UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2014
P. Gourinchas/D. Romer

Problem Set 3
Due in lecture Tuesday, September 23

1. Romer, Problem 2.7.

2. In the Ramsey-Cass-Koopmans model, the one-time destruction of half of the economy's capital stock:

A. Shifts the $\dot{c}=0$ locus to the left and does not affect the $\dot{k}=0$ locus.
B. Shifts the $\dot{k}=0$ locus down and does not affect the $\dot{c}=0$ locus.
C. Shifts the $\dot{c}=0$ locus to the left and shifts the $\dot{k}=0$ locus down.
D. Does not affect either the $\dot{c}=0$ locus or the $\dot{k}=0$ locus.

3. Piketty argues that a fall in the growth rate of the economy is likely to lead to an increase in r – g. This problem asks you to investigate this claim in the context of the Ramsey-Cass-Koopmans model:

a. Romer, Problem 2.6, part (a).
b. Romer, Problem 2.6, part (b).
c. At the time of the change, does *c* rise, fall, or stay the same, or is it not possible to tell? Explain your answer.
d. At the time of the change, does r – g rise, fall, or stay the same, or is it not possible to tell? Explain your answer.
e. In the long run, does r – g rise, fall, or stay the same, or is it not possible to tell? Explain your answer.

4. (Zero discounting.) In his famous 1928 *Economic Journal* article on optimal saving, Frank Ramsey argued that it is morally indefensible to discount the welfare of future generations. He therefore argued that a benevolent economic planner should:

$$\text{Maximize } V = \int_{t=0}^{\infty} u[c(t)]dt$$

$$\text{subject to } \dot{k}(t) = f[k(t)] - c(t),\ k(0)\text{given.}$$

(Ramsey assumed zero population growth.) You may be able to see right away the problem with this formulation: <u>any</u> path that approaches a constant steady-state consumption level will yield an <u>infinite</u> value of *V*. Thus, it is not clear how to compare such paths and identify one as "optimal".

Ramsey finessed the problem in the follow way. He defined $\bar{c}$ to be the "bliss" or maximal steady-state consumption level (the existence of which presupposes that *f(k)* eventually becomes decreasing in k or asymptotes to a finite maximum as k goes to ∞). He then redefined his problem as that of <u>minimizing</u> $\int_{t=0}^{\infty}\{u(\bar{c}) - u[c(t)]\}dt$, society's cumulative distance from "bliss", subject to the above constraints. Note that this integral can be finite if $c(t) \to \bar{c}$ as $t \to \infty$ (and if it isn't, it's not the optimum we seek in any case).

a. Use the Maximum Principle to derive necessary conditions for a solution to the Ramsey problem. (You can assume a depreciation rate of 0 for capital.) The resulting Euler condition is sometimes called the Keynes-Ramsey rule because J. M. Keynes, a friend of Ramsey's and editor of *Economic Journal*, helped him to interpret it intuitively.) Show that the economy indeed should converge to "bliss" (also known as the "golden rule" in growth theory). Interpret the model's intertemporal Euler condition. You can do so by addressing the follow question: Suppose the economy starts with $k(0)$ less than the level that maximizes $f(k)$. Since Ramsey believed in intergenerational equality, why isn't it optimal in his view for each generation simply to consume $f[k(0)]$?

b. Let $\{c^*(t)\}_{t=0}^{\infty}$ denote the Ramsey consumption path starting from an initial capital stock $k(0)$, and let $\{c(t)\}_{t=0}^{\infty}$ be any other consumption path. Show that the Ramsey path *overtakes* any other feasible consumption path starting from $k(0)$, in the following sense: there exists a finite time J such that for all $T > J$, $\int_0^T u[c^*(t)]dt > \int_0^T u[c(t)]dt$.

c. Ramsey states the Keynes-Ramsey rule as: *"rate of saving multiplied by marginal utility of consumption should always equal bliss minus actual rate of utility enjoyed."* (By "bliss" he meant $u(\bar{c})$.) Can you derive this rule?

EXTRA PROBLEMS (NOT TO BE HANDED IN/ONLY SKETCHES OF ANSWERS WILL BE PROVIDED)

5. Romer, Problem 2.10, parts (a), (b), and (c).

6. Romer, Problem 2.11.

7. Romer, Problem 2.12.

8. Romer, Problem 2.6, parts (d) and (e).

9. Romer, Problem 2.9.

10. Romer, Problem 2.10, parts (d), (e), and (f).