A Political-Economy Theory of Trade Agreements*

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Abstract

We develop a model where trade agreements – in addition to correcting terms-of-trade externalities – help governments to commit vis-à-vis domestic industrial lobbies. The model allows us to explore how the characteristics of the political environment affect the structure of the trade agreement and the extent of trade liberalization. The model also highlights the role of intersectoral capital mobility in determining trade liberalization. In a dynamic extension of the model, we explore the extent to which trade liberalization occurs gradually, and how its speed depends on the fundamentals of the problem.

Keywords: trade agreements, domestic commitment, terms of trade, lobbying for protection, capital mobility.

JEL classification: F13, D72

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1 Introduction

The history of trade liberalization after World War II is intimately related with the creation and expansion of the GATT (now WTO), and with the signing of countless bilateral and regional trade agreements. Clearly, there are strong forces pushing countries to sign international trade agreements, and it is important for economists and political scientists to understand what these forces are. Why do countries engage in trade agreements? What determines the extent and form of liberalization that takes place in such agreements?

The standard theory of trade agreements dates back to Johnson (1954), who argued that, in the absence of trade agreements, countries would attempt to exploit their international market power by taxing trade, and the resulting equilibrium (trade war) would be inefficient for all countries involved. International trade agreements can be seen as a way to prevent such a trade war. This idea was later formalized in modern game-theoretic terms by Mayer (1981).

Grossman and Helpman (1995a) and Bagwell and Staiger (1999) have extended this framework to settings where governments are subject to political pressures. In these models, even politically-motivated governments engage in trade agreements only to correct for terms of trade externalities. Thus, "politics" does not affect the motivation to engage in trade agreements.

In this paper we present a theory where politics is very much at the center of trade agreements. In particular, we consider a model where trade agreements help governments to deal with a time-inconsistency problem in their interaction with domestic lobbies. Maggi and Rodríguez-Clare (1998) showed how such a time-inconsistency problem may emerge in a small open economy when capital is fixed in the short run but mobile in the long run. The present paper builds on this idea to develop a fuller theory of trade agreements.

We start by reviewing the logic behind the domestic-commitment problem that is at the basis of our theory. This logic is easily illustrated for the case of a small economy. According to the modern political-economy theory of trade policy, it is not clear why a small-country government would want to "tie its hands" and give up its ability to grant protection. For example, in Grossman and Helpman (1994), lobbies compensate the government for the distortions associated with trade policy, and hence there is no reason why the government would want to commit not to grant protection. In fact, if the government is able to extract rents from the political process it is strictly better off in the political equilibrium than under free trade. But this may no longer be true when one takes into account that capital can move across sectors.
This is because, given the expectation of protection in a given sector, there will be excessive investment in that sector. Since this happens before the government and lobbies negotiate over protection, the government is not compensated for this "long-run" distortion. This allocation distortion is the essence of the domestic-commitment problem.

Next we explain how we develop a political-economy theory of trade agreements based on the domestic-commitment problem just described. We consider two large countries whose respective governments are subject to pressures from import-competing lobbies, and where capital is fixed in the short run but mobile in the long run. In this setting, the noncooperative equilibrium entails two types of inefficiency: a domestic time-inconsistency problem and a prisoner's dilemma arising from the terms-of-trade externality. Starting from this situation, the two countries get a chance to sign a (perfectly enforceable) trade agreement that imposes constraints on the trade policies that can be chosen in the future.

We distinguish between "ex-ante lobbying," which influences the selection of the trade agreement, and "ex-post lobbying", which influences the choice of trade policies subject to the constraints set by the agreement. Of course, the notion of ex-post lobbying is meaningful only if the agreement leaves some discretion in the governments' choice of trade policies after the agreement is signed. This is the case, for example, if the agreement imposes a tariff ceiling, so that a government is free to choose a tariff below the ceiling level. We note that this way of thinking about trade agreements is a significant departure from the existing models, where agreements leave no discretion to governments, and which therefore cannot make a meaningful distinction between ex-ante and ex-post lobbying.

Another novel feature of our model is that it integrates both existing motives for trade agreements, namely terms-of-trade externalities and domestic-commitment problems. Moreover, the model leads to several predictions that appear consistent with casual empirical observations.

First, our model can explain why trade agreements typically specify maximum tariff levels (tariff ceilings) rather than exact tariff levels. Tariff ceilings and exact tariff commitments have very different implications. With exact tariff commitments, lobbying effectively ends at the time of the agreement, since the agreement leaves no discretion for governments to choose tariffs in the future. With tariff ceilings, on the other hand, governments retain the option of setting tariffs below their maximum levels, and this will invite lobbying also after the agreement is signed. The optimal form of the agreement depends crucially on the strength of ex-ante lobbying. We find that, if ex-ante lobbying is not too strong, tariff ceilings are
preferred to exact tariff commitments. Thus the model may help explain why trade agreements are incomplete contracts, without relying on the traditional causes of contract incompleteness, such as contracting costs or nonverifiable information.

Second, in our model the degree of capital mobility is a key determinant of the extent of trade liberalization. We find that trade liberalization is deeper when capital is more mobile across sectors. To understand this result, consider the extreme case in which capital can be freely reallocated after the agreement has been signed. In this case, the lobby suffers no loss from trade liberalization, since capital can exit the affected sector and avoid any losses associated with lower domestic prices. With imperfect capital mobility, however, trade liberalization does generate losses for the lobby, and as a consequence ex-ante lobbying is stronger. Although our model generates only a comparative-statics result, it nevertheless suggests a cross-sectional empirical prediction: we should observe deeper trade liberalization in sectors where capital is more mobile. We are not aware of any empirical work exploring the link between factor mobility and trade liberalization, but casual observations seem to be in line with our model’s prediction: for example, trade liberalization has been very limited in the agricultural sector, which is intensive in resources that are not very mobile (e.g. land).

Third, when we extend the model to a continuous-time setting, we find that the optimal agreement is made of two components: an immediate slashing of tariffs relative to their noncooperative levels, and a subsequent, gradual reduction of tariffs. The immediate tariff reduction is due to the terms-of-trade motive for the trade agreement, while the domestic-commitment motive is reflected in the gradual component of trade liberalization. We also find that the speed of trade liberalization is higher when capital is more mobile. While our model is not the only one that can explain gradual trade liberalization (see discussion of the related literature at the end of this introduction), the explanation proposed here, based on domestic commitment problems and imperfect capital mobility, is novel and – we feel – empirically plausible.

Our model also generates interesting results regarding the impact of "politics" on trade liberalization. We find that trade liberalization is deeper when governments are more politically motivated (in the sense that they care more about political contributions), provided capital is sufficiently mobile in the long run. This contrasts with the "standard" theory of trade agreements, where trade liberalization tends to be less deep when governments are more politically motivated.\(^1\) The difference arises from the fact that the domestic commitment motive for a

\(^1\)The reason is that, if governments are more politically motivated, the non-cooperative equilibrium is char-
trade agreement in our model is more acute when governments are more politically motivated. Also, in our model trade liberalization tends to be deeper when governments have less bargaining power vis-à-vis domestic lobbies, and when lobbies have less influence on the negotiation of the agreement.

We want to emphasize that most of our insights follow from our structural modeling of the lobbying game, in which interest groups and governments exchange contributions for trade protection. If we modeled political pressures with a reduced-form approach, by assuming that governments attach a higher weight to producer surplus than to the other components of welfare, and we kept lobbies and contributions in the background, we would lose most of our results. For example, one might be tempted to model the domestic-commitment problem by assuming that there is a divergence between ex-ante and ex-post government objectives (e.g. at the stage of signing the agreement governments maximize welfare, while ex-post they maximize a combination of welfare and industry profits). This reduced-form setup would not be equivalent to our structural setup: for example, in the reduced-form setup there would be no role for tariff ceilings, and there would be no gradualism in trade liberalization.

This paper is related to two literatures: first, the literature on trade agreements motivated by terms-of-trade externalities (see the papers cited at the beginning of this introduction); and second, the literature on trade agreements motivated by domestic-commitment problems. In this second group, Maggi and Rodríguez-Clare (1998) and Mitra (2002) have highlighted the role of politics in creating demand for commitment, while Staiger and Tabellini (1987) have highlighted purely economic considerations. However, these three papers focus on a single small economy and do not attempt a full-fledged analysis of trade agreements. One important disadvantage of a small country model is that it does not allow one to study the interaction between the terms-of-trade motive and the domestic-commitment motive for a trade agreement.\footnote{We also note that in Maggi and Rodriguez-Clare (1998) the government is only allowed to choose between two extreme options, namely free trade or no commitment at all. If we want to study what determines the extent of trade liberalization, we need to allow governments to commit to intermediate levels of trade protection – which we do in the present paper. Moreover, that paper does not consider the possibility that lobbies might influence the shaping of the trade agreement ("ex-ante" lobbying), which plays an important role in the present paper. Finally, in this paper we allow for imperfect capital mobility, whereas Maggi and Rodríguez-Clare (1998) only consider the case of perfect capital mobility.}

A recent paper that considers a two-country model of trade agreements in the presence of domestic commitment problems is Conconi and Perroni (2005). They consider a self-enforcing acterized by a lower trade volume. Since a lower volume of trade entails a weaker terms-of-trade externality, correcting this externality requires a smaller reduction in tariffs.
agreement between a large country and a small country, where the only motive for a trade agreement is a domestic commitment issue that affects the small country.\textsuperscript{3} In contrast, our model integrates both motives for trade agreements, namely terms-of-trade externalities and domestic-commitment problems. Another important difference is that they take a reduced-form approach where there is a divergence between ex-ante and ex-post objectives of the governments. As we pointed out above, this approach is not equivalent to our structural approach where lobbying and contributions are explicitly modeled; most of our points could not be made with a reduced-form approach. In any event, Conconi and Perroni’s paper makes very different points from ours, as they focus on the implications of the self-enforcement constraints and argue that they can explain the granting of temporary Special and Differential treatment to developing countries in the WTO.

