_t}
\]

This pins down the exponent on the home production technology.

### 4.5 The R & D Problem

The R & D firm issues debt (or equity), \( I_t \), each period and chooses \( x_{it} \) to maximize:

\[
\frac{\alpha A_t^{1-\frac{1}{\alpha}}}{\eta} \left( \int_0^{A_t} x_{it}^{\alpha} di \right)^{\frac{1}{\alpha}} + \lambda_t (I_t - \int_0^{A_t} p_{it} x_{it} di)
\]

This gives:

\[
x_{it}^R = \frac{p_{it}^{\frac{1}{1-\alpha}}}{\left( \int_0^{A_t} p_{it}^{\frac{1}{1-\alpha}} di \right)} I_t
\]
4.6 Input Firms

4.6.1 Production Sector

Given the production function, \( y_{it} = A_t l_{it} \), for all firms \( i \in [0, A_t] \), profit is (where \( x_{it} \equiv x_{it}(p_{it}) \)):

\[
\pi_{it} = p_{it} x_{it}^d - \frac{W_t}{A_t} x_{it}^d
\]

since \( l_{it} = \frac{x_{it}^d}{A_t} \)

\[
\pi_{it} = x_{it}^d (p_{it} - \frac{W_t}{A_t})
\]

Now, recalling that consumer demand for input \( i \in [0, A_t] \) is:

\[
x_{C_{it}} = \frac{p_{it}^{-1}}{\int_0^{A_t} p_{it}^{-\alpha} di} C_t
\]

R & D demand is:

\[
x_{R_{it}} = \frac{p_{it}^{-1}}{\int_0^{A_t} p_{it}^{-\alpha} di} I_t
\]

Total demand for input \( i \in [0, A_t] \) is now:

\[
x_{C_{it}} + x_{R_{it}} = \frac{p_{it}^{-1}}{\int_0^{A_t} p_{it}^{-\alpha} di} (C_t + I_t)
\]

From the resource constraint, we have \( Y_t = W_t l_t + \eta \dot{A} \), so \( \frac{\dot{A}}{A_t} = \frac{\dot{Y_t}}{Y_t} = \frac{\dot{W_t}}{W_t} \). marginal cost is constant. On the balanced growth path consumption growth equals wage growth; and this also ensures a constant labor supply, given a constant growth of real wages. I can then set marginal cost as the numeraire so:

\[
\frac{W_t}{A_t} = 1 \quad \forall t
\]

Each firm takes \( \int_0^{A_t} p_{it}^{-\alpha} di \) as given, so faces demand of elasticity \( -\frac{1}{1-\alpha} \).

The markup pricing rule is now:

\[
p_{it} = \frac{1}{\alpha} \frac{W_t}{A_t} = \frac{1}{\alpha} \equiv p
\]

This also gives labor demand and real wage, \( \frac{W_t}{p} = \alpha A_t \). So:

\[
\frac{W_t}{p} = \alpha A_t
\]
The worker is thus paid less than his marginal value product; this will create a distortion in the labor/leisure choice.

Henceforth, given symmetry, I drop the firm subscripts $i$. Note that equal prices across firms implies equal quantities produced. In symmetric equilibrium, all prices are equal and constant, so the quantity produced by each firm is:

$$x = \frac{C_t + I_t}{pA_t}$$

Since $Y_t = A_t l_t^s$, $Y$ and $A$ grow at the same rate on the balanced growth path.$^{26}$

### 4.6.2 Profits

On the balanced growth path we have profits:

$$\left( \frac{1}{\alpha} - 1 \right) \frac{C_t + I_t}{pA_t} = \frac{(1 - \alpha) C_t + I_t}{\alpha pA_t} = (1 - \alpha) \frac{C_t + I_t}{A_t}$$

The $pdv$ of all future profits is equal to:

$$\int_0^\infty (1 - \alpha) \frac{C_t + I_t}{A_t} \exp(-r_t t) \, dt$$

which implies:

$$\frac{\pi_t}{r_t} = (1 - \alpha) \frac{C_t + I_t}{r_tA_t}$$

Since $\frac{Y_t}{A_t}$ is constant on the balanced growth path, so is $r_t$ so I put $r_t \equiv r$. But this costs $\eta$ (in terms of numeraire) so we have:

$$\eta = \frac{\pi_t}{r} = \frac{(1 - \alpha) C_t + I_t}{rA_t} \quad \Rightarrow r = (1 - \alpha) \frac{C_t + I_t}{A_t \eta}$$

