Stepping Off the Wage Escalator: A Theory of the Equilibrium Employment Rate

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ABSTRACT

This paper develops a theory of equilibrium labor supply based on the lifelong return to work. This lifelong return to work is the product of the general wage level and the return to experience. The paper shows how the return to work has different effects from general wage growth because it creates a wedge between the return to work and not working. The model of the paper thereby generates a powerful relationship between employment rates and productivity growth. Calculations based on the model are able to replicate the comovements in employment rates and productivity growth for low-skill workers in the United States based on the observed changes in the return to experience and in the rate of general productivity growth.
I. Introduction

The employment and unemployment rates vary substantially both across time and space. Explaining this variation has been a central question for labor and macroeconomics and for public policy for several decades. There is broad agreement among economists and policymakers that institutional arrangements for employment and unemployment, particularly the benefits associated with being unemployed and out of the labor force, account for long-term differences in employment and unemployment rates. Much of the scholarly and policy debate since the mid-1970s has focused on how the more pro-employment institutional arrangements in the United States compared to Europe account for the lower unemployment rate in the United States.

There is, however, a problem with the institutional explanations of the changes in rates of unemployment and employment. It is very difficult to find changes in institutions that could account for changes in the labor market. There were no fundamental changes in the U.S. labor market institutions that could account for the very persistent decline in the employment rate that began in the mid-1970s. Similarly, the institutions in Europe that prevailed as employment rates persistently declined had largely been in place for decades. The institutional explanation can not be the solely account for the evolution of the steady-state in the employment market. Indeed, some institutions (decline in unionization rates in the U.S., labor market reforms in Europe) were evolving in ways that could suggest convergence at higher employment rates over time and across countries. The rate of convergence has, however, been quite slow.

The observation that institutions had not changed in ways that account for the shifts in labor market equilibrium has been widely understood. Instead, the argument
was that adverse shocks accounted for the deterioration of labor market outcomes (e.g. Bruno and Sachs, 1985) or that these shocks interacted with institutional arrangements (e.g. Blanchard and Summers, 1986) to yield very persistent shifts in labor market equilibrium. The case that shocks interacted with institutions has been made recently and systematically by Blanchard and Wolfers (2000). That work leaves open the question of the mechanism that underlies the powerful interaction of institutions and shocks that appears to be necessary for accounting for the persistent difference in labor market outcomes across time and space.

This paper presents a theory of equilibrium employment that accounts for the interaction of institutions and shocks. In particular, we show that there is a powerful interaction between the return to work and the benefits of nonemployment that can lead to steady-state changes in the employment rate. Finding a link between productivity growth and labor market outcomes has been the holy grail of this literature. The decline in productivity growth—both in magnitude and timing—seems the most obvious shock to account for the persistent deterioration in labor market equilibrium beginning in the early to mid-1970s. With the acceleration of the productivity in the mid-1990s, it is also a candidate for the recent improvement in labor market outcomes. Hence, the empirical case for a linkage between productivity growth and equilibrium employment seems strong.

The powerful empirical connection between labor market outcomes and productivity growth is shown in Figure 1A that plots the smoothed productivity growth rate against the smoothed unemployment rate. There is a striking negative correlation between productivity growth and unemployment. This figure is adopted from Staiger,
Stock, Watson (2001, Figure 1.9). Stock has posted this “intriguing graph” on his WWW page for the last number of years as an implicit challenge to the economics profession to explain it.\(^1\) The stark, negative correlation between productivity growth and labor market outcomes becomes much less clear when we use the nonemployment rate instead of the unemployment rate as the measure of labor market outcome in the intriguing graph. See Figure 1B. As Juhn, Murphy, and Topel (1991, 2002) stress, the overall nonemployment rate does not show the same declines since 1980 as the unemployment rate. To anticipate our main result, our analysis will provide an explanation for the labor market/productivity growth correlation found in Figure 1 that is consistent with focusing on nonemployment. Specifically, we will show—in theory and in U.S. data—that productivity growth and the nonemployment rate of marginal workers are negatively correlated.

There is, superficially, a strong case of a theoretical link between productivity growth and employment rates: Should it be unsurprising that there is less employment when the returns to work have fallen? Yet, the theoretical link between productivity growth and equilibrium employment has, however, been elusive. Blanchard surveys various approaches to mapping productivity growth into wages. He concludes that these approaches “deliver, to a first order, long run neutrality of unemployment to productivity growth.” He points out that existing theories may have different implications for the short and medium run effects of productivity growth, but concludes our understanding of the link between productivity and employment is weak. “The truth is we do not know. And this is a serious hole in knowledge” (Blanchard, 2007, p.416). As our theory will make clear, traditional models of employment determination have the implication that permanent changes in productivity growth leave equilibrium employment rates

\(^1\) See [http://ksghome.harvard.edu/~JStock/](http://ksghome.harvard.edu/~JStock/)
unchanged. Why? In any model with a steady state, changes in productivity growth should equally affect the returns to work and the returns to not working. In particular, the social insurance for the nonemployed should keep pace upward or downward with the wage rate for the marginal worker.

Imposing the sensible restriction that the returns to nonwork move upwards and downwards with the level of productivity is therefore a straightjacket on models seeking to find a productivity growth/employment rate linkage. (Later in the paper, we survey various attempts to work around the implications of this restriction.) Our theory shows that incorporating returns to labor market experience into the model is a powerful way to escape this straightjacket. We show that the return to experience interacts with productivity growth (general wage growth) and with the value of nonemployment to provide a strong interaction of shocks and institutions that broadly accounts for the persistent movements in employment rates across time and space.

The return to experience works through two channels:

First, a decline in the return to experience has a direct effect on the decision of a marginal work to seek lifetime employment. If the wage escalator flattens, the payoff to being engaged in the workforce over a lifetime falls, so the option of accepting the nonemployment benefit becomes more attractive. This effect will operate even if the value of the nonemployment option is increasing with the rate of productivity growth. In contrast, if there is no return to experience, a flattening of the wage escalator arising from a decline in the general rate of productivity growth will not affect the margin between employment and nonemployment.
Second, the returns to experience affects the employment/nonemployment through an interaction with productivity growth (wage growth). Over a working life, a positive return to experience acts like compound interest on wage growth. Hence, a constant, positive return to experience levers the effect of productivity growth on the return to employment. Under the reasonable assumption that the benefits of nonemployment do not carry a return to experience, the existence of a positive return to experience creates the interaction between productivity growth and the employment rate.

The plan of the paper is as follows. In Section II, we present the theory. A key element of the theory is to allow for a distribution of individual skills and find the individual who is marginal for being employed. In addition to delivering the interaction between growth shocks and institutions discussed in the introduction, this theory also provides results on the effects of widening the distribution of skills. In Section III, we present empirical results on the changes in the return to experience by the skill-level of works and how it relates to our model. Better evidence is available for the United States than for Europe. The evidence provides support for the notion that changing returns to experience were sufficient large to shift employment rates. Moreover, we show that given the return to experience, there is a powerful relationship between trend in productivity and employment rates of marginal workers. In Section IV, we discuss how this paper relates to the literature. In Section V, we offer conclusions.

II. Model

We consider a simple environment in which there are two employment states, employment and nonemployment. At any point in time, workers choose whether they
want to supply their labor or not. The critical addition that we explore relative to previous literature is to allow for two forms of wage growth—growth in starting wages, and returns to labor market experience—as well as growth in the flow payoff from nonemployment.