Another literature that is related to our paper is that on gradual trade liberalization. In most of these papers, e.g. Staiger (1995), Furusawa and Lai (1999), Bond and Park (2004), Conconi and Perroni (2005) and Lockwood and Zissimos (2005), gradual trade liberalization is a consequence of the self-enforcing nature of the agreements. In these models trade liberalization occurs at once if players are sufficiently patient. The explanation we propose in the present paper does not rely on self-enforcement considerations, but rather on the interaction between frictions in capital mobility and lobbying by capital owners.

Finally we should mention two papers that offer alternative explanations for the fact that trade agreements specify tariff ceilings rather than exact tariff commitments. Horn, Maggi and Staiger (2005) examine the optimal structure of trade agreements in the presence of verification costs. They show that, in order to save on verification costs, it may be optimal to specify rigid (i.e. noncontingent) tariff ceilings. Bagwell and Staiger (2005) propose a model where tariff ceilings are motivated by the presence of privately observed – and therefore nonverifiable – shocks in the political pressures faced by governments. The explanation for tariff ceilings proposed in the present paper is quite different from those proposed in the above two papers, since it does not rely on the presence of verification problems. We feel that these two types of explanation are complementary and may both be relevant in reality, but this is an open

\textsuperscript{3}Conconi and Perroni (2004) consider a model of self-enforcing international agreements between two large countries where there is both a domestic commitment problem and an international externality. This paper is different from ours in that it analyzes issues of self-enforcement in a model with very little structure and thus offers no implications for the extent of trade liberalization brought about by trade agreements, which is the focus of our present paper.
empirical question.

The paper is organized as follows. The next section presents the basic two-period model. Section 3 extends the model to a continuous time setting to explore gradual trade liberalization. Section 4 considers some extensions of the model. Section 5 concludes.

2 The basic model

2.1 The economic structure

There are two countries, Home and Foreign, and three goods: one numeraire good, denoted by \( N \), and two manufacturing goods, denoted by \( M_1 \) and \( M_2 \).

There are two types of capital, type 1 and type 2. The \( M_1 \) good is produced one-for-one from type-1 capital, the \( M_2 \) good is produced one-for-one from type-2 capital. Each country is endowed with one unit of each type of capital. The only difference between the two countries is in the technology to produce the \( N \) good: in country H, the \( N \) good is produced one-for-one from type-1 capital, while in country F, the \( N \) good is produced one-for-one from type-2 capital. Given these assumptions, under free trade Home exports good \( M_2 \) and Foreign exports good \( M_1 \). The reason we chose this particular technology structure is that it generates a simple symmetric setup where, in each country, capital mobility is relevant only between the import-competing sector and the numeraire sector. This in turn ensures that in each country the domestic-commitment motive for trade agreements concerns the import-competing sector but not the export sector, a feature that simplifies the analysis considerably.

In both countries preferences are given by

\[
U = c_N + \sum_{i=1}^{2} u(c_i)
\]

where \( u(c_i) = vc_i - c_i^2 / 2 \). Thus the demand function for good \( M_i \) is

\[
d(p_i) = v - p_i
\]

Note that the above assumptions generate a structure that is partial equilibrium in nature, except for one crucial general-equilibrium feature, namely that capital is mobile between the import-competing sector and the numeraire sector.
Home chooses a specific tariff $t$ on $M_1$ and Foreign chooses a specific tariff $t^*$ on $M_2$. Thus, if tariffs are not prohibitive, the domestic price of good $M_1$ in Home is given by $p_1 = p_1^* + t$. Similarly, the domestic price of good $M_2$ in Foreign is $p_2^* = p_2^* + t^*$.4

Let $x$ ($x^*$) denote the level of capital allocated to sector $M_1$ ($M_2$) in country H (F). Welfare (i.e., utility of the representative agent) is given by factor income plus tariff revenue plus consumer surplus. Thus, welfare in Home and Foreign, respectively, is given by:

\[
W = (1 - x) + (p_1 x + tm_1 + s_1) + (p_2 + s_2)
\]

\[
W^* = (1 - x^*) + (p_2^* x^* + t^* m_2^* + s_2^*) + (p_1^* + s_1^*)
\]

where $m_i$ ($m_i^*$) denotes Home (Foreign) imports of good $i$ and $s_i$ ($s_i^*$) represents Home (Foreign) consumer surplus derived from good $i$. Note the separability between sectors $M_1$ and $M_2$. Specifically, note that we can express $W$ as the sum of two components: the first one, $(1 - x) + (p_1 x + tm_1 + s_1)$, depends on $t$ and $x$; and the second one, $p_2 + s_2$, depends on $t^*$ and $x^*$. The same separability applies to foreign welfare. Together with symmetry, this separability implies that we can focus on sector $M_1$; the equilibrium in sector $M_2$ will be its mirror image. Thus, to simplify notation, we drop the subscript 1 from now on, and simply refer to sector $M_1$ as the "manufacturing" sector.

The international market clearing condition for manufacturing is:

\[
d(p) + d(p^*) = x + 1
\]

This yields

\[
p^*(t, x) = v - \frac{1}{2}(x + 1 + t)
\]

and

\[
p(t, x) = v - \frac{1}{2}(x + 1 - t)
\]

4In this paper we do not consider export subsidies and taxes. If the agreement takes the traditional form of exact tariff and subsidy commitments, this restriction is innocuous, because only net protection (i.e. the difference between import tariff and export subsidy in a given sector) matters for the optimal agreement, therefore $t$ and $t^*$ can be reinterpreted in terms of net protection in the two sectors. If the agreement takes the form of tariff and subsidy ceilings, on the other hand, not only net protection but also the levels of import tariffs and export subsidies matter, and this makes the analysis substantially more complex. Maggi and Rodriguez-Clare (2005) study optimal agreements when both import and export instruments are allowed but there is no capital mobility. We also note that assuming away export instruments is relatively common in the existing literature on trade agreements: see for example Grossman and Helpman (1995b), Krishna (1998), Maggi (1999) and Ornelas (2004).
where we emphasize the dependence of equilibrium prices on the tariff and the capital allocation in the home country.

Letting \( m = d(p) - x \) denote imports of manufactures by Home, then:

\[
m(t, x) = \frac{1}{2}(\Delta x - t)
\]

where \( \Delta x \equiv 1 - x \) is the difference in supply between the two countries.

Given this notation, welfare in Home is:

\[
W(t, x) = (1 - x) + p(t, x)x + tm(t, x) + s(t, x) + [\cdot]
\]

where \([\cdot]\) does not depend on \( t \) and \( x \). Analogously, Foreign welfare is

\[
W^*(t, x) = p^*(t, x) + s^*(t, x) + [\cdot]
\]

Next we describe the political side of the model.

### 2.2 The political structure

We assume that, in each country, the capital owners in the import-competing sector get organized as a lobby and offer contributions to their government in exchange for protection.\(^5\)

We model the interaction between lobby and government in a similar way as Grossman and Helpman (1994). The government’s objective function is

\[
aW + C
\]

where \( C \) denotes contributions from the import-competing lobby. The parameter \( a \) captures (inversely) the importance of political considerations in the government’s objective: when \( a \) is lower, "politics" are more important.

The lobby maximizes total returns to capital net of contributions:\(^6\)

\[
px - C
\]

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\(^5\)We are implicitly assuming that the export sector and the numeraire sector are not able to get organized. This is a simple lobby structure that generates trade protection in the political equilibrium.

\(^6\)This is a shortcut. To be more precise, we should specify the lobby’s objective as the aggregate well-being of its lobby members, but this would give rise to the same results. Letting \( \alpha \) be the fraction of the population that owns some capital in the import-competing sector, the lobby’s objective is \( px + \alpha(tm + s) - C \), so the joint surplus of government and lobby is proportional to \( \frac{\alpha + \alpha}{1 - \alpha}W + px \), an expression that has the same qualitative structure as the one we derive below.
The lobby collects contributions in proportion to the amount of capital, thus total contributions are given by $C = cx$, where $c$ is the contribution per unit of capital.

The timing of the non-cooperative game is the following. In the first stage, investors allocate their capital. The value of $x$ summarizes the choices of investors in the first stage. In the second stage, the government and the import competing lobby in each country bargain efficiently over tariff and contributions. For simplicity we assume that the lobby has all the bargaining power (we relax this assumption in a later section). An equivalent assumption would be that the lobby makes a take-it-or-leave-it offer to the government that consists in a tariff level and a contribution level.

### 2.3 The short-run noncooperative equilibrium

To determine the subgame perfect equilibria of the game we proceed by backward induction, starting with the determination of equilibrium tariffs and contributions given the allocation of capital. This is the equilibrium of the subgame, or the "short-run" equilibrium.

