Dynamic Merger Review*

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Abstract

We analyze the optimal dynamic policy of an antitrust authority towards horizontal mergers when merger proposals are endogenous and occur over time. Approving a currently proposed merger will affect the profitability and welfare effects of potential future mergers, the characteristics of which may not yet be known to the antitrust authority. We show that, in many cases, this apparently difficult problem has a simple resolution: an antitrust authority can maximize discounted consumer surplus by using a completely myopic merger review policy that approves a merger today if and only if it does not lower consumer surplus given the current market structure.

1 Introduction

The traditional approach to the review of horizontal mergers stresses the tradeoff between market power and efficiencies. Mergers, which cause firms to internalize pricing externalities among former rivals, increase the exercise of market power, and therefore tend to reduce social welfare. On the other hand, since they can create efficiencies, horizontal mergers may instead increase welfare. This

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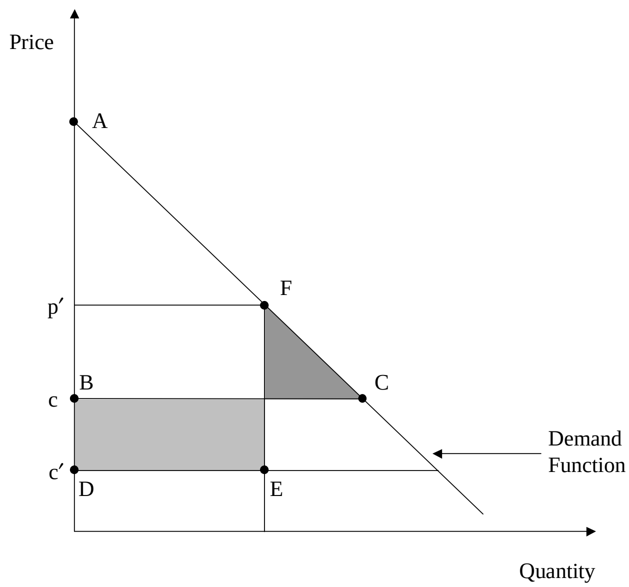

Figure 1: The Williamson tradeoff of merger approval: deadweight loss of market power (dark-shaded triangle) *vs.* efficiency gain (light-shaded rectangle).

tradeoff was first articulated by Williamson [1968] for the case of an antitrust authority who wants to maximize aggregate surplus, using a diagram like Figure 1. In the diagram, a competitive industry merges to become a monopolist that charges the price p', but lowers its marginal cost of production from c to c'. Whether aggregate surplus increases or not depends on whether the dark-grey deadweight loss triangle exceeds the light-grey efficiency gain. A similar, though even more straightforward tradeoff arises when an antitrust authority instead applies a consumer surplus standard to merger approval decisions.[1] In that case, the marginal cost reduction must be large enough that the price does not increase for the merger to be approved.

More recently, Farrell and Shapiro [1990] (see also McAfee and Williams [1992]) have provided a more complete and formal analysis of this tradeoff for the context of Cournot competition. Farrell and Shapiro provide a necessary and sufficient condition for a merger to increase consumer surplus, as well as a sufficient condition for a merger to increase aggregate surplus.

With few exceptions, however, the literature on merger review has focused on the approval decision for a single merger. Yet, in reality, mergers are usually

[1] Note that in both the U.S. and the EU, the legal standard of merger policy is close to being a consumer surplus standard rather than an aggregate surplus standard.

not one-time events.[2] That is, one proposed merger in an industry may be followed by others. In that case, approval of a merger today based on current conditions, as in the Farrell and Shapiro test, appears inappropriate. Rather, an antitrust authority in general needs to determine the welfare effect of the current proposed merger given the potential for future merger approvals, and given the fact that today's merger approval decision may alter the set of mergers that are later proposed.

In this paper, we show that in some cases this apparently difficult problem has a very simple resolution: an antitrust authority who wants to maximize consumer surplus can accomplish this objective by using a completely myopic merger review policy that approves a merger today if and only if it does not lower consumer surplus given the current market structure.

We begin in Section 2 by describing our basic model and establishing some preliminary characterizations of consumer surplus-enhancing mergers and their interactions. Our central results focus on a model of Cournot competition with constant returns to scale. Most importantly, we show in this section that there is a form of complementarity among consumer surplus-enhancing mergers in this setting.

Section 3 contains our main result. There we imbed our Cournot competition framework in a dynamic model in which merger opportunities arise, and may be proposed, over time. We show that if the set of possible mergers is disjoint, and if mergers that are not approved in a given period may be approved at a later date, then a completely myopic consumer surplus-based approval policy maximizes discounted consumer surplus for every possible realization of the set of feasible mergers.

In Section 4, we discuss the robustness of this result, considering other models of competition (homogeneous and differentiated product price competition), nonconstant returns to scale, firm entry and exit, continuing innovation, nondisjointness of mergers, and the use of an aggregate surplus criterion.

Section 5 concludes.

2 Mergers in the Cournot Model

2.1 Cournot Oligopoly

Consider an industry with n firms producing a homogeneous good and competing in quantities. Let $N \equiv \{1, 2, ..., n\}$ denote the set of firms. Firm i's cost of producing q_i units of output is given by $C_i(q_i) = c_i q_i$. Thus, for now, we restrict attention to firms producing under constant returns to scale. The inverse market demand is given by the twice differentiable function $P(Q)$, where

[2] Motta and Vasconcelos [2005] is the one paper we are aware of that considers merger review in a dynamic context. Nilssen and Sorgard [1998], Gowrisankaran [1999], Fauli-Oller [2000], and Pesendorfer [2005] are among the articles that study equilibrium merger decisions in dynamic settings without considering merger policy (and, usually, without allowing for efficiencies).

$Q \equiv \sum_{i \in N} q_i$ is industry output. We make the following (standard) assumption on demand.[3]

Assumption 1 *For any $Q > 0$ such that $P(Q) > 0$:*

(i) $P'(Q) < 0$;

(ii) $P'(Q) + QP''(Q) < 0$;

(iii) $\lim_{Q \to \infty} P(Q) < \min_i c_i$;

(iv) $\lim_{Q \to 0} P(Q) > \min_i c_i$.

Part (i) of the assumption says that demand is downward-sloping, part (ii) implies that quantities are strategic substitutes and that each firm's profit maximization problem is strictly concave, parts (iii) and (iv) imply that the equilibrium aggregate output is positive but bounded.

Let $Q_{-i} \equiv \sum_{j \neq i} q_j$ denote the aggregate output of all firms other than i. Firm i's best-response function, $b(Q_{-i}; c_i)$, is

$$b(Q_{-i}; c_i) = \arg\max_{q_i} [P(Q_{-i} + q_i) - c_i] q_i. \tag{1}$$

As is well known (see e.g., Farrell and Shapiro [1990]), Assumption 1 implies that each firm's best-response function $b(\cdot; c_i)$ satisfies $\partial b(Q_{-i}; c_i)/\partial Q_{-i} \in (-1, 0)$ at all Q_{-i} such that $b(Q_{-i}; c_i) > 0$.

Under Assumption 1, there is a unique Nash equilibrium. Let Q^* and q_i^* denote industry output and firm i's output in equilibrium. From the first-order condition for problem (1), output levels in this equilibrium satisfy

$$q_i^* = -\frac{P(Q^*) - c_i}{P'(Q^*)} \tag{2}$$

if $c_i < P(Q^*)$, and $q_i^* = 0$ otherwise. Assumption 1 also implies that the equilibrium is "stable," so that comparative statics are "well behaved." For example, we will make use of two comparative statics properties: First, a reduction in an active firm's marginal cost increases its equilibrium output and profit, reduces the output of each of its active rivals, and increases aggregate output. Second, following any change in the incentives of a subset of firms, the equilibrium aggregate output increases [decreases] if and only if the equilibrium output of that set of firms increases [decreases].[4]

2.2 The CS-Effect of Mergers

Consider a merger between a subset $M \subseteq N$ of firms. The post-merger marginal cost is denoted $\overline{c}_M$. Aggregate output before the merger is Q^*, and after is $\overline{Q}^*$.

[3] We assume that this assumption holds for all possible market structures and cost positions that may emerge through the merger review process.

[4] See Farrell and Shapiro [1990]'s Lemma, p. 111.

We are interested in the effect of the merger on consumer surplus, $CS(\overline{Q}^*) - CS(Q^*)$, where

$$CS(Q) = \int_0^Q [P(s) - P(Q)]\, ds.$$

Since $CS'(Q) = -QP'(Q) > 0$, a merger raises consumer surplus if and only if it induces an increase in industry output. We will say that a merger is *CS-neutral* if the merger does not affect consumer surplus. Similarly, we will say that a merger is *CS-increasing* [*CS-decreasing*], if consumer surplus following the merger is higher [lower] than before. Finally, a merger is *CS-nondecreasing* [*CS-nonincreasing*] if it is not CS-decreasing [CS-increasing].

We will say that a merger involves active firms if at least one of the merging firms is producing a positive quantity before the merger [and hence has $c_i < P(Q^*)$] . Observe that a merger involving only inactive firms is always CS-nondecreasing and weakly profitable. The following result catalogs some useful properties of CS-neutral mergers among active firms.

Lemma 1 *If a merger involving active firms is CS-neutral then*

1. *it causes no changes in the output of any nonmerging firm nor the total output of the merging firms;*

2. *the merged firm's margin at the pre-merger price $P(Q^*)$ equals the sum of the active merging firms' pre-merger margins:*

$$P(Q^*) - \overline{c}_M = \sum_{i \in M} \max\{0, P(Q^*) - c_i\}; \tag{3}$$

3. *the merged firm's marginal cost is no greater than the marginal cost of the most efficient merger partner: $\overline{c}_M \leq \min_{i \in M}\{c_i\}$, and it is strictly less if the merger involves at least two active firms;*

4. *the merger is profitable (it weakly raises the joint profit of the merging firms), and is strictly profitable if it involves at least two active firms.*

Proof. To see Property 1, observe that under Assumption 1 there is a unique output level for each non-merging firm i that is compatible with any given level of aggregate output Q [since there is a unique Q_{-i} such that $Q_{-i} + b(Q_{-i}; c_i) = Q$]. Since aggregate output is unchanged by a CS-neutral merger, all nonmerging firms' outputs are unchanged. In turn, this implies that the total output of the merging firms must be unchanged as well. For Property 2, a central feature in Farrell and Shapiro [1990]'s analysis, note that the merged firm's first-order condition is

$$P(Q^*) - \overline{c}_M + \left(\sum_{i \in M} q_i^*\right) P'(Q^*) = 0. \tag{4}$$

Summing up the pre-merger first-order conditions of the active merger partners yields

$$\sum_{i \in M_+} [P(Q^*) - c_i] + \sum_{i \in M_+} q_i^* P'(Q^*) = 0 \tag{5}$$

where $M_+ = \{i \in M : q_i^* > 0\}$. Since for all $i \in M \backslash M_+$, we have $P(Q^*) \leq c_i$ and $q_i^* = 0$, it follows that

$$\sum_{i \in M} \max\{0, P(Q^*) - c_i\} + \left(\sum_{i \in M} q_i^*\right) P'(Q^*) = 0 \tag{6}$$

Combining equations (4) and (6), yields (3). Property 3 follows directly from Property 2. Property 4 holds since the merging firms' joint output has not changed (Property 1), but its margin has weakly increased, and has strictly increased if the merger involves at least two active firms (Property 2). ∎

The following useful corollary follows from Properties 2 and 4 of Lemma 1 plus the fact that the post-merger aggregate output, $\overline{Q}^*$, and the profit of the merged firm are both decreasing in the merged firm's marginal cost, $\overline{c}_M$:

Corollary 1 *A merger involving active firms is CS-neutral if*

$$\overline{c}_M = \widehat{c}_M(Q^*) \equiv P(Q^*) - \sum_{i \in M} \max\{0, P(Q^*) - c_i\},$$

CS-increasing if $\overline{c}_M < \widehat{c}_M(Q^*)$, and CS-decreasing if $\overline{c}_M > \widehat{c}_M(Q^*)$. Moreover, any CS-nondecreasing merger is profitable for the merging firms, and is strictly profitable if it is CS-increasing or involves at least two active firms.