### 4.6.3 General Equilibrium

Now, in capital market equilibrium, we have the stock of assets: $b_t = \eta A_t \Rightarrow \dot{b}_t \equiv I_t = \eta \dot{A}_t$.

$$\dot{b}_t = r_t b_t + W_t l_t - C_t$$

Note that on $bgp$, from the budget constraint:

$$\dot{b}_t = W_t l_t + r b_t - C_t \Rightarrow C_t = W_t l_t + r b_t - \eta \dot{A}_t = W_t l_t + r \eta A_t - \eta g A_t$$

---

$^{26}$We can also see this from the resource constraint, $Y = C + \eta \dot{A}$. 
implying that:

\[ C_t + \dot{b}_t = W_t l_t + r b_t \Rightarrow C_t + \eta \dot{A}_t = W_t l_t + r b_t = W_t l_t + r \eta A \]

So the (nominal) demand for each firm is, noting in GE that \( b_t = A_t, \ l_t \equiv l, \ \dot{b} = I_t = \eta \dot{A} \) and \( r \) is constant.

\[ \frac{C_t + I_t}{A_t} = \frac{C_t + \dot{b}_t}{A} = \frac{W_t l_t + r \eta A_t}{A_t} = l + r \eta \]

\[ r = (1 - \alpha) \frac{Y_t}{A_t \eta} = (1 - \alpha) \frac{l + r \eta}{\eta} \]

\[ r = (1 - \alpha) \left( \frac{l}{\eta} + r \right) \]

\[ r = \frac{(1 - \alpha)}{\alpha} \frac{l}{\eta} \]

And the Keynes-Ramsey rule reduces to:

\[ \frac{\dot{C}_t}{C_t} = \frac{(1 - \alpha)}{\alpha} \frac{L}{\eta} - \rho \]

### 4.6.4 Resource Constraint

The resource constraint is:

\[ Y_t = C_t + \dot{A}_t \eta \]

that is, income is comprised of consumption and investment (savings). Dividing across by \( A_t \) indicates that, on a balanced growth path, \( Y, C \) and \( A \) all grow at the same rate.\(^{27}\)

### 4.7 Labor Condition

Using the first order condition for labor:

\[ \frac{p^{\sigma - 1} n_t^\sigma}{A_t^{(\sigma - 1)(\frac{1}{\alpha} - 1)} C_t^\sigma} = \lambda_t \]

The labor/leisure condition (assuming linear disutility for simplicity) is:

\(^{27}\)We also have \( Y_t = W_t l_t + r_t A_t \eta \). This is our income equation, that is, income is equal to wages plus return on assets. Note that given \( \frac{\pi_t}{\dot{r}_t} = \eta \), we equivalently have \( Y_t = W_t l_t + \dot{A}_t \pi_t \); income is equal to wages plus profits.
\[ \lambda_t W_t = \beta \]
\[ W_t \frac{p^{\sigma-1}n_t^\sigma}{A_t^{(\sigma-1)\left(\frac{n}{\sigma}-1\right)}} = \beta \]
\[ W_t \frac{p^{\sigma-1}n_t^\sigma}{A_t^{(\sigma-1)\left(\frac{n}{\sigma}-1\right)}} = \beta (r_t b_t + W_t d_t - \dot{b}_t)^\sigma \]

Then, in G.E (noting from the firm’s problem that the real wage is \( \alpha A \)) and dropping time scripts:
\[ W_t \frac{p^{\frac{1}{\sigma}-1}n_t^{\frac{1}{\sigma}}}{A_t^{(1-\frac{1}{\sigma})\left(\frac{n}{\sigma}-1\right)}} = \beta (r_t A + W_t - gA) \]

Noting that \( g = r - \rho \Rightarrow \rho = r - g \):
\[ \frac{(\alpha A)^{\frac{1}{\sigma}-1}n_t^{\frac{1}{\sigma}}}{\beta A^{(1-\frac{1}{\sigma})\left(\frac{n}{\sigma}-1\right)}} - \eta \rho = l \]
\[ l = \frac{n}{\beta \alpha^{1-\frac{1}{\sigma}} A^{\frac{n}{\sigma}-1}} - \eta \rho \]

In steady state, we have:
\[ \dot{n} = \sigma - 1 \frac{\dot{A}}{A} \]

Then recalling the form of the household technology:
\[ n_t = \phi A_t^\gamma \]

in a balanced growth path we have the parameter restriction:
\[ \gamma = \frac{\sigma - 1}{\sigma \alpha} \]

Then steady state labor is:
\[ l^* = \frac{\phi}{\beta \alpha^{1-\frac{1}{\sigma}}} - \eta \rho \in [0, 1] \]