We can focus on the Home country. Given the assumption of efficient bargaining, the government (G) and the lobby (L) choose $t$ to maximize their joint surplus:

$$J_{SR}(t, x) = aW(t, x) + p(t, x)x$$

This yields

$$t = t^J(x) \equiv (1/3)(\Delta x + 2x/a)$$

The noncooperative tariff $t^J$ can be decomposed in two parts. The component $\Delta x/3$ captures the incentive to distort terms of trade: when the supply difference $\Delta x$ is bigger, the volume of imports is larger, and hence this incentive is stronger. The component $2x/3a$ captures the political influence exerted by the lobby. This component is more important when the sector is larger ($x$ is higher) and when the government’s valuation of contributions relative to welfare is higher ($a$ is lower). We let the national welfare maximizing tariff (given $x$) be denoted by

$$t^W(x) \equiv \lim_{a \to \infty} t^J(x) = \Delta x/3$$

For future reference, we define $c(t, x)$ as the contributions per unit of capital such that G is just willing to impose tariff $t$, or in other words, such that G is kept at its reservation utility given tariff $t$. In the absence of contributions, G would choose the welfare maximizing tariff
given $x$, that is $t^W(x)$, so G’s reservation utility is $W(t^W(x), x)$. Since the short-run equilibrium tariff given $x$ cannot be below $t^W(x)$, we only need to focus on the case $t \geq t^W(x)$. For the government to choose a tariff $t \geq t^W(x)$, total contributions would have to be equal to

$$a \left[ W(t^W(x), x) - W(t, x) \right] = - \int_{t^W(x)}^{t} aW_1(t, x) dt$$

Thus, the function

$$c(t, x) \equiv (3a/8) \left( t - t^W(x) \right)^2$$

determines the contributions per unit of capital necessary to induce the government to choose tariff $t \geq t^W(x)$. Note that we define this function only for $t \geq t^W(x)$, since the lobby would never be willing to pay the government to impose a tariff lower than it would choose on its own.

### 2.4 The long-run noncooperative equilibrium

In this section we examine the long-run non-cooperative equilibrium, where $x$ is endogenous and is determined according to investors’ expectations about future protection in the absence of a trade agreement.

Before we proceed, however, it is useful to derive the free trade long-run equilibrium. Under free trade and perfect capital mobility, the domestic (and international) price of the $M$ good must be equal to one. Thus the free trade allocation of capital, $x^{ft}$, is determined by the condition $p(0, x) = 1$, or

$$v - \frac{1}{2}(x^{ft} + 1) = 1$$

To ensure that under free trade Home is incompletely specialized and imports good $M$ we need $0 < x^{ft} < 1$, which holds as long as $3/2 < v < 2$. We maintain this assumption throughout the rest of the paper. Note that, because of the symmetry of the model, under free trade there is no trade in the numeraire sector.

We can now turn to the long-run political equilibrium. The equilibrium conditions are:

$${t = t^t(x)}$$

$$p(t, x) - c(t, x) = 1$$

(1)
The second condition requires that the return to capital net of contributions be equal in the import-competing sector and in the numeraire sector.\(^7\) This equal-returns condition implicitly defines a curve in \((t, x)\) space, that we label \(x^{er}(t)\) (we will sometimes use \(t^{er}(x)\) for its inverse). Note that, since we defined the function \(c(t, x)\) only for \(t \geq t^W(x)\), the curve \(x^{er}(t)\) is defined only in the region \(t \geq t^W(x)\).

We let \((\hat{t}, \hat{x})\) denote a solution to the above system. Also, we let \((t^W, x^W)\) denote the intersection of the curves \(t^W(x)\) and \(x^{er}(t)\). The proof of the following proposition, together with all the other proofs of the paper, can be found in Appendix.

**Proposition 1** If \(a > (6v - 7)/6(2 - v)\) there exists a unique long-run noncooperative equilibrium. In this equilibrium each country imposes a positive but non-prohibitive tariff \(\hat{t}\). The equilibrium tariff \(\hat{t}\) is decreasing in \(a\), and approaches \(t^W\) as \(a \to \infty\).

Figure 1 illustrates the long-run noncooperative equilibrium. In the figure, the \(t^J(x)\) curve is increasing, but nothing would change if it were decreasing. To understand the shape of the \(x^{er}(t)\) curve, note that since the lobby has all the bargaining power and extracts all the joint surplus, \(t^J(x)\) maximizes the net returns to capital in the M sector (i.e. \(p - c\)). Given concavity of \(J_{SR}\) in \(t\), this implies that \(p - c\) is increasing in \(t\) below the \(t^J(x)\) curve and is decreasing in \(t\) above the \(t^J(x)\) curve. Under the condition assumed in the proposition, entry into the M sector has the intuitive effect of reducing net returns to capital there (i.e., \(p - c\) is decreasing in \(x\)). It follows that, under this condition, the equal-returns curve \(t^{er}(x)\) is increasing below the \(t^J(x)\) curve and decreasing above it, with an infinite slope at the \((\hat{t}, \hat{x})\) point.

We will maintain the assumption \(a > (6v - 7)/6(2 - v)\), which ensures the existence and uniqueness of the long-run equilibrium, throughout the paper.

Not surprisingly, for any positive but finite level of \(a\), the non-cooperative tariff is higher than the national welfare maximizing tariff: \(\hat{t} > t^W\). Also, from inspection of figure 1, it is clear that the noncooperative equilibrium allocation \(\hat{x}\) exceeds the allocation that would result in the absence of politics (i.e. when \(a \to \infty\)), that is \(x^W\). As we will show formally in a later section, this excess of \(\hat{x}\) above \(x^W\) represents an overinvestment problem, that is a

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\(^7\)An alternative way to find the long-run equilibrium, perhaps more standard from a game theoretical point of view, would be to derive the equilibrium of the subgame given \(x\) and then proceed by backward induction to derive the equilibrium level of \(x\). More specifically, given \(x\), the subgame equilibrium contribution and tariff are respectively given by \(c(x) = (3a/8x) (t^J(x) - t^W(x))^2\) and \(t(x) = t^J(x)\). The equilibrium \(x\) can then be found as the one that solves \(p(t^J(x), x) - c(x) = 1\). The procedure we follow in the text turns out to be more convenient for the analysis of the optimal trade agreement.
“long-run” distortion associated with the government’s lack of commitment vis-à-vis domestic investors. Each government is compensated by its lobby for the short-run distortion associated with protection (i.e. the consumption distortion given the allocation $x$), but is not compensated for the long-run allocation distortion. For this reason a government may value a commitment to a lower level of the tariff. This is the heart of the domestic-commitment motive for trade agreements, which operates alongside the standard terms-of-trade motive. We are now ready to examine the optimal agreement.

### 2.5 The optimal trade agreement

We suppose that, before capital is allocated, the two governments and the two lobbies determine the trade agreement. Maggi and Rodríguez-Clare (1998) assumed that lobbies do not influence the selection of the trade agreement, i.e. there is no ex-ante lobbying. Here we allow for ex-ante lobbying by assuming that the agreement maximizes the ex-ante joint surplus of the two governments and the two lobbies.\(^8\) To capture the strength of ex-ante lobbying, we weigh the lobbies’ part of the joint surplus with a parameter $\delta$. The agreement maximizes the following objective:

$$
\Psi = U^G + U^{G*} + \delta(U^L + U^{L*})
$$

where $U^G$, $U^{G*}$, $U^L$ and $U^{L*}$ denote the second-stage payoffs of the governments and lobbies as viewed from the ex-ante stage. We will be more explicit about these payoffs shortly, but first we want to discuss the interpretation of $\delta$.

A lower level of $\delta$ is interpreted as a situation where lobbies have less influence on the shaping of the ex-ante agreement. The case $\delta = 1$ corresponds to the benchmark case in which ex-ante lobbying is just as strong as ex-post lobbying. The case $\delta = 0$ corresponds to the case in which there is no ex-ante lobbying, as we assumed in our previous paper. We can offer two justifications of $\delta$ in terms of more fundamental parameters:

1. A direct interpretation of $\delta$ would be as the discount factor of the lobbies relative to that of the governments. We have in mind that an agreement is a long-run commitment that shapes

\(^8\)This efficient-agreement approach can be justified as equivalent to a more structural game between governments and lobbies. One possibility would be to consider a game along the lines of Grossman and Helpman (1995). They assume that lobbies offer (differentiable) contribution schedules to their respective governments and then governments bargain efficiently given the contribution schedules, and show that the equilibrium outcome maximizes the joint surplus of governments and lobbies. More generally, any negotiation procedure between governments and lobbies that yields a joint-surplus-maximizing outcome would be equivalent to our approach.
the political game for a long time to come, therefore discounting considerations are potentially important for the determination of the agreement. The discount factor of a government can be thought of as determined by political factors such as the probability of re-election, and the discount factor of a lobby is determined by economic factors such as the probability of bankruptcy.

2. An alternative story for $\delta$ is the following. Suppose that governments and lobbies discount the future in the same way, but contributions may be more effective in influencing day-to-day policy decisions than they are in influencing the negotiation of the agreement.\footnote{For example, it might be the case that the US trade representatives involved in the negotiation of trade agreements attach less value to contributions than policymakers involved in day-to-day decisions on trade policy (e.g. members of Congress). This is plausible if contributions are used to finance electoral campaigns, since trade negotiators are not elected officials. Another possibility is that, since a trade agreement is a long-run commitment, the magnitude of contributions required to influence it is higher than that required to influence day-to-day policy choices, and there may be a political cost associated with paying larger contributions because they are more visible to voters.} To capture this idea we can write the home government’s ex-ante objective as $U^{G \text{ex-ante}} = U^G + \delta C^{\text{ex-ante}}$, with an analogous expression holding for the foreign government. Here $\delta$ captures the weight of ex-ante contributions, which can be different from that of ex-post contributions. We can write the domestic lobby’s ex-ante objective as $U^{L \text{ex-ante}} = U^L - C^{\text{ex-ante}}$, and analogously for the foreign lobby. Multiplying the lobbies’ payoffs by $\delta$ and summing up, the ex-ante joint surplus can then be written as $\Psi = U^G + U^L + \delta(U^L + U^L*)$.