Note that the phenomenon we are aiming to model is the life-long choice that a worker makes to be committed to the labor market and therefore accrue the returns to experience. Consequently, we abstract from frictional episodes of unemployment between jobs. Adding frictional unemployment would complicate the model, but not alter its central messages.

*Technology.* To concentrate on the labor supply effects that we highlight in this paper, we keep the demand side of the model as simple as possible. To this end, denote the flow product of a job filled by a worker $i$ with labor market experience $x$ at time $t$, as $w_i(x,t)$. As anticipated by the notation, we assume that the labor market is fully competitive, so that the wage of a worker $i$ with experience $x$ at time $t$ is also equal to $w_i(x,t)$. We make three assumptions about the nature of $w_i(x,t)$:

- The level of wages at all experience levels $x$, and for all individuals $i$, grows over time according to $\frac{\partial w_i(x,t)}{\partial t} = g(t)w_i(x,t)$, where $g(t)$ is the rate of aggregate productivity growth at time $t$.
- The wage also grows as an individual $i$ accumulates labor market experience according to $\frac{\partial w_i(x,t)}{\partial x} = g_x(t)w_i(x,t)$, where $g_x(t)$ is the return to experience at time $t$. 
• Individual wages may be rewritten as \( w_i(x,t) = \omega_i w(x,t) \), where \( \omega_i \) reflects an individual worker’s time-invariant skill, and \( w(x,t) \) is the average wage profile faced by a worker.

Preferences. Potential workers are infinitely lived and at each point in time choose their labor supply to maximize their lifetime wealth. The preferences of a potential worker \( i \) with experience \( x \) therefore are given by

\[
U_i(x,t) = \int_t^\infty e^{-r(s-t)} y_i(x,s) ds,
\]

where \( y_i(x,s) \) reflects the worker’s income at time \( s \). The latter depends on whether the worker is employed or not. From above, we know that if the worker is employed at time \( t \), she is paid a flow wage of \( w_i(x,t) \).

If a worker with experience \( x \) is nonemployed at time \( t \), we assume that she receives a flow payoff \( b_i(x,t) \). The latter partly reflects unemployment insurance that she is eligible to receive, but it also reflects her preference for leisure, the value of home production, as well as the generosity of public health insurance and social security and housing benefits. In particular, it includes much more that unemployment compensation. We make the following assumptions on \( b_i(x,t) \):

• The flow payoff from nonemployment at all experience levels \( x \), and for all individuals \( i \), grows over time according to \( db_i(x,t)/dt = g_b(t)b_i(x,t) \). Note that, in contrast to the growth of wages, the latter represents the total time derivative of the
payoff from nonemployment. Because experience stops accumulating when workers are nonemployed, there is no return to experience in $b_i(x,t)$.

- Analogous to our notation for wages, we rewrite $b_i(x,t) = \beta_i b(x,t)$, where $\beta_i$ reflects an individual worker’s idiosyncratic preference for leisure, and $b(x,t)$ is the payoff from nonemployment for an average worker if she has accumulated experience of $x$ at time $t$.

*Reservation Wages.* In order to derive aggregate labor supply, we first determine the reservation wage of an individual worker in this environment. The appendix shows that the reservation wage for a given worker, $i$, is given by

\[ w_{Ri}(x,t) = \alpha(t)b_i(x,t) \text{ where } \alpha(t) = 1 - \frac{g_x(t)}{r - g(t)}. \]  

(2)

The reservation wage has some very intuitive properties. First, we see that it will lie below a worker’s flow payoff from nonemployment if there is a positive return to experience. The reason is simple: If workers anticipate positive returns to experience, they will forgo earnings in the short run in order to reap the returns to experience in the long run.

A corollary of this observation is that increases in the return to experience will reduce reservation wages even further below $b$. The reason is that increases in the return to experience raise the present discounted value of earnings from working relative to not working. Moreover, such changes in the return to experience are likely to have high-powered effects on work incentives. Intuitively, this is for the familiar reason that changes in the rate of growth of an income stream have large effects on the present
discounted value of that stream, which is what matters for employment incentives in the
model, due to compounding. Mechanically, this can be verified by noting that the
denominator, \( r - g(t) \), in equation (2) is likely to be small for reasonable values of the
rate of time preference, \( r \), and aggregate productivity growth, \( g \).

A final key lesson from equation (2) is that the model affords a role for changes in
the rate of aggregate productivity growth, \( g \), in explaining changes in workers’
reservation wages. In particular, if the return to experience is positive, the model predicts
that increases in the rate of aggregate productivity growth will lead to a reduction in
reservation wages, and thereby an increase in work incentives. The simple reason is that
greater aggregate wage growth interacts with the return to experience by compounding
the rate of wage growth relative to the growth of the payoff from nonemployment.

It is important to note that the latter effect of aggregate productivity growth on
incentives to supply labor is absent in traditional models of aggregate employment
determination which assume that \( g_x = 0 \). The perceptive reader will observe, however,
that the effect of productivity growth in our model is driven by our specification that
experience is multiplicative, not additive, in determining wages, i.e., that the Mincerian
wage equation be specified in logarithms rather than in levels. The specification that
experience and productivity are multiplicative is, however, much deeper than a functional
form restriction. If the returns to experience were additive in wages, i.e., a fixed amount
rather than fixed percentage, then the returns to experience would become vanishingly
small over time if there is a positive trend to productivity. So an additive specification
for experience is asymptotically equivalent to assuming no return to experience at all.
**Steady State Equilibrium.** In steady state, the growth rates $g$, $g_x$, and $g_b$ will be constant, and so the reservation wage coefficient $\alpha$ also will be constant. To identify the steady state supply of labor to the economy, note that, by definition, any given worker will work so long as the offered wage, $w_i(x,t) = \omega_i w(x,t)$, exceeds her reservation wage,

$$w_{ri}(x,t) = \alpha \beta_i b(x,t).$$

Thus, an individual $i$ will choose to work if and only if the ratio of her skill to her leisure preference, $\omega_i / \beta_i$, exceeds $\alpha$ times the ratio of the flow payoff from nonemployment to wages, $b(x,t)/w(x,t)$. We assume that the flow payoff from nonemployment does not vary with experience, so $b(x,t) = b(t)$. Additionally, the benefit of nonemployment is proportional to the general wage level, not the individual-specific marginal product of working. Hence, the replacement rate $\rho$ will decline with experience. Specifically,

$$b(x,t)/w(x,t) = \rho(x,t) = e^{-g,\omega x} \rho(t). \quad (3)$$

In what follows, we refer to $\rho(x,t)$ as the replacement rate for workers of experience $x$ and $\rho(t)$ as the replacement rate for newborn workers that prevails at time $t$.²

The fraction of newborn workers in the economy that wishes to work at a given point in time $t$ is therefore given by

$$L(0,t) = \Pr[\omega_i / \beta_i \geq \alpha \rho(t)] = 1 - \Omega[\alpha \rho(t)], \quad (4)$$

² It is possible that the flow payoff from nonemployment may rise somewhat with work experience. For example, unemployment insurance in the United States and elsewhere replaces a fraction of an unemployed worker’s previous income. However, it does so only up to a point. Moreover, we are interpreting the flow payoff from nonemployment, $b$, as including much more than unemployment compensation. It reflects preferences for leisure, the value of home production, public health insurance and social security and housing benefits. These are less likely to accumulate with an individual worker’s experience.
where $\Omega$ is the c.d.f. of the ratio of skill to leisure preference, $\omega/\beta$, among a cohort. It follows from (4) that the rate of growth of the flow payoff from nonemployment $g_b$ must equal the rate of aggregate wage growth $g$ in steady state. To see why, imagine this were not true; e.g. imagine that $g_b > g$. In this case, the replacement rate for newborn workers $\rho(t)$ would trend upward over time, and aggregate labor supply would trend toward zero in the long run as the payoff from nonemployment eventually dominates the payoff from employment for (almost) all workers. The opposite is true if $g_b < g$. In that case the replacement rate for newborn workers trends downwards and labor supply converges to the point where (almost) all individuals wish to work. Thus, in steady state, it must be that $g_b = g$.

