Problem Set 2
Due in lecture Tuesday, September 16

1. Show that on the balanced growth path of the Solow model, K/Y = s/(n+g+δ).

2. Consider the Solow model. Find an expression for the elasticity of the balanced-growth-path level of output per unit of effective labor with respect to n+g+δ. Simplify your expression as much as possible.

3. Saving rates may be higher at higher levels of income. This problem asks you to investigate the consequences of this possibility for economic growth.

Consider the Solow model without technological progress. For simplicity, assume that A is one, so that y and k are income per worker and capital per worker.

Now suppose that, in contrast to our usual assumptions:
- The saving rate is zero if income per worker is less than some critical level, $f(\tilde{k})$.
- The saving rate is s (where s > 0) if income per worker exceeds $f(\tilde{k})$.
Finally, assume that $sf(\tilde{k})$ is greater than $(n + \delta)\tilde{k}$.

a. Describe how, if at all, this change affects our usual diagram for the Solow model – that is, the diagram showing actual investment per worker and break-even investment per worker as functions of capital per worker.

b. Describe the behavior of output per worker over time if:
i. The initial level of capital per worker, k(0), is between 0 and $\tilde{k}$.
ii. The initial level of capital per worker, k(0), is slightly greater than $\tilde{k}$.

4. Romer, Problem 2.3.

5. (From last year's final exam.) Consider an infinitely-lived household. The household's initial wealth, A(0) is zero; its labor income is constant and equal to $\bar{Y}, \bar{Y} > 0$; and the real interest rate is constant and equal to $\bar{r} > 0$. The household's flow budget constraint is therefore $\dot{A}(t) = \bar{r}A(t) + \bar{Y} - C(t)$, and, as usual, the present discounted value of the household's consumption cannot exceed the present discounted value of the its lifetime resources.

In contrast to our usual model, however, the household obtains utility not only from consumption, but also from holding wealth. Specifically, its objective function is

$$\int_{t=0}^{\infty} e^{-\rho t}[u(C(t)) + v(A(t))]dt,$$

where $u'(\bullet) > 0$, $u''(\bullet) < 0$, $v'(\bullet) > 0$, $v''(\bullet) < 0$, and $\rho > 0$.

a. For this part only, assume $\rho = \bar{r}$. Without doing any math, explain whether $C(0)$ will be less than, equal to, or greater than $\bar{Y}$, or whether it is not possible to tell.

b. What is the present value Hamiltonian?

c. Find the conditions that characterize the solution to the household's maximization problem.

EXTRA PROBLEMS (NOT TO BE HANDED IN/ONLY SKETCHES OF ANSWERS WILL BE PROVIDED)

6. Consider an economy described by the Solow model that is on its balanced growth path. Assume that the saving rate is s_0. Now suppose that from time t_0 to time t_1, the saving rate rises gradually from s_0 to s_1 (where $s_1 > s_0$), and then remains at s_1.
    Sketch the resulting path over time of log output per worker. For comparison, also sketch on the same graph: (i) the path that log output per worker would have followed if the saving rate had remained at s_0; (ii) the path that log output per worker would have followed if the saving rate had jumped discontinuously from s_0 to s_1 at time t_0 (and remained at s_1).
    Explain your answer.

7. In the Solow model, the long-run effect of an increase in g is to:
    A. Increase the growth rate of total output, the growth rate of output per worker, and the level of capital per unit of effective labor.
    B. Increase the growth rate of total output and the level of capital per unit of effective labor, but not the growth rate of output per worker.
    C. Increase the growth rate of total output and the growth rate of output per worker, but not the level of capital per unit of effective labor.
    D. Increase the level of capital per unit of effective labor, but not the growth rate of total output or the growth rate of output per worker.

8. Romer, Problem 1.10.

9. Romer, Problem 2.2.

10. Romer, Problem 2.4.

EXTRA EXTRA PROBLEM (NOT TO BE HANDED IN/NO ANSWER WILL BE PROVIDED)

11. Romer, Problem 1.12.