In principle, ex-ante lobbying might be stronger than ex-post lobbying ($\delta > 1$), for example if governments are more shortsighted than lobbies. For this reason we will allow $\delta$ to take any positive value.

Agreements are assumed to be perfectly enforceable. In the concluding section we will discuss how the insights of our model might extend to a setting of self-enforcing agreements.

We assume that the inherited level of $x$ at the agreement stage is equal to $\hat{x}$, the long-run equilibrium allocation in the absence of an agreement. The interpretation is that the commitment opportunity comes as a surprise to the private sector.

Following the agreement, each capital owner gets a chance to move its unit of capital with probability $z \in [0, 1]$. Thus, a fraction $z$ of the capital has the opportunity to move. The case $z = 0$ captures the case of fixed capital, whereas the case $z = 1$ captures a situation in which capital is perfectly mobile in the long run but fixed in the short run. With a slight abuse of terminology, from now on we refer to this case simply as perfect capital mobility, and to the
case $z < 1$ as imperfect capital mobility.

To recapitulate, the timing of the model is as follows:

1. The agreement is selected;
2. Capital is reallocated (when feasible);
3. Given the capital allocation and the constraints (if any) imposed by the agreement, each
government-lobby pair chooses a tariff.

Again, given separability across the two manufacturing sectors, we can analyze them indi-
ependently, knowing that the agreement for sector $M_2$ will be the mirror image of that for
$M_1$. Moreover, given symmetry, the optimal agreement must maximize the joint surplus of the
two governments and the import-competing lobby in each sector. Thus, just as in the previous
subsections, we can focus on sector $M_1$ (omitting subscripts) and find the optimal agreement
by maximizing the joint surplus of the two governments and Home’s import-competing lobby
in this sector.

We consider two forms of agreement: agreements that specify tariff ceilings, that is con-
straints of the type $t \leq \bar{t}$, and agreements that specify exact tariffs, that is constraints of the
type $t = \bar{t}$. The main difference between these two types of agreement is that in the case
of exact tariff commitments the lobby will not have to pay contributions to obtain protection
ex-post, since such protection will effectively be part of the agreement. Under tariff ceilings, on
the other hand, the government can credibly threaten to impose its unilateral best tariff $t^W(x)$.
Thus, the lobby would have to compensate the government for deviating from this tariff, and
there would be positive contributions ex-post. Clearly, whether tariff ceilings or exact tariffs
are preferred crucially depends on the value of $\delta$, since this determines how ex-ante joint surplus
depends on ex-post contributions. The following proposition establishes conditions on $\delta$ under
which there is no loss of generality in focusing on tariff ceilings:

**Proposition 2** There exists a $\bar{\delta} \geq 1$ such that, if $\delta \leq \bar{\delta}$, tariff ceilings perform at least as well
as exact tariff commitments. If capital is perfectly mobile, this is true for any $\delta$.

This result states that, if ex-ante lobbying is not too strong, there is no loss of generality
in focusing on tariff ceilings. The intuition is simple. Consider first the case of perfect capital
mobility. Tariff ceilings are preferable to exact tariffs for two reasons: (1) If the agreement
imposes exact tariffs, clearly there will be no ex-post lobbying, and hence no ex-post contrib-
utions. On the other hand, tariff ceilings may induce ex-post contributions: if the ceiling for
the tariff is sufficiently high, the lobby will offer contributions to convince the government to raise the tariff towards the ceiling. From the point of view of the ex-ante joint surplus, ex-post contributions are desirable: the government values contributions, while the lobby is indifferent, because free entry ensures that the net return to capital will be equal to one regardless. (2) An additional reason why tariff ceilings may be superior to exact tariffs is that the presence of ex-post contributions mitigates the overinvestment problem, since positive contributions reduce net returns to capital in manufacturing.

If capital is imperfectly mobile, tariff ceilings are better than exact tariffs only if $\delta$ is relatively low. To see this, consider the extreme case in which capital is fixed. Then the only difference between tariff ceilings and exact tariffs is that the former induce ex-post contributions while the latter do not. From the ex-ante point of view, a dollar of contributions received by the government has more weight than a dollar of contributions paid by the lobby if and only if $\delta < 1$.

This proposition highlights that our model is able to explain the use of tariff ceilings, which is pervasive in real trade agreements. From another perspective, the model helps explain why trade agreements are not complete contracts, and in particular why they leave some discretion to governments. It is worth highlighting that none of the "usual" causes of contract incompleteness – e.g. nonverifiable information, costs of writing contracts, unforeseen contingencies – are present in our model. The reason why the optimal agreement may be incomplete here is that the agreement cannot specify the contributions that the lobby will have to pay in the future (as we implicitly assumed). If the agreement could specify both tariffs and contributions, a complete contract would be optimal. But since the contract cannot specify contributions, it may be optimal to leave the contract partially incomplete also in the other dimension, that is tariffs.

Given our interest in explaining the prevalence of agreements with tariff ceilings, in this section we focus on the case in which the condition of Proposition 2 is satisfied. In a later section we will examine how results change if the agreement specifies exact tariff commitments.

Next we characterize the optimal agreement. It is instructive to start with the case of perfect capital mobility ($z = 1$), and then extend the analysis to imperfect capital mobility.
2.5.1 Perfect capital mobility

Recall that the optimal agreement maximizes the ex-ante joint surplus of the two governments and the importing lobby in each sector. Given that \( \hat{x} \) is the inherited allocation of capital, this objective function can be written as:

\[
\Psi = aW(t, x) + xc + aW^*(t, x) + \delta[xp(t, x) - xc + (\hat{x} - x)]
\] (2)

This expression is valid only for \( x \leq \hat{x} \), but we do not need to consider the alternative case \( x > \hat{x} \), because this can never be the case in equilibrium.

To gain better understanding about the objective function above, focus on the special case of \( \delta = 1 \). In this case \( \Psi = J^{SR}(t, x) + aW^*(t, x) + (\hat{x} - x) \). There are two extra terms relative to the short-run objective \( J^{SR} \): the term \( aW^*(t, x) \), which takes into account terms of trade externalities, and the term \( (\hat{x} - x) \), which captures the rents of those lobby members that will move to the \( N \) sector in the following period.

Next we derive the ex-ante objective as a reduced-form function of the tariff ceiling \( \bar{t} \) and the allocation \( x \). This is the objective function when the equilibrium of the third stage (i.e., given \( \bar{t} \) and \( x \)) has been "rolled back" by backward induction. To this end, it is convenient to derive the functions \( t(\bar{t}, x) \) and \( c(\bar{t}, x) \) that give the equilibrium tariffs and contribution per unit of capital conditional on \( \bar{t} \) and \( x \), respectively.\(^{10}\) To derive \( t(\bar{t}, x) \), notice that this is the tariff that maximizes \( J^{SR}(t, x) \) subject to the constraint \( t \leq \bar{t} \), and recall that \( J^{SR}(t, x) \) is concave in \( t \) and maximized at \( t^J(x) \). Therefore, if \( \bar{t} \geq t^J(x) \) the tariff ceiling is not binding, hence \( t(\bar{t}, x) = t^J(x) \); and if \( \bar{t} < t^J(x) \) the ceiling is binding, hence \( t(\bar{t}, x) = \bar{t} \). Summarizing:

\[
t(\bar{t}, x) = \min\{\bar{t}, t^J(x)\}
\]

Turning to \( c(\bar{t}, x) \), the key observation is that, if \( \bar{t} > t^W(x) \), then the home government will get contributions, because its outside option in the negotiation with the lobby is given by the tariff \( t^W(x) \), and the lobby has to compensate \( G \) to raise the tariff up to the ceiling \( \bar{t} \); on the other hand, if \( \bar{t} < t^W(x) \) no contributions will be forthcoming, because \( G \) has no credible threat. Thus

\[
c(\bar{t}, x) = \begin{cases} 
(3a/8x)(t(\bar{t}, x) - t^W(x))^2 & \text{if } \bar{t} \geq t^W(x) \\
0 & \text{if } \bar{t} < t^W(x)
\end{cases}
\]

\(^{10}\)Note that we are using the same notation \( c() \) as for the contribution schedule in the noncooperative equilibrium, even though this is not the same function. This is an abuse of notation, but the reader can distinguish the two functions because the first argument is \( t \) in one case and \( \bar{t} \) in the other.
Note that there is no loss of generality in focusing on agreements in which tariff ceilings are binding, that is \( \tilde{t} \leq t^J(x) \). Letting \( \Psi(\tilde{t}, x) \) denote the ex-ante objective as a function of \( \tilde{t} \) and \( x \), we have:

\[
\Psi(\tilde{t}, x) = aW(\tilde{t}, x) + c(\tilde{t}, x)x + aW^*(\tilde{t}, x) + \delta[xp(\tilde{t}, x) - c(\tilde{t}, x)x + (\tilde{x} - x)]
\]  

(3)

The next step is to move back to the second stage and derive the equilibrium allocation conditional on \( \tilde{t} \). Clearly, if \( \tilde{t} > \hat{t} \) then the tariff ceiling is not binding, and the equilibrium will be given by \((\hat{t}, \hat{x})\), just as if there was no agreement. On the other hand, if \( \tilde{t} \leq \hat{t} \) then the equilibrium allocation is implicitly defined by the equal-returns condition

\[ p(\tilde{t}, x) - c(\tilde{t}, x) = 1 \]

We let \( x^{er}(\tilde{t}) \) denote the solution in \( x \) to the above equation for \( \tilde{t} \leq \hat{t} \). Figure 2 illustrates the curve \( x^{er}(\tilde{t}) \). Below the \( t^W(x) \) curve, this is a line with slope one (because in this region the condition that defines it is \( p(\tilde{t}, x) = 1 \)), and between the curves \( t^W(x) \) and \( t^J(x) \) it coincides with the equal-returns curve in the absence of agreements (which is depicted in figure 1).