The replacement rate must therefore be time invariant in steady state, so that $\rho(x,t) = \rho(x) = e^{-\rho x} \rho$ for all $t$. Because the replacement rate declines as a worker accumulates experience, it follows that a worker who chooses to work at the beginning of her life will continue to work forever. Combining this with equation (4) in turn implies that the aggregate supply of labor in steady state is simply equal to

$$L^* = 1 - \Omega(\alpha \rho)$$

where $\alpha = 1 - \frac{g_s}{r - g}$. (5)

Figure 2 illustrates the qualitative properties of steady state labor market equilibrium in the model, which has a very simple structure. Given our assumptions on the production technology, the wage is pinned down entirely by a horizontal demand curve for labor. Aggregate productivity growth and returns to experience shift the demand curve upward over time. Similarly, from equation (5), employment is determined entirely by a vertical steady state supply of labor, and lies below the working
age population, which we normalize to one. The gap between steady state employment and one is therefore the steady state nonemployment rate.

The model implies therefore that only variables that shift the supply of labor will affect the steady state nonemployment rate. From equation (5), we see that the replacement rate, $\rho$, is one of these variables: A higher replacement rate renders nonemployment more attractive and reduces steady state labor supply. The latter effect is a very conventional long run property of models of equilibrium employment (see Blanchard, 2000; Layard, Nickell and Jackman, 1991, among others). But equation (5) also implies additional potential supply shifters that are less common in the literature: the distribution of the ratio of skill to leisure preference, $\Omega$, and the variable $\alpha$, which in turn is driven by the rate of aggregate productivity growth, $g$, and the return to labor market experience, $g_x$. We now explore these effects in more detail.

*Wage Inequality and Steady State Employment.* An influential explanation for the increased rate of joblessness in the US from the 1970s on has been the reductions in the wages of less skilled workers associated with widening wage inequality (Juhn, Murphy and Topel, 1991, 2002). We now show that this is a natural prediction of the model described above. Widening inequality of wages arises in the form of an increase in the dispersion of skill, $\omega_i$, in the model, which in turn affects the distribution $\Omega$ in equation (5).

To be concrete, assume that the distributions of workers’ skill and leisure preference are independent and log-normally distributed, $\ln \omega_i \sim N(\mu_\omega, \sigma^2_\omega)$ and $\ln \beta_i \sim N(\mu_\beta, \sigma^2_\beta)$, so that we can rewrite steady state employment as
\[ L^* = 1 - \Omega(\alpha \rho) = 1 - \Phi \left( \frac{\ln(\alpha \rho) - \mu}{\sigma} \right), \]  

where we define \( \mu \equiv \mu_\omega - \mu_\beta \), \( \sigma^2 \equiv \sigma_\omega^2 + \sigma_\beta^2 \), and \( \Phi \) is the c.d.f. of the standard normal.

It follows that the marginal effect of an increase in wage inequality is equal to

\[ \frac{\partial \ln L^*}{\partial \sigma_\omega} = \varepsilon^* \left( \frac{\sigma_\omega}{\sigma} \right) \Phi^{-1}(1 - L^*), \]

where \( \varepsilon^* \) is the elasticity of the supply of labor with respect to wages evaluated in steady state implied by equation (4),

\[ \varepsilon^* = \alpha \rho \frac{\Omega'(\alpha \rho)}{1 - \Omega(\alpha \rho)}. \]

Increased wage inequality therefore leads to a reduction in steady state employment in this environment if the steady state employment rate exceeds fifty percent, as it does empirically. Intuitively, when the employment rate, \( L^* \), exceeds one half, workers on the margin of employment are more likely to be low skilled. Widening wage inequality reduces the wages of these marginal less skilled workers, thereby reducing the steady state employment rate. This, of course, is exactly the explanation for the trends in US nonemployment put forward in Juhn, Murphy and Topel (1991, 2002).

*Wage Growth and Steady State Employment.* We now consider the effects of the variable \( \alpha \) as another supply shifter. From equation (5), we see that \( \alpha \) has an effect on labor supply that is completely symmetric to the effect of the replacement rate, and is determined by the rate of aggregate productivity growth, \( g \), and the return to experience, \( g_x \). A higher return to experience reduces \( \alpha \), as does a higher rate of aggregate productivity growth if the return to experience is positive, which in turn raises steady state labor supply.
More precise expressions for the effects of changes in \(g\) and \(g_x\) on steady state labor supply can be obtained from differentiating (5) and are given as follows,

\[
\frac{\partial \ln L^*}{\partial g} = \frac{\varepsilon^*}{r - g - g_x} \frac{g_x}{r - g},
\]

\(9\)

\[
\frac{\partial \ln L^*}{\partial g_x} = \frac{\varepsilon^*}{r - g - g_x}.
\]

(10)

To get a sense of the magnitudes of these effects, Table 1 reports the marginal effects of aggregate productivity growth, \(g\), and the return to experience, \(g_x\), for a range of values of the underlying parameters. We set \(r = 0.1\) to target an annual discount factor of approximately 0.9. In addition, we set \(\varepsilon^*\) equal to 1 for simplicity, noting that different values for \(\varepsilon^*\) simply scale the marginal effects up or down proportionately. Table 1 presents the marginal effects for values of \(g\) between 0.01 and 0.03 and values of \(g_x\) between 0 and 0.06.\(^3\)

The coefficients reported in Table 1 are the logarithmic change in employment following a onepercentage point change in either \(g\) or \(g_x\). Thus, a onepercentage point reduction in aggregate productivity growth \(g\) (e.g. from .02 to .03) reduces steady state employment by \(0 – 35.7\) log points for the values in Table 1. Likewise, a onepercentage point reduction in \(g_x\) reduces steady state employment by between 11 and 50 log points according to Table 1. These effects, therefore, are possibly very large.

Note that when the return to experience \(g_x\) equal to zero, Table 1 shows there is no effect of changing the growth rate of productivity \(g\) on employment. As noted in the introduction, this result is an implication of the restriction that the value of the not

\(^3\) The values used for \(g_x\) are obtained from a linearization of the observed concave log earnings experience. Specifically, we find the value of \(g_x\) that sets the present discounted value of earnings up to forty five years of experience equal to the value observed in Census data for a discount rate of \(r = 0.1\). A value of \(g_x\) of around 0.05 achieves this.
working keeps pace with the trend growth in wages. Table 1 illustrates the powerful role the return to experience has loosing the straightjacket that prevents changes in growth rates from affecting the employment rate.