Thus, an antitrust authority concerned with maximizing consumer surplus and confronted with a single merger involving active firms in set M would want to approve the merger if and only if $\overline{c}_M \leq \widehat{c}_M(Q^*)$. Moreover, any merger the antitrust authority would want to approve is profitable for the merging parties and hence will be proposed.

Observe also that the threshold $\widehat{c}_M(Q^*)$ is nondecreasing in Q^*; the larger is Q^* (and the lower is the current price), the more likely it is that a merger is CS-nondecreasing. This fact will play a central role in the next subsection when we look at interactions among mergers. To see the intuition for this result, consider a proposed merger between symmetric firms, each of whom has a pre-merger marginal cost c and produces $q > 0$ units. Since the firms are choosing their outputs optimally before the merger, a lower pre-merger margin $P(Q^*) - c$ (due to a larger pre-merger aggregate output) implies a smaller pre-merger value of $P'(Q^*)q$ [see (2)]. The incentives of the merged firm to raise price, however, depend on a comparison of the merger's marginal cost reduction $\Delta c = (c - \overline{c}_M)$ to the market power effect, $P'(Q^*)q$, which reflects the internalization of the pricing externality between the merging firms. With a CS-nondecreasing merger, the first effect weakly exceeds the second. A smaller pre-merger price preserves this relation and therefore the CS-nondecreasing effect of the merger.

Figure **??** illustrates the cases of CS-neutral, CS-increasing, and CS-decreasing mergers. The figure considers a merger involving the firms in set M_1. The complementary set of firms is denoted $M_2 \equiv N \backslash M_1$. The axes in the figure measure the joint outputs of the two sets of firms. The curves labeled r_{M_1} and r_{M_2} depict what we call the "group-reaction functions" of each set of firms prior to the merger. Specifically, M_i's pre-merger group-reaction function gives the joint pre-merger Nash-equilibrium output of the firms in M_i, $r_{M_i}(q_{M_j})$, conditional on the firms in M_j jointly producing q_{M_j}. It is routine to verify that these group-reaction functions satisfy $-1 < r'_{M_i}(q_{M_j}) < 0$.

The equilibrium before the merger is point A, the intersection of the two pre-merger group-reaction curves. With a CS-neutral merger, the post-merger best-response curve of the merged firm, $b(q_{M_2}; \widehat{c}_{M_1}(Q^*))$, intersects group M_2's group-reaction curve, $r_{M_2}(\cdot)$, at point A.[5] With a CS-increasing merger, the merged firm's marginal cost is less than $\widehat{c}_{M_1}(Q^*)$, so its best-response curve lies further to the right, shifting the equilibrium to point B, where there is a larger aggregate output. In contrast, with a CS-decreasing merger, the merged firm's marginal cost is greater than $\widehat{c}_{M_1}(Q^*)$, so its best-response curve lies further to the left, shifting the equilibrium to point C where there is a smaller aggregate output.

2.3 Interactions between Mergers

We now turn to the interactions between mergers. In this subsection, we consider two potential disjoint mergers, involving firms in sets M_1 and M_2 with $M_1 \cap M_2 = \emptyset$. We'll refer to these simply as merger M_1 and merger M_2. The set of firms not involved in either merger is $N^c \equiv N \backslash (M_1 \cup M_2)$.

Our first result establishes a certain complementarity between mergers that change consumer surplus in the same direction:[6]

Proposition 1 *The sign of the CS-effect of two disjoint mergers is complementary:*

(i) *if a merger is CS-nondecreasing (and hence profitable) in isolation, it remains CS-nondecreasing (and hence profitable) if another merger that is CS-nondecreasing in isolation takes place;*

(ii) *if a merger is CS-decreasing in isolation, it remains CS-decreasing if another merger that is CS-nonincreasing in isolation takes place.*

[5] The fact that it lies to the right [left] of the pre-merger group-reaction curve of M_1 for $q_{M_2} > [<] q^*_{M_2}$ can be shown using logic similar to that leading to Corollary 1. Specifically, any post-merger best-response curve for the merged firm must cross the merged firms' pre-merger group-reaction curve once at the non-merging firms' joint output $\overline{q}_{M_2}$ such that $\widehat{c}_{M_1}(\overline{q}_{M_2} + r_{M_1}(\overline{q}_{M_2})) = \overline{c}_{M_1}$. Moreover, this crossing must be from above: at $q_{M_2} > [<] \overline{q}_{M_2}$, we have

$$\begin{aligned} b(q_{M_2}; \overline{c}_{M_1}) &= b(q_{M_2}; \widehat{c}_{M_1}(\overline{q}_{M_2} + r_{M_1}(\overline{q}_{M_2}))) \\ &> [<] b(q_{M_2}; \widehat{c}_{M_1}(q_{M_2} + r_{M_1}(q_{M_2}))) \\ &= r_{M_1}(q_{M_2}). \end{aligned}$$

[6] Proposition 1 focuses on properties we use later in this section. It is straightforward to show as well that a CS-increasing merger remains CS-increasing if a merger that is CS-nondecreasing takes place, and that a merger that is CS-nonincreasing remains CS-nonincreasing if a merger that is CS-nonincreasing takes place.

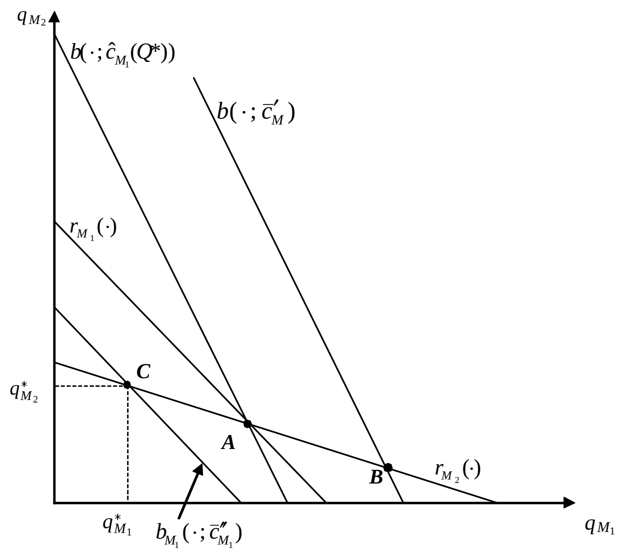

Figure 2: A merger involving the firms in M_1. Depending on the merged entity's marginal cost, the merger is CS-neutral, CS-increasing, or CS-decreasing. In the figure, $\overline{c}''_{M_1} > \widehat{c}_{M_1}(Q^*) > \overline{c}'_{M_1}$.

Proof. For part (i), suppose that mergers M_1 and M_2 are both CS-nondecreasing in isolation. Let Q^* denote aggregate output in the absence of either merger and let $\overline{Q}_i^*$ denote aggregate output if only merger i takes place. So $\overline{Q}_i^* \geq Q^*$ for $i = 1, 2$. Suppose, first, that merger M_i involves only inactive firms. Then since $P(\overline{Q}_j^*) \leq P(Q^*) \leq c_i$, it also involves only inactive firms once merger j takes place. Thus, it remains CS-nondecreasing and (weakly) profitable.

Suppose, instead, that merger M_i involves active firms. From Corollary 1 we know that $\overline{c}_{M_i} \leq \widehat{c}_{M_i}(Q^*)$ for $i = 1, 2$. Since the threshold $\widehat{c}_{M_i}(Q)$ is nondecreasing in Q, we have $\overline{c}_{M_i} \leq \widehat{c}_{M_i}(\overline{Q}_j^*)$ for $i, j = 1, 2$, $i \neq j$. That is, each merger is also CS-nondecreasing once the other merger has taken place. The argument for part (ii) follows similar lines (note that a CS-decreasing merger must involve active firms). ■

Figure **??** illustrates the complementarity between two mergers that are CS-increasing in isolation when no other firms exist ($N^c = \emptyset$). In isolation, merger M_1 moves the equilibrium from point A to point B, while merger M_2 moves the equilibrium from point A to point C. But, conditional on merger M_1 taking place, merger M_2 moves the equilibrium from point B to point D along $b(\cdot; \overline{c}_{M_1})$. Since $\partial b(\cdot; \overline{c}_{M_1})/\partial Q_{-i} \in (-1, 0)$, aggregate output must increase. That is, conditional on merger M_1 taking place, merger M_2 remains CS-increasing. Moreover, we know from Corollary 1 it also remains profitable.

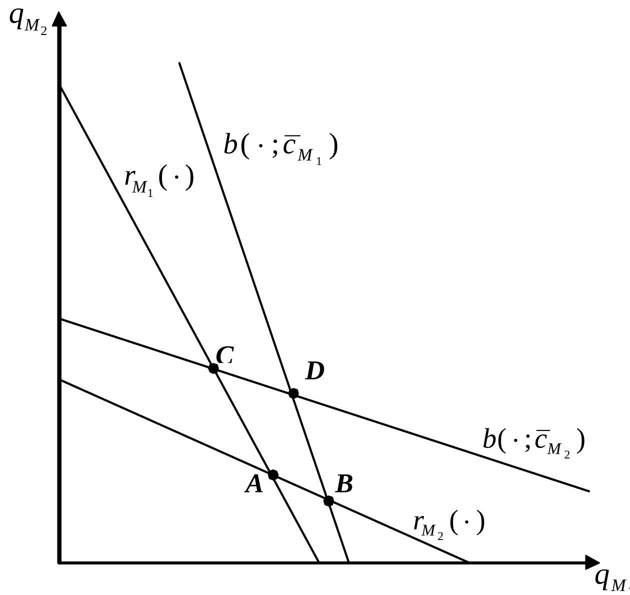

Figure 3: Each merger is CS-increasing in isolation and remains so if the other merger takes place.

Using the same type of argument, the reverse is also true: conditional on merger M_2 taking place, the merger M_1 remains CS-increasing and profitable.

Figure 4 illustrates the case in which both mergers are CS-decreasing in isolation. Conditional on the other merger taking place, each merger remains CS-decreasing. For instance, conditional on merger M_1 taking place, merger M_2 moves the equilibrium from point B to point D along $b(\cdot;\overline{c}_{M_1})$. Since $\partial b(\cdot;\overline{c}_{M_1})/\partial Q_{-i} \in (-1,0)$, it follows that the merger reduces industry output.

We now turn to the interaction between mergers that have opposite effects on consumer surplus if implemented in isolation. Specifically, suppose that merger M_1 is CS-nondecreasing (and therefore profitable) in isolation, while merger M_2 is CS-decreasing in isolation. Figure **??** illustrates that merger M_2 can become CS-increasing (and therefore strictly profitable) conditional on merger M_1 occurring. In isolation, merger M_2 moves the equilibrium from point A to point C along $r_{M_1}(\cdot)$, and thus decreases industry output. But after merger M_1 has taken place, merger M_2 moves the equilibrium from point B to point D along $b(\cdot;\overline{c}_{M_1})$, and thus increases industry output.