We now turn to the optimal trade agreement. The optimal tariff ceiling is the one that maximizes \( \Psi(\tilde{t}, x^{er}(\tilde{t})) \) for \( \tilde{t} \leq \hat{t} \). To write an expression for \( \Psi(\tilde{t}, x^{er}(\tilde{t})) \), recall that if \( \tilde{t} \geq t^W(x^{er}(\tilde{t})) \) then there are positive contributions and

\[ W(\tilde{t}, x^{er}(\tilde{t})) + C = W(t^W(x^{er}(\tilde{t})), x^{er}(\tilde{t})) \]

On the other hand, if \( \tilde{t} < t^W(x^{er}(\tilde{t})) \), then contributions are zero. Noting that \( \tilde{t} \geq t^W(x^{er}(\tilde{t})) \) if and only if \( \tilde{t} \geq t^W \), then

\[
\Psi(\tilde{t}, x^{er}(\tilde{t}))|_{\tilde{t} \leq \hat{t}} = \begin{cases} 
  aW(t^W(x^{er}(\tilde{t})), x^{er}(\tilde{t})) + aW^*(\tilde{t}, x^{er}(\tilde{t})) + \delta\tilde{x} & \text{if } \tilde{t} \geq t^W \\
  aW(\tilde{t}, x^{er}(\tilde{t})) + aW^*(\tilde{t}, x^{er}(\tilde{t})) + \delta\tilde{x} & \text{if } \tilde{t} < t^W 
\end{cases}
\]

(4)

where we used the fact that returns are equalized in the two sectors, which implies that the total lobby rents reduce to \( \tilde{x} \).

The next question is, what is the level of \( \tilde{t} \) that maximizes the objective \( \Psi(\tilde{t}, x^{er}(\tilde{t})) \)? Note that, for \( \tilde{t} \geq t^W \), Home welfare is evaluated at the tariff \( t^W(x^{er}(\tilde{t})) \), not at the ceiling \( \tilde{t} \), so the solution of this maximization problem is not immediately obvious. The next result shows that \( \Psi(\tilde{t}, x^{er}(\tilde{t})) \) is maximized at free trade:

\[ \Psi(\tilde{t}, x^{er}(\tilde{t})) \]

\[ \text{maximized at free trade:} \]

\[ 11 \]

\[ \text{Again, the notation } x^{er}(\tilde{t}) \text{ is slightly abused because this is not the same function as } x^{er}(t), \text{ the equal-returns condition in the absence of agreements.} \]
Proposition 3 In the case of perfect capital mobility, the optimal agreement is $\tilde{t}^A = 0$ (free trade) for all $\delta$ and $a$.

We have shown that, when capital is perfectly mobile, the optimal agreement is free trade even in the presence of ex ante lobbying. Intuitively, if capital is mobile, the lobby anticipates that any rents will be dissipated by entry in the ex-post stage, and hence is not willing to pay anything to compensate the government for the long run distortions associated with protection. This will of course no longer be true when capital is imperfectly mobile.

In this model there are two motives for a trade agreement: the standard terms-of-trade (TOT) externality and the domestic commitment problem. We can disentangle the two with the following thought experiment. Consider a hypothetical scenario in which the home government can commit domestically (subject to the lobby’s pressures) but acts noncooperatively vis-à-vis the foreign country. More precisely, suppose that at the beginning of the game the home government and the lobby choose a tariff ceiling without cooperating with the foreign government; then capital is allocated, and then the home government and the lobby choose the tariff given the ceiling and the capital allocation.

Let $\tilde{t}^{DC}$ be the tariff ceiling that would be chosen in this case. The objective is the same as in the previous case except that foreign welfare is not taken into account. So $\tilde{t}^{DC}$ maximizes

$$J(\tilde{t}, x^{er}(\tilde{t}))(\tilde{t}) = \begin{cases} aW(\tilde{t}^{W}, x^{er}(\tilde{t})), x^{er}(\tilde{t})) + \delta \tilde{x} & \text{if } \tilde{t} \geq \tilde{t}^{W} \\ aW(\tilde{t}, x^{er}(\tilde{t})), x^{er}(\tilde{t})) + \delta \tilde{x} & \text{if } \tilde{t} < \tilde{t}^{W} \end{cases}$$

We can think of the movement from $\tilde{t}$ to $\tilde{t}^{DC}$ as the component of trade liberalization that is due to the domestic commitment motive, and the movement from $\tilde{t}^{DC}$ to $\tilde{t}^A = 0$ as the component due to the TOT motive. Next we characterize $\tilde{t}^{DC}$ in order to say more about this decomposition.

Proposition 4 In the case of perfect capital mobility, $\tilde{t}^{DC} = \tilde{t}^{W}$.

Note that the TOT component of the agreement, i.e. the difference $\tilde{t}^{DC} - \tilde{t}^A$, is just given by the national-welfare-maximizing tariff $\tilde{t}^{W}$ (see Figure 2). Thus, in the case of governments that can commit unilaterally, the optimal agreement just removes TOT considerations from the countries’ protection levels. It is important to note that the TOT component of the agreement is independent of politics ($a$).\textsuperscript{12} On the other hand, the domestic-commitment component of

\textsuperscript{12}Straightforward algebra reveals that $\tilde{t}^{W} = (1 - v)/2$. 

the agreement, \( \hat{t} - \hat{t}^{DC} \), is larger when politics are more important (\( a \) is lower).\(^{13}\) The following corollary records this result:

**Corollary 1** In the case of perfect capital mobility, the terms-of-trade component of the agreement \( (\hat{t}^{DC} - \hat{t}^{A}) \) is independent of \( a \), while the domestic-commitment component of the agreement \( (\hat{t} - \hat{t}^{DC}) \) is decreasing in \( a \).

We next turn to the opposite benchmark case, in which capital is fixed.

### 2.5.2 Fixed capital

As a second step toward characterizing the optimal agreement for general \( z \in [0,1] \), it is instructive to examine the extreme case in which capital is fixed at some level \( x \), i.e. \( z = 0 \). In this case, the optimal agreement is given by

\[
\hat{t}^\Psi(x) \equiv \arg \max_t \Psi(\hat{t}, x)
\]

It is easy to show that the curve \( \hat{t}^\Psi(x) \) lies uniformly below the curve \( \hat{t}^J(x) \).\(^{14}\) In this case, then, the trade agreement reduces the tariff by the amount \( \hat{t}^J(x) - \hat{t}^\Psi(x) > 0 \).

Next we want to decompose the optimal agreement into its domestic-commitment and TOT components. Following the methodology described in the previous section, we consider the domestic-commitment benchmark when \( x \) is fixed. The optimal tariff ceiling in this case maximizes

\[
J(\hat{t}, x) = aW(\hat{t}, x) + \delta x[p(\hat{t}, x) - c(\hat{t}, x)] + (1 - \delta)c(\hat{t}, x) + (\cdot)
\]

We now show that for all \( \delta \) this objective is maximized by \( t^I(x) \), that is, the optimum involves no agreement at all. To see this, rewrite the objective as

\[
J(\hat{t}, x) = \begin{cases} 
aW(t^W(x), x) + \delta x[p(\hat{t}, x) - c(\hat{t}, x)] & \text{if } \hat{t} \geq t^W(x) \\
aW(\hat{t}, x) + \delta x[p(\hat{t}, x) - c(\hat{t}, x)] & \text{if } \hat{t} < t^W(x)
\end{cases}
\]

\(^{13}\)A natural question is whether there exists a domestic-commitment motive even in the absence of politics, i.e. if governments maximize welfare. Given our assumption that supply in the exporting country is fixed, the answer is no. To see this note that \( \hat{t} \) approaches \( t^W \) as \( a \to \infty \), which implies that there is no need for domestic commitment. Intuitively, with no politics, the government sets the tariff to maximize national welfare given \( x \) (i.e., \( t = t^W(x) \)), and as a consequence the investors’ allocation decisions are efficient, yielding the (unilateral) optimal point \( (t^W, x^W) \).

\(^{14}\)To see this note that, for \( \hat{t} \geq t^W \), the objective function can be written (suppressing the \( x \) argument) as

\[
\Psi(\hat{t}) = aW(t^W) + aW^*(\hat{t}) + \delta[p(\hat{t}) - c(\hat{t})]x + (\cdot)
\]

Noting that the net return to capital \( p(\hat{t}) - c(\hat{t}) \) is maximized at \( \hat{t} = t^I \) and that \( W^*_t < 0 \), it follows that the maximizer of this function is lower than \( t^I \).
For $\tilde{t} < t^W(x)$ it is direct to verify that the objective is increasing in $\tilde{t}$, and for $\tilde{t} \geq t^W(x)$ the objective is maximized by $t^J(x)$, because $p - c$ is maximized by $t^J(x)$ (as we showed earlier).