*Where Shocks Hit Hardest: The Importance of Marginal Workers.* A recurring theme in the preceding analysis is that the employment effects of shocks, be they changes in the dispersion of wages through $\Omega$, or in the rates of aggregate productivity growth and returns to experience through $\alpha$, are all increasing in the size of the elasticity of aggregate labor supply with respect to the wage, $\varepsilon^*$. The intuition for this result is simple. A small value of $\varepsilon^*$ implies that there are little incentive effects of wages on the workers choice to supply labor. This in turn extinguishes the labor supply effects of wage growth and dispersion which rely on the notion that wages incentivize labor supply.

It is natural to ask what factors might determine the size of the employment elasticity. We now show that $\varepsilon^*$ will be particularly large for workers who are low-skilled. To see this, note that we can write the steady state employment rate among workers of a given skill $\omega$ as

$$L^*(\omega) = 1 - \Lambda(\alpha \rho / \omega), \quad (11)$$

where $\Lambda(\cdot)$ is the c.d.f. of the inverse of workers’ idiosyncratic preference for leisure, $1/\beta$. It follows that the elasticity of the employment rate for workers of skill $\omega$ with respect to the wage is equal to

$$\varepsilon^*(\omega) = \frac{\alpha \rho}{\omega} \frac{\Lambda'(\alpha \rho / \omega)}{1 - \Lambda(\alpha \rho / \omega)}. \quad (12)$$
It is straightforward to verify that a sufficient condition for this elasticity to decline with skill, $\omega$, is that the modal worker of that skill is employed.\textsuperscript{4} Thus, the model predicts that low-skilled workers respond to changes in the rate of aggregate productivity growth and the return to experience to a greater extent. The simple reason is that low-skilled workers are more likely to be on the margin of the employment decision than high-skilled workers, and therefore more responsive to changes in the incentives to work.

This prediction of the model formalizes the intuition that pervades the empirical analysis of Juhn, Murphy, and Topel (1991, 2002). They show that much of the increase in joblessness in the US from the 1970s onward is concentrated among low-skilled workers. In addition, they provide estimates of the elasticity of labor supply by skill group (see Table 9 of their 1991 article and Table 10 of their 2002 article) that confirm that low-skilled labor supply is much more elastic than for higher skilled workers. Both of these results are entirely consistent with the formal implications of our model. We will see later in section III that the tight correspondence between our theoretical model and the empirical results of Juhn, Murphy, and Topel will enable us to interpret and quantify the implications of our model for observed trends in joblessness in the US over time.

\textit{Short Run Dynamics}. Until now, we have examined the long run response of the model to changes in the distribution of skill, the rate of aggregate productivity growth, and the return to experience. We have shown that a distinctive feature of our model is that it

\textsuperscript{4} To see this, note that since $\alpha \rho / \omega$ is declining in $\omega$, the elasticity of aggregate labor supply for workers with skill $\omega$ will decline with skill if $\Lambda ^{\ast}(\alpha \rho / \omega) > 0$. A sufficient condition for the latter is that the modal worker with skill $\omega$ chooses to work, so this result implicitly assumes that $\Lambda$ is unimodal.
generates a role for productivity growth in explaining variation in steady state employment that is absent in standard models of long run employment determination.

Perhaps because prevailing models predict no long run employment effects of changes in productivity growth, however, a prominent feature of previous literature has been in its emphasis on the potential short run employment effects of variation in productivity growth (see among others Blanchard, 2000; Bruno and Sachs, 1985; Ball and Moffitt, 2001). A popular idea that has been pursued is that the wage demands of workers (which correspond to reservation wages in this model) are somewhat sluggish in their response to changes in productivity growth. We now examine this possibility in the context of our model of employment determination.

Sluggish behavior of reservation wages may arise in our model if the flow payoff from nonemployment, $b$, does not immediately adjust to maintain the steady state replacement rate, $\rho$. We assume that there is a “comprehension lag” (Blanchard, 2000) such that workers do not fully comprehend that the rate of productivity growth, and thereby the sustainable rate of growth of wages for newborn workers, $g$, has declined.

To be concrete, we imagine that workers update their expectations of the rate of productivity growth, $g^e(t)$, according to

$$\frac{dg^e}{dt} = -\lambda \left[ g^e(t) - g(t) \right], \quad (13)$$

where $\lambda$ reflects the speed at which workers update. This may arise, for example, if workers find it difficult to distinguish between transitory and permanent shocks to the rate of productivity growth. Under that interpretation, $\lambda$ will depend on the relative variances of the permanent and transitory shocks.
We then consider the response to a permanent reduction in the true underlying rate of productivity growth at some point in time, say $t = 0$, from $g_0$ to $g_1$. From equation (13), it follows that workers’ expectations of the rate of productivity growth evolve according to

$$g^e(t) = e^{-\lambda t} g_0 + (1 - e^{-\lambda t}) g_1. \tag{14}$$

Given these expectations of the rate of productivity growth, workers subsequently perceive the level of the replacement rate as evolving according to

$$\ln \rho^e(t) = \ln \rho + \ln \left( \frac{g^e(t) - g_1}{g_0 - g_1} \right) t = \ln \rho + e^{-\lambda t} \left( g_0 - g_1 \right) t, \tag{15}$$

where $\rho$ is the steady state replacement rate in the economy. Moreover, workers’ perceptions of the reservation wage coefficient $\alpha$ will evolve according to

$$\alpha^e(t) = 1 - \frac{g_x}{r - g^e(t)}. \tag{16}$$

To compute the dynamic effects of a decline in productivity growth we modify slightly the steady state model presented earlier. In particular, we assume that the replacement rate does not vary with experience,

$$b(x,t)/w(x,t) = \rho(t) \tag{17}$$

for all $x$. It follows from this that the fraction of workers in the economy that wishes to work at any given point in time $t$ simply will be given by

$$L(t) = 1 - \Omega[\alpha(t) \rho(t)]. \tag{18}$$

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5 For convenience in computing the dynamics, we use the specification (17) where benefits reflect the return to experience rather than the specification (3) where they do not. Note that the steady state is the same with either specification because a newborn worker who starts to work will never quit working in steady state under either specification of the replacement rate. For the transition dynamics discussed here, specification (17) saves the trouble of keeping track of the decisions of different cohorts of workers along the transition path. Under specification (3), experienced workers would delay leaving the workforce following the change in the growth rate. These delays would slow the adjustment to the steady state.
Figure 3 illustrates an example. In the context of our model, we see that sluggish expectations of productivity growth have two implications for the short run dynamics of the model. First, they imply that workers’ perceptions of the reservation wage coefficient $\alpha$, which from equation (2) depends on the workers’ perceptions of the rate of productivity growth, converges monotonically to a higher steady state value following a productivity slowdown. Workers’ perceptions of the replacement rate $\rho$, on the other hand, rise in the short run and subsequently falls back to its original long run value as workers begin to comprehend that the rate of productivity growth has subsided.

What will be the short run effects of a productivity slowdown on the equilibrium employment rate in the economy? Thanks to the simplicity of our characterization of aggregate labor supply in equation (4), such a question is straightforward to answer: it will simply depend on the evolution of the product $\alpha(t)\rho(t)$. Consistent with this, Figure 3 shows that the employment rate, $L(t)$ in equation (4), falls slowly and converges to the new long run steady state value. As one might anticipate from the evolution of $\alpha$ and $\rho$ in Figure 3, it is possible that the employment rate can fall below and overshoot its long run steady state outcome. In fact, for the parameter values in Figure 3, the employment rate does overshoot, but only very mildly, so that it is almost impossible to discern.