When this occurs, we can say the following:

Proposition 2 *Suppose that merger M_1 is CS-nondecreasing in isolation, while merger M_2 is CS-decreasing in isolation but CS-nondecreasing once merger M_1*

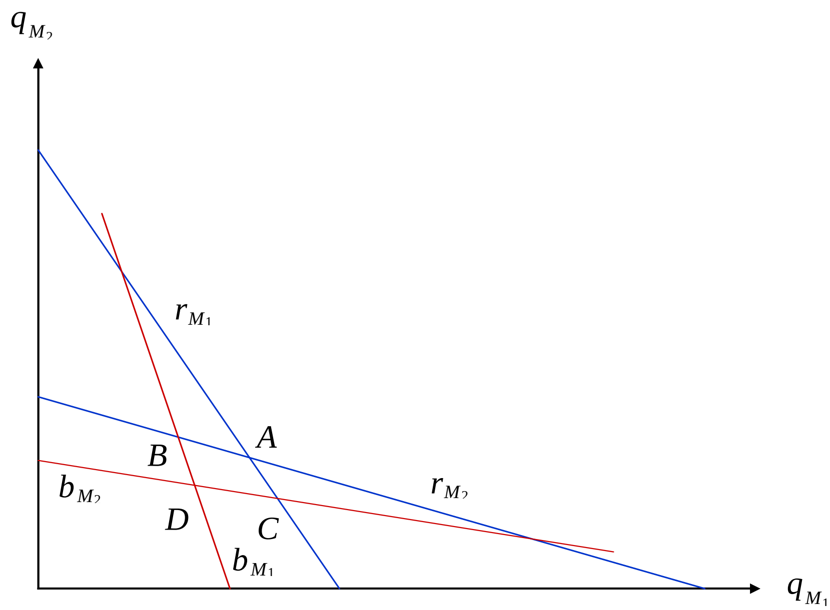

Figure 4: Each merger is CS-decreasing in isolation and remains so if the other takes place.

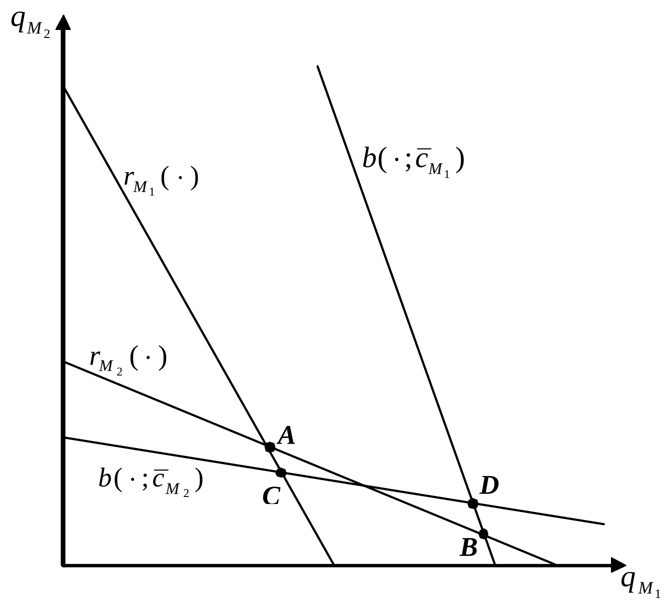

Figure 5: A CS-decreasing merger can become CS-increasing after a CS-increasing merger takes place.

has taken place. Then:

(i) *merger M_1 is CS-increasing (and therefore profitable) conditional on merger M_2 taking place;*

(ii) *the joint profit of the firms involved in merger M_1 is at least as large if both mergers take place than if neither merger takes place.*

Proof. Note, first, that if merger M_1 changes merger M_2 from CS-decreasing to CS-nondecreasing, then merger M_1 must in fact be CS-increasing in isolation [by Proposition 1(ii)]. Consider implementing merger M_1 first followed by merger M_2. By hypothesis, consumer surplus weakly increases at each step, so the combined effect on consumer surplus of the two mergers is nonnegative. Suppose we now reverse the order and implement merger M_2 first. Since the combined effect of the two mergers on consumer surplus is nonnegative while the effect of merger M_2 is strictly negative, consumer surplus must strictly increase when merger M_1 is implemented following merger M_2. Hence, part (i) must hold: merger M_1 is CS-increasing (and therefore strictly profitable) conditional on merger M_2 taking place.

To see that part (ii) holds, suppose that merger M_2 is implemented first. Since merger M_2 is CS-decreasing in isolation, it must weakly increase the profit of each firm $i \in M_1$ [the joint output of all firms other than i must decrease, otherwise the fact that $\partial b(\cdot; c_i)/\partial Q_{-i} \in (-1, 0)$ would imply that aggregate output increases]. Since merger M_1 is strictly profitable given merger M_2, the sequence of mergers must strictly increase the joint profit of the firms in M_1. ■

The result is illustrated in Figure **??**, where merger M_1 is CS-increasing (and hence strictly profitable) in isolation and remains so conditional on merger M_2 taking place: it moves the equilibrium from point C to point D along $b(\cdot; \overline{c}_{M_2})$.

Remark 1 *The proof makes clear that the merger is in fact strictly profitable in part (i), and the profit of the firms in part (ii) is strictly larger.*[7]

Remark 2 *Observe that the logic of Proposition 2 can be extended to cases with a merger M_1 that is CS-nondecreasing in isolation and a collection of mergers $M_2, ..., M_K$ that are each CS-decreasing in isolation but form a sequence that is CS-nondecreasing at each step after merger M_1 has taken place. In such cases, merger M_1 is CS-increasing (and therefore strictly profitable) given that mergers $M_2, ..., M_K$ have taken place, and the joint profit of the firms involved in merger M_1 is strictly larger if all of these mergers take place than if none do. We will use this extension of Proposition 2 in Section 3.*

[7] We state the result in this weaker form because only the weaker result is necessary for our later results, and we will later show that mergers with Bertrand price competition satisfy these same properties.

3 CS-Maximizing Merger Review

In this section, we consider the optimal merger approval policy for an antitrust authority concerned with maximizing discounted consumer surplus when multiple disjoint mergers may be proposed over time. We will show that, in the Cournot model of Section 2, a consumer surplus-oriented antitrust authority can achieve its optimal outcome using a myopic policy that approves mergers if and only if they are CS-nondecreasing at the time of approval.

As before, we denote the set of n firms by N. The set of possible mergers are those in set $\mathbb{A} \equiv \{M_1, ..., M_K\}$, where $M_k \subseteq N$ is a set of firms that may merge. We assume that possible mergers are disjoint; that is, $M_j \cap M_k = \emptyset$ for $j \neq k$. Thus, no firm has the possibility of being part of more than one merger. The merger process lasts for T periods. At the start of each period t, merger M_k may become feasible with probability $p_{kt} \in [0, 1]$, where $\sum_t p_{kt} \leq 1$.[8] Conditional on merger M_k becoming feasible in period t, the firms in M_k receive a random draw of their post-merger cost $\overline{c}_{M_k}$ according to the distribution F_{kt}. This formulation embodies another form of disjointness in merger possibilities: merger M_k receives at most one efficiency realization throughout the merger process.

Within each period t there are n stages at which the antitrust authority can approve a merger. For simplicity, at most one merger can be approved at each stage. At the start of each period, all firms with feasible mergers decide whether to propose them or not. We assume that firms split the gains from their merger in some fixed proportion (the proportion does not matter).[9] A merger is proposed if all of the merger partners agree to propose it. Given the weak dominance refinement we employ below, a merger is therefore proposed if it weakly raises the joint expected discounted expected profit of the potential merger partners. (We assume firms choose to propose a merger if indifferent.)

The antitrust authority observes that a particular merger is feasible and its efficiency (post-merger marginal cost) only once it is proposed. Firms observe their own merger possibility when it becomes feasible but, like the antitrust authority, observe the possibilities of other firms only when those mergers are proposed. Previously proposed but rejected mergers can be either withdrawn or proposed again, as can previously unproposed feasible mergers.[10] Payoffs in each period depend only on the set of mergers approved at the end of the period. The antitrust authority and the firms discount future payoffs (consumer surplus or profit) according to the discount factor $\delta \leq 1$.

[8] Since a merger that results in a post-merger marginal cost above the marginal cost of the most efficient merging firm will never be approved, this "feasibility" may be viewed as the event of receiving a cost draw below that level.

[9] For simplicity, we do not formally model bargaining among the merger partners.

[10] In the model, we do not allow previously approved mergers to be dissolved. However, it follows from our arguments that no (approved) merged firm would want to do so.

3.1 Static Merger Review

It will be useful to consider first merger review in a static setting, corresponding to the case in which $T = 1$. In this case, firms simultaneously propose mergers at the start of the period, and the antitrust authority then reviews them one at a time.

In Section 2, we examined the interaction between two mergers. For the purposes of this section, we start by noting some useful properties of the interactions among more than two mergers:

Lemma 2 ***(Incremental Gain Lemma)***

(i) Suppose that a set of mergers $\mathfrak{M} \equiv \{M_1, ..., M_{J_1}\}$ has the property that every merger $M \in \mathfrak{M}$ is CS-nondecreasing if all of the other mergers in $\mathfrak{M}$ (those in the set $\mathfrak{M} \backslash M$) have taken place. Then for any strict subset $Y \subset \mathfrak{M}$, there exists an $M' \in \mathfrak{M} \backslash Y$ that is CS-nondecreasing if all of the mergers in Y have taken place. As a result, starting from Y, there is a sequencing of the mergers in $\mathfrak{M} \backslash Y$ that is CS-nondecreasing at each step.

(ii) Suppose that a sequence of mergers $M_1, ..., M_{J_1}$ is CS-nondecreasing at each step. Then each merger $M \in \mathfrak{M} \equiv \{M_1, ..., M_{J_1}\}$ is CS-nondecreasing if all of the other mergers in $\mathfrak{M}$ (those in the set $\mathfrak{M} \backslash M$) have taken place.

Proof. (i) Suppose the result is not true, so that every $M' \in \mathfrak{M} \backslash Y$ is CS-decreasing if all of the mergers in Y have taken place. Proposition 1(ii) implies that, taking the mergers in Y as given, for any sequencing of the mergers in the set $\mathfrak{M} \backslash Y$ the merger implemented at each step, including the last step, is CS-decreasing. But this contradicts the hypothesis that the last merger in the sequence is CS-nondecreasing if all of the other mergers in the set $\mathfrak{M}$ have taken place.

Given the existence of $M' \in \mathfrak{M} \backslash Y$ that is CS-nondecreasing if all of the mergers in Y have taken place, we can update the subset Y to $Y \cup \{M'\} \subset \mathfrak{M}$ and apply the same argument again. Continuing iteratively identifies a sequencing of the mergers in $\mathfrak{M} \backslash Y$ that is CS-nondecreasing at each step starting from the subset Y.