Then growth is given by:
\[ \frac{\dot{C}_t}{C_t} = \frac{(1 - \alpha)}{\alpha} \frac{l^*}{\eta} - \rho > 0 \]

where we need:
\[ (1 - \alpha) \frac{l^*}{\eta} > \rho \]

So, altogether, we have:
\[ \frac{\dot{C}_t}{C_t} = \frac{\dot{A}_t}{A_t} = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{W}_t}{W_t} = \frac{\sigma A \cdot \dot{n}_t}{\sigma - 1 n_t} = \frac{(1 - \alpha)}{\alpha} \frac{l^*}{\eta} - \rho \]
4.7.1 Discussion

The model yields a number of insights. First, economic growth is ultimately a function of labor supply. The fruits of labor supply create a market for new innovations which is the conduit to growth in this model. One surely has a greater incentive to innovate in a world where all agents work 12 hours per day as opposed one. That is, more labor hours are a way of extending the size of the market; they yield a scale effect, and extend the size of the market. Those induced innovations are the underlying determinant of productivity, growth and welfare.

Most important, labor supply itself is endogenous. In particular, labor supply is a function of the extent to which new consumption opportunities arise. The extent of consumption opportunities, as parameterized by $\phi$ in the model, has a level effect on labor supply, and hence growth. A higher $\phi$ induces greater variety in consumption, which in turn raises the marginal utility of any given wage rate, and so raises optimal labor supply. This then has a growth effect, leading to a kind of virtuous cycle with more variety growth, and so on.

Observe that labor supply is constant in the long run; we cannot have perpetual growth in labor supply. This is due to two countervailing effects of new innovations. New innovations raise the real wage by inducing labor augmenting technical change, but at the same time, raise new consumption opportunities. Both forces act to lower and raise labor supply, respectively. In the long run, both these effects counter each other and maintain a constant labor supply. The model also incorporates structural change since, as productivity rises, workers shift to different, newer sectors. Though labor is fixed, effective labor is rising all the time along with labor augmenting technical change.

Also, what matters is market activity. Home production, as is common in Europe and developing countries, does not permit a larger market for goods, and ultimately greater variety. In this model, what induced innovation was the size of market demand. It was labor hours with provided that market, and those innovations then induced growth. Thus, moving people from the home to market activity yields two effects; for one, we have greater demand for goods since people are no longer self-sufficient, and for another, we have greater production due to specialization.

There is no allocative inefficiency in goods production, since all goods have a relative price of one. However, there is a distortion in the labor market since workers are paid less than their marginal product (specifically, $\alpha A < A$). More formally, there is a wedge between the marginal rate of substitution and the real wage. In conventional models, this distortion reduces labor supply. A surprising implication of the model, however, is that the labor market distortion will actually enhance growth. Namely, given $\sigma > 1$, a lower real wage rate (a lower $\alpha$, or more market power) will increase labor
supply due to the attendant negative income effect.

Labor supply has a positive externality in the model. A higher labor supply induces more innovations and growth. By working more and earning a higher wage, the worker is creating research incentives for new innovations - which raise wages and variety. This improves welfare for others (that is, spillover effects) who are not party to the labor contract. The worker, however, does not internalize this externality so the social planner would seek an increase in labor hours.

A notable feature of the model is that it embraces both elements of supply and demand. It is ultimately consumption demand that drives growth in the model, which then leads to improvements on the supply side.

One unsatisfactory implication of the model is that more market power (less product market competition) enhances growth. This is a feature of all new growth models, but there is much reason to be skeptical of this result. The prospect of profits is doubtless a motivating factor for new firms, which is why it is there. However, a recent literature, most notably that of Aghion, have noted that, for more established firms at the frontier, more competition promotes growth. Namely, more competition forces established firms to innovate to remain at the frontier. For new start-ups, however, which is my concern there, this feature is probably reasonable.

A higher start-up cost for firms, $\eta$, reduces labor supply. This is a standard income effect. Higher start-up costs lead to a higher value of firms in equilibrium, thereby increasing non-labor income which reduces labor supply.

Finally, welfare increases with $n$. So, this captures the fact that, despite the smaller real income (say), a poor person today is still better off than an extremely rich person a century ago. This feature is not present in conventional neoclassical or new growth models.