III. Evidence and Implications

In the previous section, we have shown that there is a powerful interaction between productivity growth and the return to experience in driving the decision of marginal workers to be employed over their lifetime. A positive, but constant, return to
experience will lead changes in productivity growth to affect employment decisions. Hence, a central contribution of the paper is to provide an explanation of the productivity growth/employment correlation that has been widely noted, but difficult to build into theoretical models.

In the first part of this section, we examine evidence for changes in the return to experience. In the second part of this section, we map those changes in experience into our model and examine the predictions they have for the change in the employment rate. In the third part of this section, we examine how much—given the positive return to experience—changes in growth rates affect employment.

A. Changes in the return to experience for low-skill workers

We now consider evidence on the changes in the returns to experience and discuss the extent to which these changes can explain changes in employment rates. We present results for the United States. As our theory makes clear, the marginal worker will be relatively low in the skill distribution. We use educational attainment as a rough and ready proxy for skill.

We present results on changes in the return to experience over time. These results are based on calculations taken from Heckman, Lochner, and Todd’s (2007) analysis of decennial censuses. Though their important study focuses on the returns to schooling, it provides valuable evidence on the returns to experience. Our figures rework their tabulations, which the authors kindly provided to us, to show the statistics of interest in this context.
Figure 4 shows log earnings as a function of potential experience, normalized to zero at zero potential experience. Each panel of the figure presents the results for different levels of education. Normalizing log earnings to zero for zero log experience abstracts from the significant differences in levels of earnings across the groups. The data are for white men and include all earnings (wage and salary and business income). We limit the analysis to men to avoid the complications arising from the trend in female labor supply and the more episodic nature of female work over the lifecycle. Within each panel, the lines correspond to different Census years. The results for high school dropouts (9-11 years of education) show a clear structural break in the returns to experience between 1970 and 1980. We focus on the dropouts because this group is likely to be at the bottom of the skill distribution and hence marginal for the employment/nonemployment decision. In Figure 4A, the shift down in the return to experience in the 1980 through 2000 compared to the 1950 through 1970. At five to ten years of experience, earnings are 30 to 40 percent lower in the later period compared to the earlier period. The gap in the return to experience that opened in between 1970 and 1980 persists for higher level of experience. There is a bit of narrowing of the gap as experience approaches 30 years, though these late-career fluctuations in earnings will have little impact on lifetime employment decisions.

For high school graduates, shown in Figure 4B, the return to experience drops at mid-career in 1980, just as for dropouts. Unlike for dropouts, there is a sharp rebound in high school graduates return to experience in 1990.

---

6 We examined similar charts for blacks and for individuals with educational attainment of less than 9 years. Even with the large sample sizes the Census allows, they were too noisy to support meaningful inferences. In the calculations presented below, we use up to 45 years of experience. We truncate the charts at 30 years of experience again because the data get noisy for higher levels of experience. In any case, the charts show the relevant range experience for a present value metric.
Workers with schooling beyond high school are unlikely to be at the point in the skill distribution where employment is a marginal decision, so there patterns of returns to experience are less relevant for them. For comparison with the low-education groups, we include results in Figure 4C, 4D, and 4E (some college, college degree, and post-graduate). For some college and college graduates, there is not much of pattern except that the 1950 look to have a higher return to experience. The post-graduates in Figure 4E show a decline in the returns to experience similar to that shown in Figure 4A for the high school dropouts, except for post-graduates, the structural break is after 1980, not after 1970. While this finding is interesting for other purpose, the level of the return to work of the post-graduates is so high in the skill distribution that the shift in the returns to experience should not affect their decision whether or not to work.

The foregoing results look at the experience-earnings profile at points in time. The same data can be examined from the cohort perspective. It is not clear whether the time or cohort perspective is the correct one, so it is worth checking whether the basic message changes by shifting perspective. Figure 5 presents the experience earnings profiles for the cohorts for individuals who have zero years of potential experience in 1950, 1960, 1970, 1980, and 1990. Since we have only decadal Census data, we get a reading on wages only once every 10 years. With the passing of each decade, the range of experience gets a decade shorter.\(^7\)

For the high school dropouts, the picture by cohort in Figure 5A tells exactly the same story as by year in Figure 4A. The returns to experience for cohorts beginning in 1980 is substantially lower than for earlier cohorts. The same pattern is evident by

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\(^7\) We could compute decadal earnings profiles for cohorts with zero experience in other years. We set these aside so as not to clutter the graphs.
cohorts for high school graduates in Figure 5B. As with the result by year in Figure 4, the stark shift between 1970 and 1980 in the returns to experience is less evident for individuals with higher education (see Figures 4C, 4D, and 4E).

B. Implications of the decline in the return to experience

The preceding discussion presents strong evidence for a reduction in the return to labor market experience for low-skilled, marginal workers. We now seek to provide a quantitative sense of the implications of this trend for work incentives and equilibrium employment. In Figure 6 we present estimates of the value of lifetime earnings for low-skilled workers (9 to 11 years of education) implied by the experience–earnings profiles in Figure 4A. Specifically, we calculate the present value of earnings over a forty-five year working life for a range of values for the discount rate, and for each Census year. Figure 6 plots these calculations as a multiple of the starting wage for a worker with zero labor market experience at each date.

It is clear from Figure 6 that high school dropouts faced a large reduction in their prospective lifetime earnings after 1970, consistent with the impression from the experience–earnings profiles in Figure 4A. Moreover, it is clear that the magnitude of this reduction was substantial. Expected lifetime earnings for high school dropouts fell between twenty and thirty percent from 1970 and before to 1980 and after.

We now consider the implications of this observed reduction in the returns to experience for low-skilled employment rates over time through the lens of the model of Section II. To do so, we modify the model of Section II to incorporate finite worker lifetimes. There are several reasons to move to a finite-lifetime model. In the infinite-
horizon model, the returns to experience compound indefinitely. In reality, the returns to experience end at death or retirement. Moreover, as will be apparent in our empirical results, the returns to experience flatten later in working lives. For these reasons, the infinite horizon specification overstates the value of experience. Consequently, the derivatives calculated in Table 1, though they give the right qualitative picture, overstate the response of employment to changes in experience and are therefore not comparable to what we would expect to see in the data.

We therefore consider a specification of the model with a finite lifetime. In the appendix we show that, if workers live for \( X \) years, steady state labor supply in the modified model is given by

\[
L^* = 1 - \Omega \left[ \alpha(X) \rho \right] \quad \text{where} \quad \alpha(X) = \frac{r - g - g_s}{r - g} \frac{1 - e^{-X(r-g)}}{1 - e^{-X(r-g-g_s)}}.
\]  

Recall that changes in the return to experience are summarized by the parameter \( g_s \) in the model. In the model, we assumed that \( g_s \) is independent of the level of experience. Hence, we need to map the concave log earnings-experience profiles observed in Figure 4 into a single value of \( g_s \). For each Census year, we find the constant value of \( g_s \) that generates a present value of earnings from the model’s perspective that equals the empirical present discounted value of earnings reported in Figure 6 over a forty-five year lifetime, i.e. for \( X = 45 \).