(ii) Consider an arbitrary merger M_j in sequence $M_1, ..., M_{J_1}$. We will show that M_j is CS-nondecreasing given that all of the mergers in $\mathfrak{M} \backslash M_j$ have taken place. For $k \geq 0$, define the set $\mathfrak{M}^k = \{M_i : i \leq k\}$. Suppose that (a_k) merger M_j is CS-nondecreasing given $\mathfrak{M}^k \backslash M_j$ and that (b_k) merger M_{k+1} is CS-nondecreasing given $\mathfrak{M}^k$. Observe that property (a_k) is true by hypothesis for $k = j$, and that property (b_k) holds by hypothesis for all k. We claim that properties (a_k) and (b_k) imply property (a_{k+1}): M_j is CS-nondecreasing given $\mathfrak{M}^{k+1} \backslash M_j$. To see this, observe that if merger M_{k+1} is CS-nondecreasing given $\mathfrak{M}^k \backslash M_j$, property (a_{k+1}) follows from Proposition 1(i), while if merger M_{k+1} is CS-decreasing given $\mathfrak{M}^k \backslash M_j$ [which can hold only if $j \leq k$] then property (a_{k+1}) follows from Proposition 2(i) [and the assumption that M_{k+1} is CS-nondecreasing given $\mathfrak{M}^k$]. Applying induction we find that merger M_j is CS-nondecreasing given that all of the mergers in $\mathfrak{M} \backslash M_j$ have taken place [property (a_{J_1})]. ■

Lemma 3 *Suppose that two distinct sets of mergers* $\mathfrak{M}_1 \equiv \{M_1, ..., M_{J_1}\}$ *and* $\mathfrak{M}_2 \equiv \{M_1, ..., M_{J_2}\}$ *with* $\mathfrak{M}_1 \nsubseteq \mathfrak{M}_2$*, not necessarily disjoint, each have the property that every merger* $M \in \mathfrak{M}_i$ *is CS-nondecreasing if all of the other mergers in* $\mathfrak{M}_i$ *(those in the set* $\mathfrak{M}_i \backslash M$*) have taken place. Then:*

(i) there is a merger $M_1' \in \mathfrak{M}_1 \backslash (\mathfrak{M}_1 \cap \mathfrak{M}_2)$ *that is CS-nondecreasing given that all of the mergers in* $\mathfrak{M}_2$ *have taken place;*

(ii) the set of mergers $\mathfrak{M}_1 \cup \mathfrak{M}_2$ *results in a level of consumer surplus that is at least as great as that of either set* $\mathfrak{M}_1$ *or set* $\mathfrak{M}_2$*.*

Proof. (i) Part (i) of the Incremental Gain Lemma [Lemma 2(i)] implies that there exists a merger $M_1' \in \mathfrak{M}_1 \backslash (\mathfrak{M}_1 \cap \mathfrak{M}_2)$ that is CS-nondecreasing given that all of the mergers in $\mathfrak{M}_1 \cap \mathfrak{M}_2$ have taken place. It also implies that there is a sequencing of the mergers in $\mathfrak{M}_2 \backslash (\mathfrak{M}_1 \cap \mathfrak{M}_2)$, say $M_{21}, ..., M_{2J_2}$, that is CS-nondecreasing at each step, given that the mergers in $\mathfrak{M}_1 \cap \mathfrak{M}_2$ have taken place. Let $\mathfrak{M}^k = \{M_{2i} : i \leq k\}$. Proposition 1(i) implies that if merger M_1' is CS-nondecreasing given that all the mergers in $(\mathfrak{M}_1 \cap \mathfrak{M}_2) \cup \mathfrak{M}^k$ have taken place, then [since by hypothesis merger $M_{2,k+1}$ is also CS-nondecreasing given that all of the mergers in $(\mathfrak{M}_1 \cap \mathfrak{M}_2) \cup \mathfrak{M}^k$ have taken place] M_1' is also CS-nondecreasing given that all of the mergers in $(\mathfrak{M}_1 \cap \mathfrak{M}_2) \cup \mathfrak{M}^{k+1}$ have taken place. Since merger M_1' is CS-nondecreasing if all of the mergers in $(\mathfrak{M}_1 \cap \mathfrak{M}_2) \cup \mathfrak{M}^0 = (\mathfrak{M}_1 \cap \mathfrak{M}_2)$ have taken place, applying induction yields the result (taking $k = J_1$).

(ii) We argue first that every merger in set $\mathfrak{M}_2 \cup \{M_1'\}$ is CS-nondecreasing if all of the other mergers in that set have taken place. Part (i) implies that this is true for merger M_1'. Now consider any merger $M_2' \in \mathfrak{M}_2$. By hypothesis, merger M_2' is CS-nondecreasing given that all the mergers in set $\mathfrak{M}_2 \backslash M_2'$ have taken place. If merger M_1' is also CS-nondecreasing if all of the mergers in set $\mathfrak{M}_2 \backslash M_2'$ have taken place, then Proposition 1(i) implies that merger M_2' is CS-nondecreasing if all of the mergers in $(\mathfrak{M}_2 \backslash M_2') \cup \{M_1'\} = (\mathfrak{M}_2 \cup \{M_1'\}) \backslash M_2'$ have taken place. If, instead, merger M_1' is CS-decreasing if all of the mergers in set $\mathfrak{M}_2 \backslash M_2'$ have taken place, then Proposition 2(i) implies that this same property holds. This establishes that every merger in $\mathfrak{M}_2 \cup \{M_1'\}$ is CS-nondecreasing if all of the other mergers in that set have taken place. Moreover, by part (i), the level of consumer surplus with set $\mathfrak{M}_2 \cup \{M_1'\}$ is at least as large as with set $\mathfrak{M}_2$.

Next, note that sets $\mathfrak{M}_1$ and $\mathfrak{M}_2 \cup \{M_1'\}$ satisfy the hypotheses of the Lemma. So we can apply the argument again for these two sets. Continuing iteratively in this fashion we establish the result by adding to $\mathfrak{M}_2$ a sequencing of the mergers in $\mathfrak{M}_1 \backslash (\mathfrak{M}_1 \cap \mathfrak{M}_2)$ that is CS-nondecreasing at each step. This establishes that the level of consumer surplus is at least as high with set $\mathfrak{M}_1 \cup \mathfrak{M}_2$ as with set $\mathfrak{M}_2$. We also need to show that the level of consumer surplus in $\mathfrak{M}_1 \cup \mathfrak{M}_2$ is at least as large as in set $\mathfrak{M}_1$. If $\mathfrak{M}_1 \supseteq \mathfrak{M}_2$, so that $\mathfrak{M}_1 \cup \mathfrak{M}_2 = \mathfrak{M}_1$, this follows immediately. If instead $\mathfrak{M}_1 \nsupseteq \mathfrak{M}_2$, then we can repeat the argument above with the roles of $\mathfrak{M}_1$ and $\mathfrak{M}_2$ reversed to establish the result. ■

We now define what we mean by a myopic merger review policy:[11]

[11] Note that, for simplicity, we resolve indifference in favor of approval.

Definition 1 *A* ***myopic CS-based merger policy*** *approves at each stage a proposed merger that is CS-nondecreasing given the market structure at the start of that stage, if any such mergers exist.*

A myopic CS-based merger policy results in a sequence of merger approvals that are each CS-nondecreasing at the time of their approval, and all proposed mergers that are not approved are CS-decreasing given the final set of approved mergers.

In the static (one-period) setting, the only thing that matters for the antitrust authority's payoff is the set of approved mergers at the end of the period. This, of course, depends on the realization of the set of feasible mergers (including their cost realizations). The antitrust authority does not know which mergers are feasible. It only sees proposed mergers. Nonetheless, we will see that any myopic CS-based rule maximizes consumer surplus for each realization of the set of feasible mergers. That is, the antitrust authority does as well as if it knew the set of feasible mergers and could implement whichever ones it wanted. To this end, we introduce the following notion of ex-post optimality, defined relative to a particular realization of the set of feasible mergers.

Definition 2 *Let $\mathfrak{F}$ denote the set of feasible mergers (including their cost realizations). A set of approved mergers $\mathfrak{M}$ is* ***CS-maximizing for*** *$\mathfrak{F}$ if it maximizes consumer surplus given $\mathfrak{F}$. It is a* ***largest CS-maximizing set for*** *$\mathfrak{F}$ if it is not contained in any other set that is CS-maximizing for $\mathfrak{F}$.*

Observe that both the largest CS-maximizing set and the set of mergers resulting from a myopic CS-based merger policy have the property that every approved merger is CS-nondecreasing given the set of approved mergers, and all unapproved mergers are CS-decreasing given the set of approved mergers. This is immediate for the largest CS-maximizing set: otherwise we could either raise consumer surplus by no longer approving a merger that is CS-decreasing given the other approved mergers, or we could find a larger CS-maximizing set by approving an additional merger that is CS-nondecreasing given the other approved mergers. For the set of mergers resulting from a myopic CS-based merger policy this property follows from part (ii) of the Incremental Gain Lemma [Lemma 2(ii)], which says that any set of mergers that results from a sequence of CS-nondecreasing merger approvals has the property that each approved merger is CS-nondecreasing given all the other approved mergers.

The properties of merger interactions we have identified imply that the largest CS-maximizing set is unique and increasing in the set of feasible mergers:

Lemma 4 *For each set of feasible mergers $\mathfrak{F}$ there is a unique largest CS-maximizing set $\mathfrak{M}^*(\mathfrak{F})$. Moreover, if $\mathfrak{F} \subset \mathfrak{F}'$ then $\mathfrak{M}^*(\mathfrak{F}) \subseteq \mathfrak{M}^*(\mathfrak{F}')$.*

Proof. Suppose that $\mathfrak{M}$ and $\mathfrak{M}'$ are both largest CS-maximizing sets for $\mathfrak{F}$. Then the sets $\mathfrak{M}$ and $\mathfrak{M}'$ satisfy the hypothesis of Lemma 3. So, by Lemma

3(b), $\mathfrak{M} \cup \mathfrak{M}'$ is CS-maximizing as well, contradicting the assumption that $\mathfrak{M}$ and $\mathfrak{M}'$ are largest CS-maximizing sets for $\mathfrak{F}$.

For the second claim, suppose $\mathfrak{M}^*(\mathfrak{F}) \not\subseteq \mathfrak{M}^*(\mathfrak{F}')$. The sets $\mathfrak{M}^*(\mathfrak{F})$ and $\mathfrak{M}^*(\mathfrak{F}')$ satisfy the hypothesis of Lemma 3 and, since $\mathfrak{M}^*(\mathfrak{F}) \subseteq \mathfrak{F} \subset \mathfrak{F}'$, all mergers in set $\mathfrak{M}^*(\mathfrak{F})$ are feasible under $\mathfrak{F}'$. Thus, under $\mathfrak{F}'$, approval of the mergers in set $\mathfrak{M}^*(\mathfrak{F}) \cup \mathfrak{M}^*(\mathfrak{F}')$ is feasible and by Lemma 3(b) is also CS-maximizing, contradicting $\mathfrak{M}^*(\mathfrak{F}')$ being the largest CS-maximizing set for $\mathfrak{F}'$. ■

We next relate the outcome of a myopic CS-based approval policy to largest CS-maximizing sets:

Lemma 5 *Suppose the antitrust authority follows a myopic CS-based merger policy and $T = 1$. Then if $\widehat{\mathfrak{F}}$ is the set of proposed mergers, the set of approved mergers is $\mathfrak{M}^*(\widehat{\mathfrak{F}})$.*

Proof. Suppose that the set of proposed mergers is $\widehat{\mathfrak{F}}$. Let $\mathfrak{M} \subseteq \widehat{\mathfrak{F}}$ denote the set of mergers resulting from a myopic CS-based merger policy. If $\mathfrak{M} \not\subseteq \mathfrak{M}^*(\widehat{\mathfrak{F}})$, then Lemma 3(b) implies that the set $\mathfrak{M} \cup \mathfrak{M}^*(\widehat{\mathfrak{F}})$ is also CS-maximizing but larger than set $\mathfrak{M}^*(\widehat{\mathfrak{F}})$, a contradiction to $\mathfrak{M}^*(\widehat{\mathfrak{F}})$ being the largest CS-maximizing set under $\widehat{\mathfrak{F}}$. Hence, $\mathfrak{M} \subseteq \mathfrak{M}^*(\widehat{\mathfrak{F}})$. If $\mathfrak{M} \subset \mathfrak{M}^*(\widehat{\mathfrak{F}})$, Lemma 3(a) implies that once the mergers in $\mathfrak{M}$ have been approved, there is a merger in $\mathfrak{M}^*(\widehat{\mathfrak{F}}) \backslash (\mathfrak{M} \cap \mathfrak{M}^*(\widehat{\mathfrak{F}}))$ that is CS-nondecreasing given that the mergers in $\mathfrak{M}$ have taken place, contradicting $\mathfrak{M}$ being the result of a myopic CS-based merger policy. ■