We can conclude that, when $x$ is fixed, the domestic-commitment component of the agreement is nil, and the whole tariff cut is coming from the TOT component. A domestic-commitment motive for trade agreements is present only if capital is mobile ($z > 0$).

At this point it is useful to relate this case of fixed capital with the standard TOT story, and more specifically with Grossman and Helpman’s (GH) 1995 model. Note that, if $\delta = 1$, this case is essentially a simplified version of GH’s model. To see this, note that for $\delta = 1$, $\Psi(\tilde{t}, x)$ reduces to

$$\Psi(\tilde{t}, x)|_{\delta=1} = aW(\tilde{t}, x) + aW^*(\tilde{t}, x) + xp(\tilde{t}, x) + (\cdot)$$

where we omit the term in $(\cdot)$ because it is constant in $\tilde{t}$. This is the joint surplus of the two governments and the lobby. As in GH’s model, the optimal agreement maximizes this joint surplus. Note also that in this case an exact tariff is equivalent to a tariff ceiling, because contributions wash out in the ex-ante objective.

Next consider the impact of the political parameter $a$ on the extent of trade liberalization. It is easy to show that the agreed-upon tariff cut is given by $t^J(x) - t^\Psi(x) = 2m(t^J(x), x)$. Thus the tariff cut is deeper when the noncooperative import volume is higher. This is intuitive: when imports are larger, the TOT externality is more important, thus the inefficiency in the Nash equilibrium is stronger, and hence the trade agreement will cut the tariff by a larger amount.

The above observation has a straightforward implication for the comparative-statics effect of changes in $a$: when politics are more important ($a$ is lower), the noncooperative tariff $t^J$ is higher, hence the import volume is lower, and as a consequence the agreed-upon tariff cut is less deep.

This prediction on the impact of "politics" on the extent of trade liberalization is not specific to the GH model, but holds more generally when trade agreements are motivated by TOT considerations, as emphasized by Bagwell and Staiger (2001). This contrasts sharply with our earlier finding in the case of perfect capital mobility, where we found that the extent of trade liberalization is decreasing in $a$. This result points to an important insight: when the domestic-commitment motive for a trade agreement is important, the impact of "politics" on the extent of trade liberalization is essentially opposite the one predicted by the standard TOT
Before moving on, we make a remark on how the results of GH’s model are affected if \( \delta \) is allowed to be different than one. In particular, consider the case in which ex-ante lobbying is weaker than ex-post lobbying, that is \( \delta < 1 \). Then the results of the GH model may change in an important way, because tariff ceilings may be preferred to exact tariff commitments. To see this, note that the objective \( \Psi(\tilde{t}, x) \) becomes:

\[
\Psi(\tilde{t}, x) = aW(\tilde{t}, x) + aW^*(\tilde{t}, x) + \delta xp(\tilde{t}, x) + (1 - \delta)c(\tilde{t}, x)x + (\cdot)
\]

If \( \tilde{t} \) and \( x \) are such that \( \tilde{t} < t^W(x) \), then there are no contributions, and the objective is qualitatively the same as in the case \( \delta = 1 \) (except that the relative weight of welfare vs. profits increases). But if the tariff ceiling \( \tilde{t} \) exceeds \( t^W(x) \), then there are positive contributions, and the objective is qualitatively different than in the GH case. The optimal tariff ceiling may be above or below \( t^W(x) \), depending on the parameters; if it is above \( t^W(x) \), then tariff ceilings are strictly better than exact tariffs, because contributions enter positively in the objective \( \Psi(\tilde{t}, x) \). Thus, if the optimal tariff ceiling is above \( t^W(x) \), it suffices to perturb GH’s model by lowering \( \delta \) below one (even slightly), to find a role for tariff ceilings.

This allows us to disentangle the role of \( \delta \) from the role of \( z \) in explaining why we get different results from the standard model: when the only departure from the standard GH model is that ex-ante lobbying is weaker than ex-post lobbying (\( \delta < 1 \)), the only motive for trade agreements is still the TOT externality, but a role for tariff ceilings may appear. On the other hand, independently of the value of \( \delta \), a domestic commitment motive for trade agreements emerges if and only if there is capital mobility (ie, \( z > 0 \)).

### 2.5.3 Imperfect capital mobility

We are now in a position to characterize the optimal agreement for the more general case \( z \in [0, 1] \). Let us start by considering the equilibrium conditional on a given tariff binding \( \tilde{t} \). To develop intuition, suppose that \( \tilde{t} < \hat{t} \) and \( z \) is small. From the analysis of the previous section, we know that if capital were perfectly mobile, the equilibrium allocation would be the one that

\[ t^W = t^W(x^W) \]

The reader might wonder why in our model with perfect capital mobility the TOT component of the tariff cut (\( t^W \)) is independent of \( a \), as Corollary 1 states. How can this be reconciled with the observation in the text that, in the standard TOT model, the tariff cut is increasing in \( a \)? The key is that with perfect capital mobility, the TOT component of the tariff cut is also proportional to the import volume \( m(t, x) \), but evaluated at the point \( (t^W, x^W) \), not at the noncooperative equilibrium. Since \( m(t^W, x^W) \) is independent of \( a \), so is the TOT component of the tariff cut.
equalizes returns given \( \bar{t} \), that is \( x^{\text{er}}(\bar{t}) < \hat{x} \). But if \( z \) is small, capital will not be able to exit the import-competing sector in sufficient amount to equalize net returns to capital across sectors. The allocation will then be \( x_z = (1 - z)\hat{x} \) and the rate of return will be higher in the N sector. In general, the equilibrium allocation conditional on \( \bar{t} \) is \( \max \{ x^{\text{er}}(\bar{t}), x_z \} \equiv \bar{x}^{\text{er}}(\bar{t}) \) if \( \bar{t} \leq \hat{t} \) and \( \hat{x} \) otherwise. This is simply the equal-returns curve truncated at \( x_z \).

This result implies that the optimal agreement is the one that maximizes \( \Psi(\bar{t}, \bar{x}^{\text{er}}(\bar{t})) \) for \( \bar{t} \leq \hat{t} \). Note that, since investors are risk neutral, what matters for the lobby is only the total expected future returns for the lobby members, which are given by \( x(p - c) + (\hat{x} - x) \), thus the same expression we had for \( \Psi(\bar{t}, x) \) with perfect mobility is valid also with imperfect mobility. The key is that the parameter \( z \) enters the problem only through its effect on \( x \).

We are now in a position to characterize the optimal agreement for general \( z \). Recall from the previous analysis that, if \( z = 0 \), the optimal tariff ceiling is given by \( t^\Psi(\hat{x}) \), and if \( z = 1 \), the optimal tariff ceiling is zero. The next proposition "connects the dots" between these two extremes. Let \( t^{\text{er}}(x) \) be the inverse of \( x^{\text{er}}(\bar{t}) \) (in the relevant region \( x^{\text{er}}(\bar{t}) \) is increasing, so its inverse exists).

**Proposition 5** (i) The optimal tariff ceiling is given by

\[
\bar{t}^A = \begin{cases} 
\min(t^{\text{er}}(x_z), t^\Psi(x_z)) & \text{for } x_z \geq x^{ft} \\
0 & \text{for } x_z < x^{ft}
\end{cases}
\]

(ii) The tariff cut \( \hat{t} - \bar{t}^A \) is (weakly) increasing in \( z \).

(iii) The tariff cut \( \hat{t} - \bar{t}^A \) is increasing in \( a \) for low values of \( z \) and decreasing in \( a \) for high values of \( z \).

(iv) The tariff cut \( \hat{t} - \bar{t}^A \) is (weakly) decreasing in \( \delta \).

Figure 3 illustrates how the optimal agreement point \( A \) depends on \( z \). Consider two values of \( z \), say \( z' \) and \( z'' \). For point \( z' \) the agreement is given by \( A' \), located on the \( t^\Psi(x) \) curve, whereas for point \( z'' \) the agreement is given by point \( A'' \), located on the equal-returns curve. Thus, as \( z \) increases from zero, point \( A \) travels along the \( t^\Psi(x) \) schedule until it hits the equal-returns curve \( t^{\text{er}}(x) \), and then travels down along the \( t^{\text{er}}(x) \) curve until it reaches the free trade point (this path is marked in bold in figure 3). Note that both \( t^\Psi(x) \) and \( t^{\text{er}}(x) \) are increasing in \( x \), so the optimal tariff binding decreases as \( z \) increases. As a consequence, the tariff cut \( \hat{t} - \bar{t}^A \) increases with \( z \), as stated in point (ii) of the proposition.
This result suggests an empirical prediction: trade agreements should lead to deeper trade liberalization in sectors where factors of production are more mobile. Although our basic model cannot make cross-sectoral predictions because there is a single organized sector, it would not be hard to write a multi-sector model that delivers a genuinely cross-sectoral prediction along these lines.

Point (iii) of Proposition 5 focuses on the impact of the political parameter $a$ on the extent of trade liberalization. Recall from the previous analysis that, if $z = 0$, the tariff cut is deeper when politics are less important ($a$ is higher). By continuity, this is the case also if $z$ is sufficiently small. On the other hand, we saw that, if $z = 1$, the tariff cut is deeper when politics are more important ($a$ is lower). Again, by continuity this is the case whenever $z$ is sufficiently high. Thus the model highlights that, if the domestic-commitment motive is important enough, the prediction of the standard TOT model – that trade liberalization is deeper when politics are less important – gets reversed.