We can now infer the change in the nonemployment rate for low skilled workers implied by the model using the results of Section II as a function of these estimates of changes in the returns to experience. Note that, analogous to equation (10), we can write the semi-elasticity of the employment rate for workers of a given skill \( \omega \) with respect to \( g_s \), as
\[
\frac{\partial \ln L'(\omega)}{\partial g_x} = \varepsilon^*(\omega) \frac{\partial \ln \alpha(X)}{\partial g_x},
\]

where \(\varepsilon^*(\omega)\) is the wage elasticity of labor supply for workers of skill \(\omega\). It follows that, in order to compute the nonemployment rate for low-skilled workers implied by the model, we need a measure of \(\varepsilon^*(\omega)\) for low-skilled workers. Such estimates are reported in Juhn, Murphy and Topel (1991, 2002) who show that this elasticity is approximately equal to 0.3 for low-skilled workers.\(^8\)

Figure 7 plots the nonemployment rate for high school dropouts implied by the model and by the different values of \(g_x\) decade by decade. To isolate the effects of the return to experience, we hold the productivity growth rate fixed at its average level of 0.02. The figure includes separate lines for different discount rates. The model predicts a substantial rise in the nonemployment rate for low-skilled workers given the size of the observed reduction in their lifetime earnings. In particular, the model predicts an 8 to 9 percentage point increase in the nonemployment rate between 1970 and 1980 and after. Hence, a substantial increase in the nonemployment rate—at least among the low-skill group—can be attributed to the decline in the return to experience.

\(^8\) Note that the elasticity \(\varepsilon\) is a function of a number of elements of the model—the distribution of skill and leisure preference, productivity growth and the return to experience, and the replacement rate. It is not a utility function parameter. (There is no intensive margin in the model; disutility of work in the model is reflected entirely in the parameter \(\beta_1\).) The institutional-shock interaction discussed in the introduction is captured in the elasticity. Changes in the replacement rate, with its implications for changes in the elasticity, are probably not that relevant for the United States, but could be more important in Europe. We will explore this implication of our model in future work. These considerations suggest that econometric estimates of labor supply elasticity, such as in Juhn, Murphy, and Topel, need to take into account that parameter is not structurally invariant to changes in replacement rates.
C. Productivity growth and the employment rate revisited

Our theory aims to explain the relationship between productivity growth and employment rates. The Staiger-Stock-Watson intriguing graph (Figure 1A), using the unemployment rate, strongly supports the view that productivity growth rate changes are adverse shocks for employment. As we discuss in the introduction, the picture is different for the nonemployment rate. As Juhn, Murphy, and Topel (1991, 2002) have taught us, the decline in unemployment rates from the peaks in the early 1980s has not led to corresponding increases in employment rates. Consequently, though the decline in the employment rate in the 1970s matches the fall in productivity growth, there is not a rebound on employment in the 1990s when productivity growth recovered (Figure 4A).

In this subsection, we use the theory developed in Section II and the empirical results of Section IIIA to reassert the strong comovement on labor market outcomes and productivity growth even when nonemployment rather than unemployment is the measure of labor market equilibrium. First, the theory points strongly to the employment rate (as opposed to the unemployment rate) as the appropriate metric for examining the link between productivity growth and labor market outcomes. Changes in productivity growth have long-term implications for whether an individual supplies labor to the market. Second, the theory shows how movements in productivity growth will have powerful effects on the labor supply decisions of marginal workers, while having little effect on those at the higher reaches of the skill distribution. Accordingly, we should be looking for a relationship between the employment rate of marginal workers and productivity growth.
We present such evidence in Figure 8. It shows the same smoothed productivity growth as shown in Figure 1. In Figure 8, the nonemployment rate is shown separately for low wage workers (lowest decile of the wage distribution) and for high wage workers (top four deciles of the wage distribution). These nonemployment rates by wage percentile group, taken from Juhn, Murphy, and Topel (2002), are based on the March CPS interviews (retrospective for the previous calendar year). Figure 8 shows that the nonemployment rate for high wage workers is roughly constant at a low level. In contrast, the nonemployment rate for low wage workers moves substantially—from 15 percent in the 1960s to 35 percent in the 1980s and then recovering to 25 percent by the end of the 1990s. As in the intriguing graph, nonemployment is roughly the mirror image of productivity growth. In particular, it does appear that the productivity growth rebound of the late 1990s did have the predicted effect on the labor market experience of marginal workers.\footnote{Nonemployment of other low wage groups, e.g., the 10-20 and 20-30 deciles have similar, though muted patterns.} The Juhn, Murphy, and Topel data end in 2000, so we do not know whether this improvement persisted. With the recession of 2001, the slow recovery of employment, and the slowdown in productivity growth, it will be hard to know what is happening to this relationship in any case. But for the 1990s it is clear that the improvement marginal workers had an increased rate of employment, just as our theory would predict given the rebound of productivity growth.\footnote{Juhn, Murphy, and Topel (1991) predicted that the employment rate for low-wage workers would be permanently depressed. This prediction was predicated on a presumption that wages would not recover for this group. In fact, these low-wage workers appear to have had modest wage gains in the late 1990s (see Juhn, Murphy, and Topel (2002, Figure 1). That, combined with the steepening of the escalator from the improvements in the return to working that we have documented, can account for the increase in the employment rate shown in Figure 8.}

To provide a quantitative sense of the likely effects of these movements in aggregate productivity growth on the nonemployment rates of low-skilled workers...
implied by our model, we consider a simple numerical example based on the results of Section II. The analysis is similar to quantitative assessment of the effects of changes in the return to experience above. We first note that, analogous to equation (9), the finite lifetime model we used for calibration above implies that the semi-elasticity of the employment rate of individuals of skill $\omega$ is given by

$$\frac{\partial \ln L^* (\omega)}{\partial g} = \epsilon^* (\omega) \frac{\partial \ln \alpha (X)}{\partial g}. \quad (21)$$

Recall that $\epsilon^* (\omega)$ is the wage elasticity of labor supply for workers of skill $\omega$. We again set the latter equal to 0.3 based on Juhn, Murphy and Topel’s analysis.

To get a sense for the effects of productivity growth, Figure 9 plots the evolution of the nonemployment rate implied by the model for fixed returns to experience and for the same range of discount rates used in our analysis of changes in the return to experience. In particular, for each discount rate we fix the return to experience $g$, to equal the time average of its calibrated value for that discount rate. We then feed into the model the observed low frequency changes in the rate of productivity growth pictured in Figure 1. The implied effects of the observed changes in aggregate productivity growth on low-skilled nonemployment are substantial in the model. The model predicts that the slowdown in productivity growth would have led to an increase in the nonemployment rate of low skilled workers of 5 to 6 percentage points between 1970 and 1980. It also suggests that the rebound in productivity growth in the late 1990s would have led to a symmetric decline in the nonemployment rate of high school dropouts.

The combined visual impression of the implications of changes in the return to experience in Figure 7 and of changes in the rate of productivity growth in Figure 9
suggest that allowing both \( g \), and \( g \) to vary over time may go a long way toward explaining the overall trend in nonemployment for low skilled workers. Figure 10 confirms this impression. It plots the change in the nonemployment rate for high school dropouts predicted by the model if both the return to experience and productivity growth vary as the data suggest. Figure 10 also superimposes the actual trend nonemployment rate of low skilled workers from Juhn, Murphy, and Topel (2002) reported in Figure 8. The results are very encouraging. The model predicts almost exactly the large observed increase in the nonemployment rate observed between 1970 and 1980. It is also in line with the subsequent decline in low skilled nonemployment in the late 1990s.