Lemma 5 implies that, when $T = 1$, if firms propose all feasible mergers then the outcome of any myopic CS-based merger policy is exactly the largest CS-maximizing set given the realized set of feasible mergers. The remaining issue is whether firms have an incentive to propose feasible mergers. The following proposition establishes that they do:

Proposition 3 *Suppose the antitrust authority follows a myopic CS-based merger policy and $T = 1$. It is then a weakly dominant strategy for firms with a feasible merger to propose it. When firms adopt this weakly dominant proposal strategy, the set of approved mergers is $\mathfrak{M}^*(\mathfrak{F})$, for every $\mathfrak{F}$.*

Proof. Given Lemma 4, we need only show that regardless of the proposal strategies being followed by other firms, the firms in merger M_k maximize their expected payoff by proposing their merger given that the antitrust authority uses a myopic CS-based policy. Let $\widehat{\mathfrak{F}}$ denote a realization of the set of proposed mergers if merger M_k is proposed (firms in other mergers may be using mixed strategies) and let $\widehat{\mathfrak{F}}_{-k} \equiv \widehat{\mathfrak{F}} \backslash M_k$ denote that realization without merger M_k included. By Lemma 4, $\mathfrak{M}^*(\widehat{\mathfrak{F}}_{-k}) \subseteq \mathfrak{M}^*(\widehat{\mathfrak{F}})$. If $\mathfrak{M}^*(\widehat{\mathfrak{F}}_{-k}) = \mathfrak{M}^*(\widehat{\mathfrak{F}})$, then the payoffs of firms in M_k are independent of whether they propose. Suppose instead that $\mathfrak{M}^*(\widehat{\mathfrak{F}}_{-k}) \subset \mathfrak{M}^*(\widehat{\mathfrak{F}})$. We distinguish between two cases. First, suppose that $\mathfrak{M}^*(\widehat{\mathfrak{F}}_{-k}) \cup M_k = \mathfrak{M}^*(\widehat{\mathfrak{F}})$. In this case, proposing merger M_k does not affect the other mergers that will be approved. Since $M_k \in \mathfrak{M}^*(\widehat{\mathfrak{F}})$, the merger is CS-nondecreasing and hence [by Corollary 1] profitable, given the other mergers [those in the set $\mathfrak{M}^*(\widehat{\mathfrak{F}})\backslash M_k = \mathfrak{M}^*(\widehat{\mathfrak{F}}_{-k})$] that will be approved. Second, suppose that $\mathfrak{M}^*(\widehat{\mathfrak{F}}_{-k}) \cup M_k \subset \mathfrak{M}^*(\widehat{\mathfrak{F}})$. By part (i) of the Incremental Gain Lemma [Lemma 2(i)], there is a sequencing of the mergers in $\mathfrak{M}^*(\widehat{\mathfrak{F}})\backslash\mathfrak{M}^*(\widehat{\mathfrak{F}}_{-k})$ that is CS-nondecreasing at each step. However, since all of the mergers in this set other than M_k must be CS-decreasing given that the mergers in $\mathfrak{M}^*(\widehat{\mathfrak{F}}_{-k})$ have taken place [otherwise they would have been in $\mathfrak{M}^*(\widehat{\mathfrak{F}}_{-k})$], merger M_k must be CS-nondecreasing given that the mergers in $\mathfrak{M}^*(\widehat{\mathfrak{F}}_{-k})$ have occurred and must be the first merger in this sequence. By Remark 1, the firms in M_k have a greater profit when all of the mergers in $\mathfrak{M}^*(\widehat{\mathfrak{F}})\backslash\mathfrak{M}^*(\widehat{\mathfrak{F}}_{-k})$ are approved than when none are. Hence, it is strictly more profitable in this case for the firms to propose merger M_k. As a result, each firm involved in merger M_k has a weakly dominant strategy to propose it. ■

3.2 Dynamic Merger Review

We now extend the analysis to the dynamic setting, where $T > 1$. A realization of feasible mergers is now a sequence $\mathfrak{F} = (\mathfrak{F}_1, ..., \mathfrak{F}_T)$ where $\mathfrak{F}_t \subseteq \mathfrak{F}_{t'}$ for $t' > t$. An outcome of the merger review process is a sequence of cumulatively approved mergers $\mathfrak{M} = (\mathfrak{M}_1, ..., \mathfrak{M}_T)$ where $\mathfrak{M}_t \subseteq \mathfrak{M}_{t'}$ for $t' > t$ and $\mathfrak{M}_t \subseteq \mathfrak{F}_t$ for all t. We begin with a result characterizing approval sequences that maximize discounted consumer surplus for a given realized feasible sequence:

Lemma 6 *Given a realization of feasible mergers $\mathfrak{F} = (\mathfrak{F}_1, ..., \mathfrak{F}_T)$, the approval sequence $\mathfrak{M} = [\mathfrak{M}^*(\mathfrak{F}_1), ..., \mathfrak{M}^*(\mathfrak{F}_T)]$ maximizes discounted consumer surplus.*

Proof. Given the realized sequence of feasible mergers $\mathfrak{F} = (\mathfrak{F}_1, ..., \mathfrak{F}_T)$, consider the problem of maximizing discounted consumer surplus. If we ignore the monotonicity constraint that the set of allowed mergers cannot shrink over time, we can choose the allowed set of mergers in any period independently from the mergers allowed in every other period. It is evident that in that case the approval sequence $\mathfrak{M} = [\mathfrak{M}^*(\mathfrak{F}_1), ..., \mathfrak{M}^*(\mathfrak{F}_T)]$ is optimal since it maximizes consumer surplus in every period. However, since Lemma 4 implies that $\mathfrak{M}^*(\mathfrak{F}_1) \subseteq ... \subseteq \mathfrak{M}^*(\mathfrak{F}_T)$, the monotonicity constraint is satisfied, so this is a solution to the constrained problem. ■

Lemma 6 shows that if feasible mergers always are proposed and the antitrust authority approves the mergers in set $\mathfrak{M}^*(\mathfrak{F}_t)\backslash\mathfrak{M}^*(\mathfrak{F}_{t-1})$ in each period t, then the outcome will maximize discounted consumer surplus given the actual realization of feasible mergers, even though the antitrust authority has no knowledge in any period of future feasible mergers.

Finally, we will argue that proposing a feasible merger is a weakly dominant strategy of a sort. Specifically, we consider subgame perfect equilibria for the firms given the antitrust authority's policy. We call a profile of strategies for the firms a *weakly dominant subgame perfect equilibrium* if in every period each firm is playing a weakly dominant strategy in the one-period game induced by (weakly dominant subgame perfect) continuation play. Our main result is:

Proposition 4 *Suppose the antitrust authority follows a myopic CS-based merger policy. Then all feasible mergers being proposed in each period after any history is a weakly dominant subgame perfect equilibrium. In this equilibrium, the outcome maximizes discounted consumer surplus given the realized sequence of feasible mergers* $\mathfrak{F} = (\mathfrak{F}_1, ..., \mathfrak{F}_T)$.

Proof. We proceed by backward induction. Proposition 3 shows that the result is true in period T for any set of previously-approved mergers, $\mathfrak{M}_{T-1}$. Suppose it is true in every period $t > s$, and consider play in period s after the mergers in set $\mathfrak{M}_{s-1}$ have previously been approved. Let $\mathfrak{M}^*(\mathfrak{F}|\mathfrak{M})$ denote the largest CS-maximizing set in a period given that the mergers in $\mathfrak{M}$ have previously been approved and the set of feasible mergers (including those in $\mathfrak{M}$) is $\mathfrak{F}$. If all feasible mergers are proposed in period s, then the set of approved mergers in period s is $\mathfrak{M}^*(\mathfrak{F}_s|\mathfrak{M}_{s-1})$; if, instead, some are not proposed, then Lemmas 4 and 5 imply that the set of approved mergers is a subset of $\mathfrak{M}^*(\mathfrak{F}_s|\mathfrak{M}_{s-1})$. Moreover, by Lemma 4, $\mathfrak{M}^*(\mathfrak{F}_s|\mathfrak{M}_{s-1}) \subseteq \mathfrak{M}^*(\mathfrak{F}_t|\mathfrak{M}_{s-1})$ for all $t > s$. Since all feasible mergers are proposed in periods $t > s$, the set of approved mergers at the end of each period $t > s$ will therefore be $\mathfrak{M}^*(\mathfrak{F}_t|\mathfrak{M}_{s-1})$. In particular, the approvals in period s do not affect those in any later period. So the strategic considerations reduce to a single-period problem, and firms find proposing any feasible merger to be a weakly dominant strategy in period s for the same reasons as in Proposition 3 when $T = 1$. Applying induction establishes that it is a weakly dominant subgame perfect equilibrium for firms to propose all feasible mergers in every period after any history. Lemma 6 then implies that the outcome of this equilibrium maximizes discounted consumer surplus given the realized sequence of feasible mergers $\mathfrak{F} = (\mathfrak{F}_1, ..., \mathfrak{F}_T)$. ∎

4 Extensions and Limitations

In this section, we discuss a number of extensions of our model and also note some of the limitations of our main results.

4.1 Price competition

So far, we have assumed that firms compete in quantities. In this subsection, we show that our main results continue to hold, with a minor modification, when firms compete in prices rather than quantities.

As before, there are n firms producing a homogeneous good at constant returns to scale. Firm i's marginal cost and price are denoted c_i and p_i. Market demand is given by the nonincreasing function $Q(p)$, where p is the lowest price offered by any firm. Let $\iota(i|N) \in N$ denote the firm with the ith lowest marginal cost when the set of firms is N, i.e., $c_{\iota(1|N)} \leq c_{\iota(2|N)} \leq ... \leq c_{\iota(n|N)}$. (If a subset of firms have the same marginal cost, then the firms in this subset are ordered arbitrarily.) Assuming that $Q(c_{\iota(1|N)}) > 0$ and that Assumption 1(ii) holds, and restricting attention to the standard Bertrand pricing equilibrium[12], firm $\iota(i|N)$'s equilibrium price $p_{\iota(i|N)}$ is given by

$$p_{\iota(i|N)} = \begin{cases} c_i & \text{if } 2 \leq i \leq n, \\ \min\{p^m(c_{\iota(1|N)}), c_{\iota(2|N)}\} & \text{if } i = 1, \end{cases} \tag{7}$$

where the nondecreasing function $p^m(c) \equiv \arg\max_p (p-c)Q(p)$ is the monopoly price of a firm with marginal cost c. In equilibrium, all consumers purchase at price $p_{\iota(1|N)}$, and so consumer surplus is given by

$$CS(p_1, p_2, ..., p_n) = \int_{p_{\iota(1|N)}}^{\infty} Q(p)dp. \tag{8}$$

Note that $CS(p_1, p_2, ..., p_n)$ is independent of $p_{\iota(i|N)}$ for $i > 1$, and decreasing in $p_{\iota(1|N)}$.