The last point of Proposition 5 offers another example of how "politics" affects trade liberalization in our theory. If lobbies exert stronger ex-ante pressure on governments ($\delta$ is higher), this will lead to trade agreements with higher maximum tariffs. In other words, just as one would expect, stronger ex-ante lobbying leads to weaker trade liberalization.

Next we want to decompose the optimal agreement into its domestic-commitment and TOT components. As in the previous sections, the key step is to characterize the fictitious benchmark in which the home government (in agreement with the lobby) can commit domestically. We saw earlier that, if $z = 0$, the optimal domestic-commitment point is the same as the noncooperative equilibrium (there is no domestic commitment motive for a trade agreement), while for $z = 1$ the optimal domestic-commitment point is $(t^W, x^W)$. The next result shows that as $z$ increases, the optimal domestic-commitment point travels from the noncooperative point $(\hat{t}, \hat{x})$ to point $(t^W, x^W)$ along the equal-returns curve:

**Proposition 6** In the domestic-commitment benchmark, the optimal tariff ceiling is

$$t^{DC}(z) = \begin{cases} 
  t^{er}(x_z) & \text{for } x_z \geq x^W \\
  t^W & \text{for } x_z < x^W 
\end{cases}$$

The domestic-commitment component of the agreed-upon tariff cut, $\hat{t} - t^{DC}(z)$, is clearly increasing in $z$. What can we say about the effect of $z$ on the TOT component, $t^{DC}(z) - \hat{t}^A(z)$? In general the answer is ambiguous, but notice that for small $z$ the TOT component of the
tariff cut decreases with \( z \). To see this, consider a small increase in \( z \) from zero. Then \( \bar{t}_D(z) \) goes down with infinite slope, while \( \bar{t}_A(z) \) goes down with finite slope, therefore \( \bar{t}_D(z) - \bar{t}_A(z) \) decreases. Thus we can say that the liberalization-deepening effect of factor mobility is entirely due to the domestic-commitment motive, at least for \( z \) relatively small.

### 3 Gradual trade liberalization

In this section we consider a continuous-time extension of the model, where the agreement can determine the path of the tariff ceiling for the future. The questions we are interested in are: Does the optimal agreement entail instantaneous liberalization, gradual liberalization, or a combination of the two? If trade liberalization has a gradual component, what does the optimal tariff path look like, and what determines the speed of liberalization?

Consider the same model as above, but now assume that time is continuous, denoted by \( s \). As in the previous section, we assume that when the agreement opportunity arises (at time \( s = 0 \)) the world is sitting at the long-run noncooperative equilibrium, so the capital allocation is \( \hat{x} \). The trade agreement determines a (fully enforceable) future path for the tariff ceiling, \( \bar{t}(s) \).

We assume that, at each point in time, a fraction \( z \) of capital-owners gets a chance to exit sector \( M \). On the other hand, to simplify the exposition and the derivation of the main results, we assume that entry into the \( M \) sector is free: that is, owners of capital in the \( M \) sector can freely and instantaneously move to the \( M \) sector if they wish. A more symmetric specification, where there is friction in capital mobility also from the \( N \) sector to the \( M \) sector, would deliver exactly the same results, but the analysis would be more cumbersome.\(^{16}\) The capital allocation \( x \) will be the physical state variable of the problem.

At each point in time after the agreement is signed, the lobby makes a take-it-or-leave-it offer \( (t, c) \) to the government, taking into account the constraints set by the agreement.

Analogously to the two-period model, we assume that the agreement maximizes the weighted ex-ante joint surplus \( U^G + U^{G*} + \delta(U^L + U^{L*}) \), where \( U^j \) is interpreted as player \( j \)'s payoff in present value terms. Here for simplicity we assume that governments and lobbies discount the

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\(^{16}\)The problem is that without free entry into the \( M \) sector, our way of modeling capital mobility would lead to a discontinuity in the law of motion of \( x \). The rate of change \( dx/dt \) would go from \(-zx\) when the value of capital in the \( N \) sector is higher than in the \( M \) sector, to \( z(1 - x) \) in the opposite situation. This makes the optimal control problem harder to solve, but it can be shown that the results hold also in this case.
future in the same way, and let $\rho > 0$ denote the common instantaneous discount rate. We
note that, unlike in the two-period model, having different weights in the ex-ante joint surplus
is not equivalent to having different discount rates. We chose the first of these two approaches
because it makes the analysis simpler.\footnote{We also analyzed the case in which governments and lobbies have different discount rates. It turns out that, for the problem to be concave, we need the discount rate of the governments not to be higher than the discount rate of the lobbies. Under this assumption, we find that the optimal path for the tariff ceiling has the same qualitative features as the one we characterize in this section.}

We will focus on Markov equilibria, that is equilibria where players’ strategies depend on
the history only through the state variable $x$.\footnote{Our restriction to Markov equilibria rules out "reputational" equilibria, but this is quite natural since we are assuming that tariff agreements are perfectly enforceable. Reputational equilibria are "useful" for sustaining cooperative outcomes when there is no external enforcement of agreements. In our enforceable-agreement setting, it can be shown that reputational equilibria cannot do better than the Markov equilibrium from the point of view of the ex-ante joint surplus.}

The first step of the analysis is to derive the equilibrium paths of $x$, $t$ and $c$ for a given path
of the tariff ceiling, $\bar{t}(s)$.\footnote{Analogously to the two-period model studied in the previous section, as long as $\delta$ is not too high then tariff ceilings are preferred to exact tariffs. A sufficient condition for this is $\delta \leq 1$. We assume that this condition is satisfied, so we restrict our attention to agreements that specify tariff ceilings.} We will omit the time argument $s$ whenever this does not cause confusion.

First note that, given the Markov restriction, the equilibrium tariff as a function of $x$ is
simply $t(x) = \min\{\bar{t}, t^J(x)\}$. If $\bar{t} \leq t^J(x)$ the tariff ceiling is binding and hence the tariff is
given by $\bar{t}$, otherwise the tariff is given by $t^J(x)$. The associated contributions are given by $c(t, x)$, just as in the static model.

To characterize the equilibrium path for $x$, let $V^M$ ($V^N$) be the value of a unit of capital in
the $M$ ($N$) sector. Since there is free entry into the $M$ sector, then in equilibrium it must be
that $V^M \leq V^N$. Moreover, the following no-arbitrage conditions must hold:

\begin{align}
\rho V^M &= z(V^N - V^M) + \dot{V}^M + p - c \quad (6) \\
\rho V^N &= 1 \quad (7)
\end{align}

To understand the first of these no-arbitrage conditions, note that the flow return to a unit of
capital in the $M$ sector (on the RHS of 6) is composed of three terms: the expected capital
gain from moving to the $N$ sector, $z(V^N - V^M)$, the capital gains arising from any increase in
$V^M$, $\dot{V}^M$, plus the instantaneous profits or "dividends", $p - c$. In equilibrium this flow return
must be equal to the opportunity cost of holding an asset of value $V^M$, given by $\rho V^M$. The
second no-arbitrage condition (7) is similar, except that because of free capital mobility from N to M, there cannot be capital gains to holding a unit of capital in the N sector, hence the condition is simply that the instantaneous profits of capital in N, given by 1, be equal to the opportunity cost of holding this asset, $\rho^L V^N$.

Combining the no-arbitrage equations 6 and 7 and letting $y \equiv V^M - V^N$, we obtain:

$$\dot{y} = (\rho + z)y - (p - c - 1)$$

Letting $g(t, x) \equiv p(t, x) - c(t, x) - 1$, integrating and imposing the condition $y(s) \to 0$ as $s \to \infty$,$^{20}$ we obtain:

$$y(s) = \int_s^\infty e^{-(\rho + z)(v-s)} g(t(v), x(v)) dv \leq 0 \text{ for all } s$$

It follows from the above discussion that, given a path for the maximum tariff $\bar{t}(s)$, the equilibrium conditions for $t(s), x(s)$ and $y(s)$ are the following:

1. $t(s) = \min\{\bar{t}(s), t^J(x(s))\}$
2. $y(s)$ satisfies 9 and $y(s) \leq 0$ for all $s$
3. $\dot{x}(s) = -zx(s)$ if $y(s) < 0$ and $\dot{x}(s) \geq -zx(s)$ if $y(s) = 0$.

Condition $y(s) \leq 0$ in (2) is a consequence of the assumption that there is free entry into the M sector. Condition (3) simply states that if $y < 0$ then capital leaves the M sector as fast as possible, whereas if $y = 0$ then any reallocation is an equilibrium as long as it satisfies the physical restriction that capital cannot leave the M sector faster than at rate $-zx$.

We can now derive the optimal agreement $\bar{t}(s)$. The objective function is

$$\int_0^\infty e^{-\rho s} \Psi(t(s), x(s)) ds$$

where

$$\Psi(t, x) \equiv aW(t, x) + xc(t, x) + aW^*(t, x) + \delta [g(t, x)x + \dot{x}]$$

We say that a plan $(t(s), x(s), y(s))$ is implementable if there is an agreement $\bar{t}(s)$ such that $(t(s), x(s), y(s))$ is an equilibrium, i.e. satisfies conditions (1)-(3). We will look for the plan that maximizes 10 in the set of implementable plans, and then we will identify the agreement $\bar{t}(s)$ that implements this plan. To turn this maximization into a more standard optimal control

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$^{20}$This is a condition that there should be no "bubbles" in the asset market. We could replace this by the weaker condition that $y$ converges to a finite value as $s \to \infty$. 