5 Off balanced growth path

The general equilibrium model can be regarded as a reasonably accurate description of the very long run. In the long run equilibrium, there is a tight link between real wage growth and variety growth. Over short periods of time, these two variables likely move out of line for considerable periods. An increase in openness to trade, for instance, would likely increase variety growth rapidly at first upon a rise in imports. In this case, real wage growth would likely lag. Technological innovations that enhance real wage growth diffuse across nations, without any attendant increase in variety growth. More generally, factor price equalization across nations (as predicted by the Stolper Samuelson theorem)
conceivably affects domestic real wages in certain sectors, without having any obvious implications for domestic consumption opportunities. Institutions are another prominent example. Some induced exogenous policy changes - say, a rise in product market regulations - would depress consumption opportunities without having any effect on real wages.

It's arguable that, with increasing globalization, variety is approximately uniform across nations. This, however, is not necessarily true; among developed nations, the nontraded sector constitutes the bulk of GDP and, as such, there is no equilibrating force driving equality of consumer variety across borders. Indeed, as economies develop, consumption goods become less and less tradable. Also, there is an abundant literature on trade costs that preclude market forces from equating levels of variety across nations in tradables alone. Finally, for cultural reasons, goods in one country might have little appeal to the citizenry of other nations. Consider the case, say, of French movies in the US, and conversely.

The model can reasonably explain long run historical trends in labor supply, which show labor supply rising significantly (from prehistoric times, when people mostly engaged in leisure, having satisfied subsistence needs\(^{28}\)) at the onset of development, then falling, and subsequently levelling off. At the outset of economic development, people work hard at first to satisfy new abundant needs, since \(n\) increases rapidly initially. Then labor supply tends to fall as needs are satiated. On the balanced growth path effects roughly cancel; product variety increases, with the wage increasing proportionally.

6 Applications

6.0.2 Development

The model has applications to development. For most of economic history, growth in real GDP per capita was non-existent. It was only when commercialism started prior to the Industrial Revolution did the market economy begin to evolve. The model presented has the implication that growth in real GDP per capita is increasing in market hours worked. Market hours worked, in turn, depend on consumption opportunities. Consumption opportunities started to become plentiful just prior to the Industrial Revolution. The model, therefore, can broadly fit the very long run time series on growth. Furthermore, it would be interesting to apply this analysis to the so-called Malthusian trap and demographic transition. In centuries prior to the Industrial Revolution, increases in income induce

\(^{28}\)Males in subsistence societies (tribes etc) consume about 1,200 hours more leisure per year than in those in affluent modern America.
higher fertility. This was likely due to a dearth of other available “goods” to consume. One reason, surely, that fertility falls with growth is the abundance of goods one can avail of instead.

6.0.3 The Industrial Revolution

Work by economic historian Jan DeVries is very much in the spirit of my analysis. Sometime around fifty years prior to the Industrial Revolution (which began in 1750), there was what DeVries terms an “Industrious Revolution”. Namely, people began to move out of household production into market production. This was essentially a consequence of greater property rights and economic deregulation as a consequence of the Glorious Revolution in 1688, whence merchants became wealthy enough to check the power of the ruling elite. Before that, merchants were exposed to the constant threat of expropriation and taxation. With this new commercial culture people began producing for themselves; they began to specialize, and engage in exchange on a much grander scale than before. Consequently, the variety of goods available to consume increased appreciably. This, in turn, spurred an increase in work effort which DeVries describes as an “industrious revolution”. He mentions how many literary works dating from that period, convey impressions of commercial boom, a new fixation on material goods. DeVries claims that this “unleash[ed] a beneficial industriousness”.

It was this environment that laid the ground for the Industrial Revolution (or really, an Industrial “evolution”). After 1750 itself, work hours soared as workers and children endured long hours of factory work; commercialism was now truly now a fact of live. Moreover, at the initial stages of the industrial revolution, real wages did not rise for decades (until around 1820). It was only then that work effort fell, and women and children began to withdraw from the workforce.

In terms of the model, product variety increased markedly initially, as new products proliferated. Also, wages remained static. Consistent with the model predictions, labor hours per person rose appreciably initially. However, as needs become satisfied and wages rose, labor supply fell, again in accord with model predictions.

Finally, Shleifer et al have a model showing how a large initial demand for goods is crucial for the development process to start. On this theme, David Landes, in his work on the Industrial Revolution, claims society was too socially stratified in Continental Europe for a large demand for consumer goods to emerge. And given that the upper classes in Britain sought more humble luxuries, the lower classes sought to emulate them; in other words, the upper class in Britain seemed attainable and provoked work effort on behalf of lower classes. As regards cities, Landes remarks that while cities in Continental Europe were administrative centers, by contrast, they acted as commercial centers in
Britain. As regards the US, he mentions that the US winters were comparatively cold, so consumers were more interested in basic goods to keep warm etc. That precluded a market for emerging luxury consumer goods, which was necessary to stimulate work effort and innovation.