One important difference between the Cournot and Bertrand models is that with Bertrand competition a merger that is CS-neutral in isolation can become CS-decreasing when another merger takes place that is CS-increasing in isolation, as the following example demonstrates:

Example 1 *Suppose there are four firms, $N = \{1, 2, 3, 4\}$, with initial costs $c_1 = 5$, $c_2 = 10$, $c_3 = 15$, $c_4 = 20$, and suppose that there are two possible mergers $M_1 = \{1, 3\}$ and $M_2 = \{2, 4\}$ with $\overline{c}_{M_1} = 9$ and $\overline{c}_{M_2} = 8$. If the monopoly price for a firm with marginal cost equal to 5 is greater than 10 [i.e., $p^m(5) > 10$], then with no mergers firm 1 will set a price of 10 and make all the sales in the market. The cost-increasing merger M_1 is then CS-neutral in isolation since the post-merger price will still be 10. Merger M_2 is CS-increasing in isolation because it reduces firm 1's price from 10 to 8. However, once merger M_2 occurs, merger M_1 is CS-decreasing since it raises the price from 8 to 9.*

This problem can be traced to the fact that a merger involving the lowest-cost firm $\iota(1|N)$ that increases cost can be CS-neutral in the Bertrand model. We will say that a merger of the firms in set M is *cost increasing* if the post-merger marginal cost of the merged entity, $\overline{c}_M$, is above the marginal cost of the most efficient merger partner, i.e., $\overline{c}_M > \min_{i \in M} c_i$. Intuitively, an antitrust authority can without loss reject any cost-increasing merger, since any such merger both worsens efficiency and the extent of market power.[13] We shall henceforth focus on an antitrust authority that never approves cost-increasing mergers. Formally, this is equivalent to supposing that feasible mergers are

[12] Specifically, the limit of undominated equilibria for games with a discrete pricing grid, as the grid becomes fine.

[13] Formally, given any set of feasible mergers in a period, observe that it is possible to weakly improve consumer surplus starting from any set of approved mergers by instead rejecting all mergers that are cost-increasing. As a result, in any period, given any set of feasible mergers, the largest CS-maximizing set from among those feasible mergers that do not increase cost maximizes consumer surplus in that period.

never cost increasing, an assumption that will allow us to fit the analysis into the same framework as the Cournot model.

The following result records some properties of mergers that do not increase cost:

Lemma 7 *Consider a merger that does not increase cost among a subset M of firms in a Bertrand market.*

1. *It is weakly profitable (it weakly increases the joint profit of the firms in M).*
2. *The merger is CS-decreasing only if it involves all of the firms with cost $c_{\iota(1|N)}$, all of the firms with cost $c_{\iota(2|N)}$, and moreover $p^m(\overline{c}_M) > c_{\iota(2|N)}$.*

Proof. To see Property 1, note that, in equilibrium, only a uniquely lowest-cost firm (a firm that is the only one to have cost $c_{\iota(1|N)}$) can make a positive profit before the merger. Hence, Property 1 can fail to hold only if the merger involves a unique lowest-cost firm that is the unique firm with cost $c_{\iota(1|N)}$. Suppose it involves such a firm. In that case, a merger that does not increase cost will not affect the equilibrium prices of the firms not involved in the merger (who price at cost both before and after the merger), so the merger must be weakly profitable.

To see Property 2, let $\overline{N}$ denote the set of firms after the merger. Assume the merger is CS-decreasing, i.e.,

$$p_{\iota(1|\overline{N})} = \min\{p^m(c_{\iota(1|\overline{N})}), c_{\iota(2|\overline{N})}\} > \min\{p^m(c_{\iota(1|N)}), c_{\iota(2|N)}\} = p_{\iota(1|N)}.$$

Since the merger is not cost increasing, $c_{\iota(1|\overline{N})} \le c_{\iota(1|N)}$, so that $p^m(c_{\iota(1|\overline{N})}) \le p^m(c_{\iota(1|N)})$. Next, note that if the merger does not involve all of the firms with cost $c_{\iota(1|N)}$, then $c_{\iota(2|\overline{N})} = c_{\iota(1|N)} \le c_{\iota(2|N)}$. Since this would imply that $\min\{p^m(c_{\iota(1|\overline{N})}), c_{\iota(2|\overline{N})}\} \le \min\{p^m(c_{\iota(1|N)}), c_{\iota(2|N)}\}$, a contradiction, the merger must involve all of the firms with cost $c_{\iota(1|N)}$. Next, if the merger does not involve all of the firms with cost $c_{\iota(2|N)}$, then since it involves all of the firms with cost $c_{\iota(1|N)}$ and is not cost increasing, it must again be that $c_{\iota(2|\overline{N})} \le c_{\iota(2|N)}$, which again yields a contradiction. Hence, the merger must involve all of the firms with cost $c_{\iota(1|N)}$ and all of the firms with cost $c_{\iota(2|N)}$. Finally, suppose that $p^m(\overline{c}_M) \le c_{\iota(2|N)}$, or equivalently, $p^m(c_{\iota(1|\overline{N})}) \le c_{\iota(2|N)}$. Since the merger is not cost increasing, $p^m(c_{\iota(1|\overline{N})}) \le p^m(c_{\iota(1|N)})$. But this implies that $p_{\iota(1|\overline{N})} = \min\{p^m(c_{\iota(1|\overline{N})}), c_{\iota(2|\overline{N})}\} \le \min\{p^m(c_{\iota(1|N)}), c_{\iota(2|N)}\} = p_{\iota(1|N)}$, a contradiction. ■

We now turn to the interaction between two disjoint mergers, M_1 and M_2, where $M_1 \cap M_2 = \emptyset$. Let N denote the set of firms if neither merger takes place, N_i the set of firms after merger M_i (but not M_j, $j \ne i$) has taken place, and N_{12} the set of firms after both mergers have taken place. The key fact is that in the Bertrand model, Propositions 1 and 2 continue to hold for mergers that do not increase cost:

Proposition 5 *In the Bertrand model, the CS-effect of mergers that do not increase cost is complementary:*

(i) *if a merger that does not increase cost is CS-nondecreasing in isolation (and hence profitable), it remains CS-nondecreasing (and hence profitable) if another merger that does not increase cost and is CS-nondecreasing in isolation takes place.*

(ii) *if a merger is CS-decreasing in isolation and does not increase cost, it remains CS-decreasing if another merger that does not increase cost and is CS-decreasing in isolation takes place.*

Proof. To see part (i), suppose to the contrary that the merger, say M_1, becomes CS-decreasing if the other merger, say M_2, takes place. Since M_1 is not cost increasing, by Lemma 7 the merger must involve (after M_2 has taken place) all of the firms with cost $c_{\iota(1|N_2)}$ and all of the firms with cost $c_{\iota(2|N_2)}$, and moreover $p^m(\overline{c}_{M_1}) > c_{\iota(2|N_2)}$. Because mergers are disjoint and M_2 does not increase cost, this implies that when done in isolation, merger M_1 involves all of the firms with costs of either $c_{\iota(1|N)}$ or $c_{\iota(2|N)}$. If so, then $c_{\iota(2|N)} = c_{\iota(2|N_2)}$. But this implies that $p^m(\overline{c}_{M_1}) > c_{\iota(2|N)}$, so that merger M_1 is CS-decreasing in isolation, a contradiction.

To see part (ii), note that since each merger M_i is CS-decreasing in isolation, by Lemma 7 each must involve (when done in isolation) all of the firms with costs of $c_{\iota(1|N)}$ and all the firms with cost $c_{\iota(2|N)}$. However, since mergers are disjoint, this is impossible. ■

We now turn to the interaction between mergers that, in isolation, affect consumer surplus in opposite directions.

Proposition 6 *Consider two mergers M_1 and M_2 that are not cost increasing. Suppose that merger M_1 is CS-nondecreasing in isolation, while merger M_2 is CS-decreasing in isolation but CS-nondecreasing once merger M_1 has taken place. Then:*

(i) *Merger M_1 is CS-increasing (and therefore profitable) conditional on merger M_2 taking place;*

(iii) *The joint profit of the firms involved in merger M_1 is strictly larger if both mergers take place than if neither merger takes place.*

Proof. The proof of part (i) is identical to the proof of Proposition 2.

To see part (ii), note that since M_2 is CS-decreasing in isolation it involves all of the firms with costs equal to $c_{\iota(1|N)}$ or $c_{\iota(2|N)}$ by Lemma 7. Since it is not cost increasing, the profit of all firms not involved in this merger, including all of those in M_1, must be zero before and after merger M_2 place. Since merger M_1 is strictly profitable after merger M_2 takes place [by part (i)], the sequence of mergers must strictly increase the joint profit of the firms in M_1. ■

Given these two propositions, our main result follows exactly as in the Cournot model. This implies that an antitrust authority that myopically approves, on a merger-by-merger basis, CS-nondecreasing mergers that do not increase cost is following a dynamically optimal policy in the sense that it maximizes discounted consumer surplus for any realized sequence of feasible mergers.

4.2 Differentiated Products

The Cournot and Bertrand analyses so far assumed a homogeneous product market. Unfortunately, extending our main results to the case of differentiated products, and hence to multiproduct firms, is not straightforward. For example, think of the extreme case in which there are two differentiated products in the market. A merger might leave overall consumer surplus unchanged while raising one price and lowering the other. Since in the extreme case where the two products are independent in demand, there are two independent homogeneous goods markets, it appears that we will not be able to say anything about the complementarity of CS-nondecreasing mergers. On the other hand, our main results do extend to the case of differentiated products if "strong symmetry" is imposed on both demand and costs (in the sense that all firms that are involved in the same merger have identical marginal costs for all of their products, both pre-merger and post-merger). In that case, price effects for all goods move in the same direction and the complementarity results from our previous analyses carry over, as we now discuss. In our discussion, we will focus on the case of price competition with differentiated products, but the analysis of quantity competition with differentiated products should proceed along similar lines.

Let $Q_j(\mathbf{p}_N)$ denote the demand for product j, where $\mathbf{p}_N$ is the vector of prices, and suppose that the demand system is symmetric across products. Moreover, assume that demand is such that products are demand substitutes, prices are strategic complements, and the own-price effect dominates the cross-price effects both in terms of the level of demand as well as in terms of the slope of demand.[14]

For simplicity, suppose that, prior to merging, all firms produce a single product so that firm $j \in N$ produces product $j \in N$. After merging, the firms in the set M_k produce all of the products in M_k. We impose strong symmetry on the costs of those firms that are involved in the same potential merger. Specifically, we assume that, prior to merging, each firm $j \in M_k$ faces the same marginal cost $c_j = c_{M_k}$ while after the merger all products in M_k are produced at the same marginal cost $\overline{c}_{M_k}$. This assumption ensures that any equilibrium has the property that every firm in the set M_k charges the same price for each one of its products, $p_i^* = p_{M_k}^*$ for $i \in M_k$, both pre-merger and post-merger. In particular, this means that we can think of each firm's strategic variable being one-dimensional, and so the standard analysis of differentiated

[14]The own effect of price change dominates the cross effects in terms of the level of demand if $\sum_{j \in N} \left(\partial Q_i(\mathbf{p}_N)/\partial p_j\right) < 0$ and in terms of the slope of demand if $\sum_{i \in N} \left(\partial^2 Q_j(\mathbf{p}_N)/\partial p_j \partial p_i\right) < 0$, $j \in N$.

goods price competition with single-product firms (see Vives [1999]) extends to our setting with multiproduct firms.