IV. Related Literature

TO BE ADDED

V. Summary and Discussion

Rates of unemployment and nonemployment move persistently across time and space. Changes in the trend in productivity are a leading candidate for accounting for changes in employment rates. Productivity growth has persistent swings—strong in the 1960s, weak in the 1970s and into the 1980s, improving in the 1990s, and perhaps weakening again recently—that roughly match the low-frequency movements in labor market equilibria. Yet, despite the appeal of the changes in the rate of growth of productivity for explaining the employment rate, the connection does not emerge easily. The central issue is that a slowdown in productivity growth—while it reduces the return to working also reduces the return to not working. In theory, the value of work and
nonwork must move together to assure balanced growth. In practice, the social benefits that provide for support for nonworkers are likely to move with the general level of productivity of the economy.

This paper shows that if the return to experience is positive, changes in productivity growth will affect the employment/nonemployment margin. The return to experience creates a wedge between the value of nonwork (indexed to productivity) and the value of work (indexed to productivity times the returns to experience). This wedge leads changes in productivity growth to have significant implications for the employment rate. It also means that changes in the return to experience will affect the employment rate.

The calculations presented in this paper show that the interaction of the return to experience and the productivity trend are important for explaining movements in the nonemployment rate for marginal workers in the United States. We show that the model predicts the negative correlation between trend growth in productivity and employment outcomes of low-wage workers. The productivity slowdown in the 1970s and its reversal in the 1990s mirrors their employment rate. Our calculations show that the change in productivity growth rates from three percent to one percent would lead to a 5 to 7 percent change in the nonemployment rate, depending on the return to experience. Moreover, the sharp and persistent drop in the return to experience for low-skill workers during the 1980s also is a significant explanatory factor in the increase in the nonemployment rate. This factor has not reversed and is part of the explanation for the persistently high nonemployment of low-skill workers. It can account for approximately a 10 percentage point increase in the nonemployment rate of workers at the margin.
Taken together, the change in the return to experience and change in productivity growth rates explain much of the change in the employment rate of low-wage men over the last four decades. The nonemployment rate rose from under 15 percent in the 1960s to over 30 percent in the 1980s. It declined modestly during the 1990s economic boom. The effects implied by our model of the changes in the return to experience and productivity growth do a remarkably good job of tracking both the timing and magnitude of the nonemployment rate of low-wage workers. Because so much of the aggregate fluctuation in employment rates comes from the lower end of the wage distribution, these effects also will account for a substantial fraction of the variation in aggregate employment rates.
REFERENCES


Juhn, Chinhui, Kevin M. Murphy, Robert H. Topel. “Why has the Natural Rate of Unemployment Increased over Time?” Brookings Papers on Economic Activity (2:1991) 75-142.


Appendix

Derivation of the Reservation Wage. To derive the reservation wage, we solve a problem whose limiting case corresponds to the model of the main text. In particular, we assume that workers may choose whether to work or not with probability $\lambda \, dt$ in a small interval of time $dt$ (i.e. workers may choose to work or not at Poisson rate $\dot{\lambda}$). Given this, we can characterize the value of employment to a worker with experience $x$ at time $t$ as

$$rW(x,t) = w(x,t) + \lambda \max\left\{ B(x,t) - W(x,t) , 0 \right\} + \frac{dW(x,t)}{dt},$$

and the value of nonemployment as

$$rB(x,t) = b(x,t) + \lambda \max\left\{ W(x,t) - B(x,t) , 0 \right\} + \frac{dB(x,t)}{dt}.$$  

As $\lambda$ approaches infinity, workers may choose between employment and nonemployment at all times, and this model approaches the model of the main text.

We solve for the reservation wage that sets $W(x,t) = B(x,t) + \varepsilon$ for $\varepsilon$ greater than but approaching zero. To do this we need to solve for the value functions $B(x,t)$, and $W(x,t)$. These comprise a system of two functional equations, which we solve mutually via the method of undetermined coefficients. To this end, we conjecture that, in the neighborhood of the reservation wage, the value functions are of the form

$$B(x,t) = \alpha_1 w(x,t) + \alpha_2 b(x,t),$$
$$W(x,t) = \beta_1 w(x,t) + \beta_2 b(x,t),$$

and confirm that this form is verified in what follows. Note that, given this, we can write the worker's reservation wage as

$$w_r(x,t) = \frac{\alpha_2 - \beta_2}{\beta_1 - \alpha_1} b(x,t).$$

It thus remains to solve for the parameters $\{ \alpha_1, \alpha_2, \beta_1, \beta_2 \}$. Under the conjecture, we can rewrite the value of nonemployment as

$$(r + \lambda) \left[ \alpha_1 w(x,t) + \alpha_2 b(x,t) \right] = b(x,t) + \lambda \left[ \beta_1 w(x,t) + \beta_2 b(x,t) \right] + \alpha_2 g(t) w(x,t) + \alpha_2 g_n(t) b(x,t),$$
Note that there is no term that reflects the return to experience in the latter because experience stops accumulating when a worker is not employed. Equating coefficients, we obtain

\[ (r + \lambda) \alpha_1 = \lambda \beta_1 + \alpha_1 g(t) \quad \Rightarrow \alpha_1 = \frac{\lambda \beta_1}{r + \lambda - g(t)} \]
\[ (r + \lambda) \alpha_2 = 1 + \lambda \beta_2 + \alpha_2 g_x(t) \quad \Rightarrow \alpha_2 = \frac{1 + \lambda \beta_2}{r + \lambda - g_b(t)}. \]  
(27)

Next, we seek to obtain in a symmetric fashion the parameters of the value of employment, the \( \beta_s \). To this end, we rewrite the value of employment under the conjecture as

\[ r \left( \beta_1 w(x,t) + \beta_2 b(x,t) \right) = w(x,t) + \beta_1 \left[ g(t) + g_x(t) \right] w(x,t) + \beta_2 \left[ g_b(t) + g_x(t) \right] b(x,t). \] \( \text{(28)} \)

Note there are two forms of wage growth associated with being employed: the growth due to aggregate wage growth, \( g \), and the growth due to the accumulation of experience, \( g_x \). Equating coefficients, we obtain:

\[ r \beta_1 = 1 + \beta_1 \left[ g(t) + g_x(t) \right] \quad \Rightarrow \beta_1 = \frac{1}{r - g(t) - g_x(t)} \]
\[ r \beta_2 = \beta_2 \left[ g_b(t) + g_x(t) \right] \quad \Rightarrow \beta_2 = 0 \]  
(29)

We can now solve for the parameters of \( B(x,t) \) in closed form

\[ \alpha_1 = \frac{\lambda}{r + \lambda - g(t)} \left( \frac{1}{r - g(t) - g_x(t)} \right); \quad \alpha_2 = \frac{1}{r + \lambda - g_b(t)}. \] \( \text{(30)} \)

Recalling the expression for the reservation wage, we obtain

\[ w_r(x,t) = \frac{r + \lambda - g(t)}{r + \lambda - g_b(t)} \left( \frac{r - g(t) - g_x(t)}{r - g(t)} \right) b(x,t). \] \( \text{(31)} \)

Taking the limit as \( \lambda \) approaches infinity using l’Hopital’s rule yields the result stated in the main text.
Finite Horizon Version of the Model. In order to calibrate the model to observed data on experience-earnings profiles, we modify it by incorporating a finite lifetime of \(X\) years. We then conjecture that optimal labor supply in this environment takes the form:

**Conjecture.** An individual either works for her entire lifetime (\(X\) years), or does not work for her entire lifetime.

We show in what follows that an individual has no incentive to deviate from such a policy.

Under the conjecture, an individual anticipates that if she works at the start of her life, she will do so until date \(X\), yielding utility \(\omega_i w(0,t)\left(1-e^{-X(r-g-g_s)}\right)/(r-g-g_s)\).