6.0.4 Europe/US Labor Differentials

In his study of OECD countries, Prescott attributes the US/European labor hours discrepancy to different rates of taxation. Tax rates increased markedly in European countries over the past thirty years, while have remained fairly stable in the United States. This divergence in tax rates, he contends, is the driving force behind the trends in labor supply across the two regions. However, this explanation has a number of deficiencies. First, rates of taxation are correlated with many other factors pertaining to the labor market. Prominent among these are levels of unionization, labor market regulations, product market regulations, welfare benefits, replacement rates, and so forth. As noted by others such as Blanchard, Prescott is silent on these issues, and the issue of causality. Second, the fall in labor supply was gradual over a period of thirty years. In particular, it was not concentrated in the period of large tax increases. Third, labor elasticities in response to wage changes are nowhere near those needed to justify Prescott’s claim.

Another explanation was posited by Alesina and Sacerdote. They maintain that the lower European work hours are a result of union demands. However, this too is unsatisfactory. Unions surely represent the desires of workers. Such demands for more leisure surely just reflect a revealed preference on their behalf. French workers, for instance, readily strike upon dissatisfaction with labor issues. They seem quite content with fewer work hours. In this vein, research by Bell and Freeman show that Europeans want to work fewer hours. It seems, then, that unionization is only a proximate cause of lower labor hours, not a fundamental one.

The third prominent explanation was made by Blanchard. He maintains that differential preferences for leisure across countries. But this begs the more fundamental question of why such differences exist in the first place. Appeals to exogenous preference differences rarely constitute convincing economic analysis. In some ways, this paper addresses Blanchard’s point. My model presents a microfoundation for preference differences. If consumption opportunities are relatively scarcer, then labor supply will be lower too. Indeed, much evidence suggests that labor and product market regulations abound in mainland Europe. This leads to less product variety, but more subtly, to diminished consumption opportunities. Factors such as restrictions on opening/closing hours abound in Europe. Here is a
description of commercial culture by two European economists, Alesina and Giavazzi:29

“In many places shop opening hours are tightly regulated.....Try to shop at 2pm in most European cities outside of major tourist centers: you will find that all shop keepers are out to lunch. Office workers looking to shop on their lunch break are out of luck. Need to go to a bank on Saturday morning? Forget it. Banks are closed on Saturdays, and on weekdays banks stay open only until 3pm or 4pm. Need to buy a newspaper in Milan on a Saturday afternoon? All newsstands are closed....If you...happen to get sick on a weekend, the only place that can sell you aspirin is a pharmacy, but again only a few pharmacies are open on weekends.”

Furthermore, product market regulations restrict competition and protect incumbent firms, thereby preventing new consumption opportunities. Older firms are under little competitive pressure to compete through introducing new products. This is particularly true for the service and nontradable sectors which are less developed than the US. As economies develop, consumers demand more services, so these issues become more pressing. Additionally, unions ensure high real wage growth in Europe, at the cost of higher unemployment. This high real wage growth, coupled with restrictions on consumption opportunities both act to lower labor supply.

7 Conclusion

I have presented a model of long run growth, with especial emphasis on the labor supply over time. Given that labor (broadly speaking, encapsulating human capital) is the most important factor of production, an explicit modelling of labor supply seems important. Unlike other more conventional growth models, the model makes labor endogenous, and illustrates how the proliferation of new products affects labor hours supplied. In addition, it shows how welfare increases over time. Few macroeconomic models incorporate the reality, and attendant welfare improvement, due to an expanding product space over time.

Furthermore, the framework has a number of applications, both methodological and explanatory. In particular, it presents a utility function consistent with stable labor hours on a balanced growth path. The formulation is reasonable in that casual introspection suggests that new products increase the marginal utility of consumption. The standard neoclassical labor leisure model dictates that this

fact alone is sufficient to maintain stable labor hours. By contrast, current formulations consistent with this stylized fact seem contrived or devoid of empirical support. My formulation also offers a resolution to the risk free rate puzzle. Other applications to financial theory seem conceivable, given the stochastic discount factor is a function of marginal utility - the focus of my analysis. The model also offers an explanation of disparate labor hours across countries. In particular, it contributes to the topical debate over European/US labor supply differentials. As opposed to current explanations, this paper emphasizes distortions in European product markets. If historical experience suggests product variety, and commercialism more generally, enhances work effort, then, surely, such forces can be operative today as well. Overall, the insights of the model seem a promising path for future work.