Consider a merger amongst active firms in set M_k, and let $\mathbf{p}_N^*$ denote the vector of pre-merger equilibrium prices. Since prices are strategic complements, the merger is CS-neutral if and only if it leaves all prices unchanged, so the threshold value of post-merger marginal cost that makes this merger CS-neutral is given by

$$\widehat{c}_{M_k}(\mathbf{p}_N^*) \equiv c_{M_k} + [p_{M_k}^* - c_{M_k}] \frac{\sum_{j \in M_k, j \neq i} \frac{\partial Q_i(\mathbf{p}_N^*)}{\partial p_j}}{\sum_{j \in M_k} \frac{\partial Q_i(\mathbf{p}_N^*)}{\partial p_j}},\ i \in M_k. \tag{9}$$

Since $\partial Q_i(\mathbf{p}_N)/\partial p_j > 0$ for $j \neq i$, and $\sum_{j \in M_k} \partial Q_i(\mathbf{p}_N)/\partial p_j < 0$ for $i \in M_k$, $\widehat{c}_{M_k} < c_{M_k}$. That is, for the merger to be CS-neutral, the merger must be cost-reducing, and therefore profitable for the merging parties. Strategic complementarity implies that a decrease in post-merger marginal cost $\overline{c}_{M_k}$ induces all prices to fall. Consequently, a merger amongst active firms in M_k is CS-increasing if and only if $\overline{c}_{M_k} < \widehat{c}_{M_k}$, CS-neutral if and only if $\overline{c}_{M_k} = \widehat{c}_{M_k}$, and CS-decreasing if and only if $\overline{c}_{M_k} > \widehat{c}_{M_k}$.

While every CS-neutral merger is profitable, it is not straightforward to show that every CS-nondecreasing merger is profitable. The complication arises because a reduction in marginal cost $\overline{c}_{M_k}$ has two opposing effects on the profits of the merged entity M_k: holding fixed the prices of all other firms, the *direct* effect of a decrease in $\overline{c}_{M_k}$ is to increase the merged firm's profit; but the *strategic* effect of a decrease in $\overline{c}_{M_k}$ is to reduce the merged firm's profit as all other firms will decrease their prices in response. One therefore needs to impose conditions on demand to ensure that the direct effect outweighs the strategic effect and a decrease in its marginal cost raises that firm's equilibrium profit. It is straightforward to check that this is indeed the case when demand is linear, $Q_j(\mathbf{p}_N) = \alpha_N - \beta_N p_j + \gamma_N \sum_{i \neq j} p_i$.

Let us now turn to the interaction between mergers. Our previous result on the complementarity of those mergers that change consumer surplus in the same direction (Proposition 2) carries over to the present setting if approving a CS-increasing merger M_l raises the threshold $\widehat{c}_{M_k}$ for merger M_k, $k \neq l$ (and approving a CS-decreasing M_l reduces $\widehat{c}_{M_k}$). Since a CS-increasing merger reduces all prices, this means that our complementarity result extends if demand is such that $d\widehat{c}_{M_k}(\mathbf{p}_N^*)/dp_{M_l}^* < 0$. In the case of linear demand, for example, the (negative) term

$$\Psi_i \equiv \frac{\sum_{j \in M_k, j \neq i} \frac{\partial Q_i(\mathbf{p}_N^*)}{\partial p_j}}{\sum_{j \in M_k} \frac{\partial Q_i(\mathbf{p}_N^*)}{\partial p_j}} \tag{10}$$

in equation (9) is a constant. Since $dp_{M_k}^*/dp_{M_l}^* > 0$ (prices are strategic complements), it follows that $d\widehat{c}_{M_k}(\mathbf{p}_N^*)/dp_{M_l}^* < 0$, so our main results extend.

More generally, since $dp_{M_k}^*/dp_{M_l}^* > 0$, a sufficient condition for $d\widehat{c}_{M_k}(\mathbf{p}_N^*)/dp_{M_l}^* <$ 0, $k \neq l$, is that the term in (10), which is related to the "diversion ratio" in merger analysis, is either nonincreasing or not increasing "too fast" as prices fall (due to an decrease in the marginal cost of the products in M_l).[15]

4.3 Fixed Costs and Exit

So far, we have assumed that all fixed costs are sunk, and that mergers had no effect on these costs. Among other things, this restriction implicitly assumed that there were no costs of merger proposals. Our Cournot results extend to cases in which fixed costs are present and possibly affected by mergers provided (i) that mergers do not cause active firms to shut down and (ii) increases in fixed costs do not prevent potentially CS-increasing mergers from being proposed.[16]

If (i) is violated, for example, Proposition 2 need not hold. To see this, suppose both mergers M_1 and M_2 are CS-increasing in isolation and do not induce any firm to exit. However, if both mergers are approved, then some other firm $j \in N \backslash (M_1 \cup M_2)$ might find it optimal to exit. (This outcome is possible since, without exit, the market price after both mergers would be lower than after only one merger.) Taking the endogenous exit of firm j into account, consumer surplus after both mergers might therefore be lower than after merger M_k only, in which case merger M_l would be CS-decreasing conditional on merger M_k.

While this observation suggests that in general our main results could break down if we allowed for fixed costs and endogenous exit, the failure of our complementarity result arises only because of the "lumpiness" (or "discreteness") of exit. To see this, suppose that there is a competitive fringe of price-taking firms that do not take part in any mergers, and potential exit involves only these fringe firms. We can then construct the competitive fringe's (long-run) supply function, $S^F(p)$, which takes potential exit (and entry) of these firms into account. The residual demand of the large, strategic firms in set N is then given by $R(p) \equiv D(p) - S^F(p)$, where $D(p)$ is market demand. As long as the inverse residual market demand function $P(\cdot) \equiv R^{-1}(\cdot)$ satisfies the conditions of Assumption 1, our analysis and conclusions remain unchanged.

Regarding (ii), small costs of proposing mergers may induce delay of merger proposals but have a negligible impact on discounted expected consumer surplus. To see this, recall that CS-nondecreasing mergers are strictly profitable in the Cournot model. As long as the cost of proposing a merger is small, firms involved in a feasible merger that is CS-nondecreasing at the beginning of period t have an incentive to propose their merger in that period. However, firms involved in a feasible merger that is CS-decreasing at the beginning of period t may not propose their merger in that period if it is unlikely that the merger is approved by the antitrust authority in period t (which would occur if and only

[15] The diversion ratio from $i \in M_k$ to the other products in set M_k is the share of the lost sales of product i that are captured by the other products in M_k after an increase in the price of product i. Since symmetry implies that for $i, j \in M_k$, $i \neq j$, $\partial Q_j(\mathbf{p}_N^*)/\partial p_i^* = \partial Q_i(\mathbf{p}_N^*)/\partial p_j^*$, the diversion ratio is equal to $\Psi_i/(1-\Psi_i)$.

[16] In the Bertrand model, all but the lowest firm will exit if there are positive levels of fixed costs.

if the antitrust authority approves a CS-increasing merger that turns the CS-decreasing merger into a CS-nondecreasing one). But, since the probability of such a merger's approval is small anyway, and there is only a finite number of potential mergers, the effect of not proposing such mergers in period t on consumer surplus in period t is small. Moreover, if $T >> t$, any feasible merger that would be approved in period t in the absence of merger proposal costs will still be proposed and approved in the presence of small merger proposal costs, but in period $t' \geq t$. The social cost of such merger delays is smaller the shorter is the time span between any two adjacent periods (i.e., the larger is the discount factor δ).

4.4 Demand Shifts

While our model had a stationary demand function, Corollary 1 suggests that our main results hold provided that demand is weakly declining over time. Specifically, suppose inverse demand can be written as $P(Q;\theta_t)$, where θ_t is the demand state realized at the beginning of period t (before mergers are approved), $P(Q;\theta_t)$ is weakly increasing in θ_t, and assume that demand is weakly declining over time, $\theta_{t+1} \leq \theta_t$ for all t. The post-merger marginal cost threshold that makes a merger amongst active firms in set M CS-neutral can then be written as

$$\widehat{c}_M(Q^*;\theta_t) \equiv P(Q^*;\theta_t) - \sum_{i\in M} \max\{0, P(Q^*;\theta_t) - c_i\}.$$

Holding fixed industry output Q^*, the threshold $\widehat{c}_M(Q^*;\theta_t)$ will thus weakly increase over time (as long as the merged firm remains active). If merger M is CS-nondecreasing in period t, it will therefore remain CS-nondecreasing (and hence profitable) in all subsequent periods $t' > t$, while a merger that is CS-decreasing in period t may become CS-nondecreasing in some later period $t' > t$ even holding market structure fixed. The largest CS-maximizing set of mergers in a period t now depends not only on the set of feasible mergers but also on the demand state: it is now written $\mathfrak{M}^*(\mathfrak{F}_t, \theta_t)$. Since $\mathfrak{F}_t \subseteq \mathfrak{F}_{t+1}$ and $\theta_t \geq \theta_{t+1}$, this set is continues to be weakly increasing over time. Hence, as before, the approval sequence $\mathfrak{M} \equiv [\mathfrak{M}^*(\mathfrak{F}_1, \theta_1), ..., \mathfrak{M}^*(\mathfrak{F}_T, \theta_T)]$ maximizes discounted consumer surplus (Lemma 6), all feasible mergers being proposed is a weakly dominant subgame perfect equilibrium if the antitrust authority adopts a myopic CS-based merger approval policy, and the resulting equilibrium outcome is dynamically optimal (Proposition 4).

4.5 Entry

In our analysis above, we assumed that the set of firms is fixed, except for mergers. Would our conclusions change if we allowed for firm entry? Recall that our model implies that the equilibrium price $P(Q^*)$ falls weakly over time. This suggests that if a firm does not find it profitable to enter the market at the beginning of the first period, before any mergers have become feasible, then this firm will not find it profitable to enter the market in any later period (provided that its costs have not changed). That is, allowing for free entry of firms (with unchanging costs) should not affect our results.

Moreover, suppose that new firms periodically enter the market later, for example after discovering how to make the product. These entry events lower the market price, and leave our main result unchanged for reasons that parallel those in our discussion above of demand shifts.

4.6 Continuing Innovation

In the analysis above, we assumed that when a merger, say M_k, becomes feasible, the firms in M_k receive a (random) draw of their post-merger marginal cost $\overline{c}_{M_k}$ once and for all; if merger M_k is implemented, the marginal cost of the merged entity is $\overline{c}_{M_k}$ forever after. But it seems plausible that, over time, firms involved in a (potential) merger may have more than one idea of how to create synergies amongst them, both pre-merger and post-merger. As we now discuss, it is possible to extend our analysis to allow for continuing innovation.

Consider the following generalization of our previous setup: as before, we assume that if merger M_k becomes feasible at the beginning of period t, then the firms in M_k receive a random draw of their post-merger marginal cost from distribution function F_{kt}; however, we now assume that, at the beginning of any subsequent period $t' > t$, there is a probability $p_{ktt'} \geq 0$ that the firms in M_k receive another draw of their post-merger marginal cost. The stochastic process governing these additional cost draws (or "innovations") is independent of whether the firms in M_k have already merged or not. Crucially, we assume that firms have "perfect recall": in any given period, they are free to implement any one of the innovations they have made so far. If the firms in M_k have not yet merged, they can implement the innovation by merging. If the firms in M_k have already merged in a previous period, they can still decide to implement any one of their past or present innovations at the beginning of the period; any such decision (and the corresponding level of post-merger marginal cost) becomes common knowledge before the antitrust authority undertakes its merger reviews. These assumptions ensure that firms have no incentive to delay a merger so as to wait for a better innovation.