Likewise, if she doesn’t start work at the beginning of her life, she anticipates never working, yielding utility \(\beta b(t)\left(1-e^{-X(r-g)}\right)/(r-g)\). It follows that, under the conjecture, the optimal labor supply policy is of the form

\[
\text{work for lifetime} \quad \text{if} \quad \frac{\omega_i}{\beta_i} \geq \alpha(0,X)\rho(0),
\]

\[
\text{do not work for lifetime} \quad \text{if} \quad \frac{\omega_i}{\beta_i} < \alpha(0,X)\rho(0),
\]

where \(\alpha(x,X) = \frac{r-g-g_s}{r-g} \cdot \frac{1-e^{-(X-x)(r-g)}}{1-e^{-(X-x)(r-g-g_s)}}\), and where \(\rho(x) = e^{-g_s x}\rho(0)\) is the replacement rate at experience \(x\), with \(\rho(0) = b(t)/w(0,t)\).

The labor supply policy in equation (32) will be the optimal policy if a worker for whom \(\omega_i / \beta_i \geq \alpha(0,X)\rho(0)\) has no incentive to stop working at any point in her life, and if a worker for whom \(\omega_i / \beta_i < \alpha(0,X)\rho(0)\) has no incentive to start working at any point in her life. We now confirm that there is no incentive to deviate from the conjectured policy.

First, we show that an individual who chooses to work at the beginning of her life will choose to work at all subsequent dates. To see this, note that it is possible to show that \(\frac{\partial}{\partial x} (\alpha(x,X)\rho(x)) < 0\). It follows that, if \(\omega_i / \beta_i \geq \alpha(0,X)\rho(0)\), then it is also true that \(\omega_i / \beta_i \geq \alpha(x,X)\rho(x)\) for all \(x \geq 0\). Now consider a worker with \(\omega_i / \beta_i \geq \alpha(0,X)\rho(0)\) who starts work at the beginning of her life. After a short period of time \(dt\), the worker faces a labor supply problem that is identical to that at the beginning of her life except that she has accumulated labor market experience. Because \(\omega_i / \beta_i \geq \alpha(x,X)\rho(x)\) for all \(x \geq 0\), she will again choose to supply her labor after an interval \(dt\) if she anticipates that she will work at all subsequent points in time. By induction, she will indeed supply her labor at all future dates. Thus, she has no incentive...
to deviate from the conjectured policy: Any worker who starts working at the beginning of her life will also work at all intervening dates.

Analogously, we show that an individual who chooses not to work at the beginning of her life will choose not to work at all subsequent dates. To see this, note that \( \frac{\partial}{\partial x} \left( \alpha(0, X) \rho(0) \right) < 0 \). It follows from this that if \( \omega_i / \beta_i < \alpha(0, X) \rho(0) \), then it must also be true that \( \omega_i / \beta_i < \alpha(0, X - s) \rho(0) \) for all \( s > 0 \). By an analogous logic to that above, any worker who chooses not to start work at the beginning of her life will also choose not to start work at any intervening point in time.
Table 1. Marginal Effects of a one percentage point increase in \( g \) and \( g_x \) on log Employment

<table>
<thead>
<tr>
<th>( g_x )</th>
<th>( 0.01 \times \partial \ln L^*/\partial g )</th>
<th>( 0.01 \times \partial \ln L^*/\partial g_x )</th>
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</thead>
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<td>0.032 0.042 0.057</td>
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</tr>
<tr>
<td>0.06</td>
<td>0.222 0.375 0.857</td>
<td>0.333 0.500 1.000</td>
</tr>
</tbody>
</table>

Note: Calculations based on the infinite horizon model. See equations (9) and (10). The parameter \( \varepsilon^* = 1 \).
Note: Productivity growth is computed from the BLS output per hour for the business sector. The employment and nonemployment data are BLS series for males. All series are annual and then smoothed using the HP filter.
Figure 2. Steady State Equilibrium in the Model
Parameter values: $r = 0.1, g_s = 0.05, \rho = 0.5, \lambda = 0.2, \sigma = 1, \mu = 0$
Figure 4. Returns to Experience, By Census Year

A. 9-11 Years of Education

B. 12 Years of Education
Figure 4. Returns to Experience, By Census Year (continued)

C. 13-15 Years of Education

D. 16 Years of Education
Figure 4. Returns to Experience, By Census Year (continued)

E. More Than 16 Years of Education

Note: Authors’ tabulation based on original tabulations of decennial Census data by Heckman, Lochner, and Todd (2007). Data are for white males. Potential experience is defined as age minus education minus six. Census year earnings profiles measure the return to experience at a point in time ($g_x$ in the model).
Figure 5. Returns to Experience, By Cohort

A. 9-11 Years of Education

B. 12 Years of Education
Figure 5. Returns to Experience, By Cohort (continued)

C. 13-15 Years of Education

D. 16 Years of Education
E. More Than 16 Years of Education

Note: Authors’ tabulation based on original tabulations of decennial Census data by Heckman, Lochner, and Todd (2007). Earnings are deflated by the CPIU. Data are for white males. Potential experience is defined as age minus education minus six. Cohort earnings profiles include total return to work, that is, the sum of return to experience and general wage increase ($g + g_1$ in the model).
Note: Authors’ calculation of the present value of earnings over a forty-five year working life, discounted at rate $r$, and expressed as a fraction of the starting wage of a worker with zero labor market experience. Data used for the calculation are the experience profiles for 9–11 years of education are the same as those underlying Figure 4A.
Figure 7. Implied Response of Low-Skilled Nonemployment to Observed Changes in the Return to Experience

Note: Authors’ calculation of the change in the nonemployment rate for workers with 9 to 11 years of schooling implied by the reduction in the present value of earnings for these workers depicted in Figure 4A. Evolution of nonemployment rate is computed for a range of values for the discount rate, $r$. Productivity growth $g$ is held constant at 0.02. The nonemployment rate is normalized to 0.18 in 1970.
Note: Productivity growth is computed from the BLS output per hour for the business sector (same as Figure 1). The nonemployment rates are from Juhn, Murphy, and Topel’s (2002, Figure 9) tabulation of the March CPS retrospective annual data. Nonemployment data are for males. The “low wage” group are in the 0-10 percentiles of the wage distribution. The “high wage” group are in the 61-100 percentiles. All series are annual and then smoothed using the HP filter.
Figure 9. Implied Response of Low-Skilled Nonemployment to Observed Changes in Trend Aggregate Productivity Growth

Note: Authors’ calculation of the change in the nonemployment rate for workers with 9 to 11 years of schooling implied by the changes in trend productivity growth (g) depicted in Figure 1. For each discount rate, the evolution of nonemployment rate is computed for values of \( g_x \) equal to the time average of the values used in Figure 7. The nonemployment rate is normalized to 18 percent in 1970.
Figure 10. Implied Response of Low-Skilled Nonemployment to Observed Changes in Both Trend Aggregate Productivity Growth and the Return to Experience

Note: Authors’ calculation of the change in the nonemployment rate for workers with 9 to 11 years of schooling implied by the changes in trend productivity growth ($g$) depicted in Figure 1 and the return to experience ($g_x$). The model generated nonemployment rate is normalized to 18 percent in 1970. The dashed line superimposes the actual trend nonemployment rate for low skilled workers from Juhn, Murphy, and Topel (2002).