To see that our previous results carry over to this generalized setting, note first that any cost-reducing innovation of a merged entity is profitable for the firm involved, holding fixed market structure. As to the interaction of cost-reducing innovations, note that – from the viewpoint of the outsiders to the innovation – a cost-reducing [cost-increasing] innovation is akin to a CS-increasing [CS-decreasing] merger in that it induces the firm(s) involved to produce more [less] output. Hence, Proposition 1 carries over the case where one of the mergers is, in fact, an innovation by a merged firm. If a cost-reducing innovation of merged firm M_k triggers a merger M_l in the sense of turning a CS-decreasing merger into a CS-nondecreasing one (the analogue to Proposition 2), then the argument about profitability in the proof of Proposition 2 carries over: if the CS-decreasing merger M_l is implemented first, the profit of every other firm, including the merged firm M_k, goes up; if the cost-reducing innovation by M_k is subsequently implemented, the profit of merged firm M_k increases even further (since a firm's profit is increasing as that firm's marginal cost goes down). Hence, the merged firm M_k is better off implementing its cost-reducing innovation even if this triggers another merger. These observations also imply that if a merger is proposed, the merger partners always want to propose the merger with the lowest cost draw that they received. Applying the same arguments as before, one can therefore show that, if the antitrust authority adopts a myopic CS-based merger review policy, implementing every cost-reducing innovation by a merged firm, and proposing every feasible merger with the lowest cost draw in every period, after any history, is a weakly-dominant subgame perfect equilibrium. The equilibrium outcome maximizes discounted consumer surplus.[17]

4.7 Aggregate Surplus Standard

In our analysis above, we have assumed that the antitrust authority's objective is to maximize discounted consumer surplus. Indeed, as pointed out in the introduction, this is close to being the legal standard in the U.S. and the EU. Nevertheless, it is interesting to ask whether the antitrust authority can maximize aggregate surplus (AS) by adopting a myopic AS-based merger approval policy.

In the homogeneous-goods Bertrand model, the answer is, yes. This should not be too surprising since in that model only a firm with the uniquely lowest marginal cost makes any profit. This implies, for instance, that any CS-nondecreasing merger that is not cost-increasing is AS-nondecreasing. Moreover, if any AS-nondecreasing merger that does not increase cost is CS-decreasing, no outsider to this merger can have marginal costs lower than the two most efficient merger partners. Using these observations, one can then prove that our results on the effects of mergers in the Bertrand model, Propositions 5 and 6, continue to hold if we replace the consumer surplus criterion by the aggregate surplus criterion. Consequently, a myopic AS-based merger policy maximizes discounted aggregate surplus.

In the homogeneous-goods Cournot model, however, the complementarity of AS-increasing mergers does not hold in general. To see this, recall that, in the Cournot model, a marginal cost reduction by a highly inefficient firm (one that produces almost no output, and thus has a profit margin approximately equal to zero) necessarily reduces aggregate surplus. In contrast, a cost-reducing merger between the two most efficient firms in a market may increase aggregate surplus. Thus, complementarity might fail because a cost-reducing, AS-increasing merger by other firms in the market may transform these two firms from being the most efficient firms in the market to being the least efficient.

[17] It is important, however, that firms do not get continuing innovations that can be used if they do not merge since a firm might delay merging to see what these future cost draws will be. So, for example, the newly arriving firms discussed at the end of the previous section, must not have been able to merge with other firms before their discovery.

4.8 Nondisjoint Mergers

Perhaps the most restrictive assumption we make is that feasible mergers are disjoint.[18] This assumption substantially simplifies the analysis of dynamic merger policy. The most important reason is that it greatly simplifies firms' merger proposal decisions. Indeed, firms have a weakly dominant strategy to propose any feasible merger when the antitrust authority follows a myopic CS-based merger policy. With non-disjoint mergers, however, firms may have incentives to propose the wrong mergers from the antitrust authority's perspective as the following example illustrates:

Example 2 *Consider a Cournot market in which there are three firms, $N = \{1, 2, 3\}$, and a single period ($T = 1$). Marginal costs satisfy $c_1 < c_2 < c_3$, so absent any merger firm 1 produces the most output, and firm 3 the least. Suppose that two mergers are possible: $M_1 = \{1, 2\}$ and $M_2 = \{2, 3\}$. That is, firm 2 may merge with either firm 1 or firm 3. In this setting, firms 1 and 3 will bid for firm 2 to merge with them. The resulting outcome will be the merger that maximizes industry profit (this is a simple menu auction as in Bernheim and Whinston [1986]).*

How does the merger that maximizes industry profit compare to the merger that maximizes consumer surplus? To illustrate, suppose both mergers are CS-neutral, so that the antitrust authority is indifferent between them. Both mergers are profitable, but Lemma 1 implies that the large merger M_1 (in the sense of joint market share) is more profitable than the small merger M_2. Specifically, note first that the profits of non-merging firms are unaffected by a CS-neutral merger. So the change in industry profits equals the change in the merging firms' joint profit. The profit gain for the firms in merger M_1 is $[P(Q^*) - c_2]q_1^* + [P(Q^*) - c_1]q_2^*$ while the profit gain for the firms in merger M_2 is $[P(Q^*) - c_3]q_2^* + [P(Q^*) - c_2]q_3^*$.[19] Since $[P(Q^*) - c_1] > [P(Q^*) - c_3]$ and $q_1 > q_3$, merger M_1 is more profitable. Clearly, then, if the efficiency gain in merger M_2 were slightly larger, merger M_2 would be the better merger for consumer surplus, but the firms would propose merger M_1. A policy that approved any CS-nondecreasing merger would therefore not result in the first-best outcome for the antitrust authority.

A second issue is that firms may face a disincentive to propose a merger that today would be CS-increasing because of the effect the merger's approval would have on future merger partners.

[18] Note, however, that our result would continue to hold under the weaker assumption that each dynamic path of realized mergers involves any given firm in at most one merger.

[19] For example, the premerger profit of firms in M_1 is $[P(Q^*)-c_1]q_1^*+[P(Q^*)-c_2]q_2^*$, while their post-merger profit is $[P(Q^*)-\overline{c}_{M_1}]q_1^*+[P(Q^*)-\overline{c}_{M_1}]q_2^*$. By Lemma 1, this post-merger profit equals

$$[\{P(Q^*) - c_1\} + \{P(Q^*) - c_2\}]q_1^* + [\{P(Q^*) - c_1\} + \{P(Q^*) - c_2\}]q_2^*$$

So the change in their joint profit is $[P(Q^*) - c_2]q_1^* + [P(Q^*) - c_1]q_2^*$. A similar derivation applies for the change in joint profit of the firms in merger M_2.

We conjecture, however, extend our result to cases in which mergers can be overlapping, but have a nested structure. Specifically, suppose that the set of feasible mergers is $\mathbb{A} \equiv \{M_1, ..., M_K\}$ and that, for $i \neq j$, $M_i \cap M_j \neq \emptyset$ implies that either $M_i \subset M_j$ or $M_j \subset M_i$. (For example, with five firms it could be that three mergers are feasible: $M_1 = \{1, 2\}$, $M_2 = \{3, 4\}$, and $M_3 = \{1, 2, 3, 4, 5\}$.) Make three additional assumptions. First, that whenever a merger M_j becomes feasible, so does every merger M_i that is smaller than it (i.e., every $M_i \subset M_j$). Second, that whenever a merger M_j is proposed, the firms must propose every merger M_i that is smaller than it. This, in essence, gives the antitrust authority the right to insist on spinoffs in approving mergers. Third, each firm's share of the profit gain from a merger equals its share of the number of original firms.[20]

The basic idea is as follows: First, we can show that if all feasible mergers are proposed in every period then a myopic approval policy in which the antitrust authority approves the largest set of mergers that maximize current consumer surplus also maximizes discounted consumer surplus for any realization of the set of feasible mergers. The additional complication with the nested structure is that approval of a large merger M_2 in some period t precludes later approval of merger $M_1 \subset M_2$. But, if we view the larger merger as being comprised of two mergers, M_1 and then the merger of firm M_1 with the firms in set $M_2 \backslash M_1$, the complementarity of CS-nondecreasing mergers implies that the desirability to the antitrust authority of implementing the larger merger can only grow through time.

Once this first step is established, what remains is to show that there is an equilibrium in which the firms will propose all feasible mergers. Here there are two complications. First, when the antitrust authoity would want to approve a large merger M_2, the firms involved in the merger might have an incentive to propose instead a smaller merger $M_1 \subset M_2$. However, we can again view the larger merger as a combination of two mergers, M_1 and then the merger of firm M_1 with the firms in set $M_2 \backslash M_1$. Saying that the larger merger M_2 results in greater consumer surplus than the smaller merger M_1 is equivalent to saying that the merger of firm M_1 with the firms in set $M_2 \backslash M_1$ is CS-nondecreasing once merger M_1 has occurred. But, we know that if that is so, then the firms' joint profits are greater if they propose it.

The second complication is that firms might have an incentive not to propose a CS-nondecreasing merger today because it could worsen the terms at which they form a still larger merger tomorrow through bargaining power effects. The assumption on the bargaining process, however, rules out such a possibility by ensuring that bargaining power is unchanged by prior mergers.

While this conjectured result extends our analysis, it leaves open many interesting questions about optimal merger policy in which feasible mergers may overlap and the assumptions listed above fail. We intend to explore these in future work.

[20] For example, each divisional manager involved in a merger (where a divisional manager is a manager of one of the original units) may randomly be chosen to make a take-it-or-leave it merger proposal to all of the other firms' managers.

5 Conclusion

In this paper, we have analyzed the antitrust authority's optimal merger approval policy in a dynamic model in which merger opportunities arise stochastically over time, firms decide whether or not to propose a feasible merger, and the antitrust authority decides whether or not to approve proposed mergers. In our model, an antitrust authority who wishes to maximize discounted consumer surplus can implement the dynamically optimal solution by adopting a completely myopic policy according to which the antitrust authority approves a merger if and only if it does not lower consumer surplus given the current market structure. In fact, the antitrust authority cannot improve upon the outcome induced by the myopic policy even if it had perfect foresight about potential future mergers and had the power to break up previously approved mergers.

This surprising conclusion is based on two intermediate results in our model. First, every merger that the antitrust authority would like to approve is profitable for the merging parties and will therefore be proposed. Second, there is a fundamental complementarity in the consumer surplus effect of mergers: if each of two mergers share the same sign of their consumer surplus effect, then the sign of each one's consumer surplus effect does not change if the other merger is implemented. This complementarity result follows because an increase in the toughness of competition (due to the approval of a price-reducing merger) does not affect the "efficiency effect" of the merger but reduces the "market power effect" of the merger, implying that a merger is more likely to be consumer surplus increasing the more competitive is the industry.

We have also shown that the main conclusion – the dynamic optimality of a myopic merger approval policy – is robust in several dimensions. For instance, the conclusion does not depend on firms' and the antitrust authority's information about potential future mergers nor on whether firms compete in prices or quantities. However, we also identified a number of limitations of our result. One quite restrictive assumption of our model is that mergers are disjoint: no firm can be part in more than one potential merger. This assumption eliminates any scope for strategic bargaining amongst merger partners and, indeed, the choice between alternative mergers. In future research, we plan to analyze further what can be said about the optimal merger approval policy in a setting with nondisjoint mergers.

One interesting side implication of our model is that it provides a novel theory of merger waves (for example, see Fauli-Oller [2000]). In contrast to much of the existing literature (e.g., Jovanovic and Rousseau [2002, forthcoming]), our explanation of merger waves does not rely on aggregate shocks. Specifically, because of the complementarity of CS-nondecreasing mergers in our model, the arrival of a CS-increasing merger opportunity for some firms may have a domino effect by turning other feasible but currently CS-decreasing mergers into CS-nondecreasing mergers, and thereby triggering a merger wave.[21] An interesting

[21] To see this, suppose that, at the end of period $t-1$, there is a nonempty set of feasible

aspect of this result is the way in which a CS-based merger approval policy of the antitrust authority affects the emergence of merger waves, since complementarity of mergers does not hold in general absent this antitrust review.

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but CS-decreasing mergers. The antitrust authority will optimally not approve these mergers. Suppose now that in period t, one (or more) CS-increasing merger(s) become(s) feasible. The antitrust authority will optimally approve such a merger. The approved CS-increasing merger(s) may turn one (or more) of the feasible CS-decreasing mergers into a CS-increasing merger, which in turn will be approved by the antitrust authority, and so on.