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problem, we let \( u = \dot{x} \) and note that any implementable plan must satisfy equation 8, together with the following "relaxed" restrictions:

\[
y(s) \leq 0, \quad \lim_{s \to \infty} y(s) = 0, \quad x(0) = \dot{x}, \quad \text{and} \quad u(s) + zx(s) \geq 0 \quad \text{for all} \quad s \geq 0
\]  

(11)

Conditions 8 and 11 are necessary for implementability. Our approach is to maximize the objective 10 subject to these necessary conditions for implementability, and then verify that the solution satisfies all implementability conditions. If this is the case, then we have found the optimal plan.

To characterize the solution to this problem, we need to introduce some notation. Just as in previous sections, let \( t_\text{er}(x) \) be the tariff that equalizes net returns to capital across sectors, which is implicitly defined by \( g(t, x) = 0 \), and \( t^\Psi(x) \) the tariff that maximizes \( \Psi(t, x) \). Also, let \( x^z(s) \) represent the path of \( x \) obtained when capital exits the M sector as fast as possible until the free trade allocation is reached:

\[
x^z(s) = \max \{ \dot{x}e^{-zs}, x^ft \}
\]

In order for the problem to be well-behaved, we need to assume that \( a \) is sufficiently high. A simple and sufficient condition for our result to hold is \( a \geq \frac{6\delta(3v-4)+1}{6(2-v)} \equiv \tilde{a} \). In the Appendix we will prove the result under a weaker (but more complicated) condition. The condition that \( a \) be sufficiently high serves essentially two purposes. First, it ensures that \( t^\Psi(x) - t^\text{er}(x) \) is decreasing in \( x \), which in turn implies that the curves \( t^\Psi(x) \) and \( t^\text{er}(x) \) have a unique intersection. Second, it ensures that the problem is concave, so that we can apply sufficiency conditions from optimal control theory.

We are now ready to state the main result of this section:

**Proposition 7** Assume \( a \geq \tilde{a} \). The optimal agreement entails four phases:

(i) an instantaneous drop in the tariff from \( \hat{t} \) to \( t^\Psi(\hat{x}) \);

(ii) a first gradual liberalization phase in which \( t(s) = \psi(x^z(s)) \), and \( y(s) < 0 \);

(iii) a second gradual liberalization phase in which \( t(s) = \text{er}(x^z(s)) \), and \( y(s) = 0 \);

(iv) a steady state in which the tariff is zero.

The optimal path for the allocation is given by \( x(s) = x^z(s) \) for all \( s \).

This proposition states that the optimal trade agreement entails a discrete tariff cut at time zero, with the tariff dropping from \( \hat{t} \) to \( t^\Psi(\hat{x}) \), which is then followed by gradual trade liberalization and exit of capital from the M sector. This gradual trade liberalization is characterized
by two phases. In the first phase, the tariff is given by the best static tariff $t^\Psi(x^z(s))$ as a function of the evolving capital allocation, whereas in the second stage the tariff is just the one that equalizes net returns across sectors (given the capital allocation, $x^z(s)$). Note that in the first phase capitalists in the $M$ sector want to leave as fast as possible, since the value of a unit of capital in that sector is lower than in the $N$ sector (i.e. $y(s) < 0$); in the second phase capitalists are indifferent as to where to locate their capital (i.e. $y(s) = 0$), but the government induces exit at the fastest possible rate. After a period of adjustment, the capital stock reaches the free trade allocation, and free trade obtains thereafter.\footnote{We emphasize that the above result is far from being a corollary of the comparative-statics result of the static model, where we derived the optimal tariff ceiling as a function of $z$. As an example of the different structure of the problem, note that in this section we need a unique intersection between the $t^\Psi$ curve and the $t^\tau$ curve, a condition we did not need in the static model. In the static model, the optimal tariff ceiling always follows the lower envelope of these two curves as $z$ increases. In the dynamic problem, if the two curves intersect more than once the optimal tariff ceilings may not follow their lower envelope. We actually find it surprising that under some simple conditions the solution of the continuous-time model mirrors exactly the comparative-statics solution of the static model.}

As in previous sections, we want to understand the role of the TOT motive and of the domestic-commitment motive in the determination of the optimal trade-liberalization path. Following a similar approach as in the previous section, consider the hypothetical case in which, starting from the non-cooperative equilibrium $(\hat{t}, \hat{x})$, each government gets a chance to unilaterally commit to a future path for the tariff ceiling, but without a trade agreement.\footnote{The objective function that the government maximizes is the same as in 10 except that it doesn’t include $aW^*(t, x)$.} Let us denote the resulting path as $t^{DC}(s)$. We can think of $\hat{t} - t^{DC}(s)$ as the domestic commitment component of the trade liberalization path, with the remainder, that is $t^{DC}(s) - t(s)$, being the TOT component.

It is straightforward to show that the optimal domestic-commitment path entails $t^{DC}(s) = t^{er}(x^z(s))$ until the point $(t^W, x^W)$ is reached. One direct implication is that trade liberalization associated with the domestic commitment motive takes place gradually, with no discrete tariff reduction. The reason for this is that the government can achieve the desired reallocation of capital towards the $N$ sector without any reduction in returns to capital in the $M$ sector by following the path $t^{er}(x^z(s))$ from $(\hat{t}, \hat{x})$ to $(t^W, x^W)$. This implies that the discrete tariff drop that follows the signing of the trade agreement (from $\hat{t}$ to $t^\Psi(\hat{x})$) is entirely associated with the TOT motive, while the domestic-commitment motive is reflected in the gradual component of trade liberalization. More generally, this suggests that the gradual component of trade
liberalization should be more important, relative to the instantaneous component, when the domestic-commitment motive is more important relative to the TOT motive.\footnote{An interesting question is whether the TOT component of the trade agreement is gradual or not. The answer is that it depends. Recall that the TOT motive is responsible for (i) an instantaneous tariff drop from $\tilde{t}$ to $t^W(\tilde{x})$, and (ii) a reduction in the steady-state tariff from $t^W$ to zero. Thus, if $\tilde{t} - t^W(\tilde{x}) < t^W$, we can say that the TOT component of the tariff cut is partly instantaneous and partly gradual, but if $\tilde{t} - t^W(\tilde{x}) > t^W$, then the TOT component of the tariff cut \textit{overshoots} its steady-state level, hence we can say that this component is "anti-gradual". It is easy to find parameter values for which each of the two cases described above ($\tilde{t} - t^W(\tilde{x})$ higher or lower than $t^W$) obtains.}

Another interesting prediction of this analysis is that trade liberalization paths established in trade agreements should entail faster liberalization for sectors where exit can proceed at a faster pace. An interesting open question is whether this prediction is consistent with empirical observations.

4 Extensions

In this section we consider two extensions. First, what happens if governments have some bargaining power vis-à-vis their domestic lobbies? Second, how are the results affected if ex-ante lobbying is so strong ($\delta$ is so high) that Proposition 2 does not hold and exact tariffs are preferable to tariff ceilings? We address these extensions in the context of the two-period model of Section 2.

4.1 Governments’ bargaining power

An assumption we have maintained thus far is that in each country the lobby has all the bargaining power. This is a convenient assumption because in this case the government does not derive any rents from the political process, and hence it has a strong desire to foreclose domestic political pressures. If the government has some bargaining power, however, the domestic-commitment motive for a trade agreement is weaker. The question then is, how do our results change when we drop the extreme assumption that the lobby has all the bargaining power?

To illustrate this in the simplest way, we consider the opposite extreme, namely the case when the government has all the bargaining power. For some $x$ consider the government and lobby negotiating a tariff above $t^W(x)$. The rents obtained by the lobby would be given by:

$$x \left[ p(t, x) - p(t^W(x), x) \right]$$
Since the government has full bargaining power, it captures all these rents in the form of contributions. Thus, for $t \geq t^W(x)$,

$$c(t, x) = p(t, x) - p(t^W(x), x)$$

Net profits for capitalists in sector $M$ are then $p(t, x)$ for $t < t^W(x)$ and $p(t^W(x), x)$ for $t \geq t^W(x)$. This implies that the equal-returns (or ER) curve now becomes vertical at point $(t^W, x^W)$, so that $x^{er}(t) = x^W$ for $t \geq t^W$, and that the long-run non-cooperative equilibrium - which, as before, is given by the intersection of $t^f(x)$ and $t^{er}(x)$ - is given by $\hat{x} = x^W$, $\hat{t} = t^f(x^W)$ (see Figure 4). If $z$ is such that $x^W(1 - z) > x^ft$ (so that capital cannot exit the $M$ sector in sufficient amount to reach the free trade allocation), then the ER curve will be truncated at $x_z = x^W(1 - z)$.

Consider now what happens when governments can sign a trade agreement. The first step is to write down the ex-ante joint surplus of the two governments and the lobby. Plugging the above result for contributions into (3), we obtain:

$$\Psi(\bar{t}, x) = aW(\bar{t}, x) + xp(\bar{t}, x) + aW^*(\bar{t}, x) + \delta [xp(t^W(x), x) + (\bar{x} - x)] - xp(t^W(x), x)$$

Just as above, let $t^\Psi(x)$ denote the value of $\bar{t}$ that maximizes $\Psi(\bar{t}, x)$ for a given $x$. Dropping the terms in the objective that do not depend on $\bar{t}$, we get

$$t^\Psi(x) = \arg\max_t (aW(t, x) + xp(t, x) + aW^*(t, x))$$

Note that $t^\Psi(x)$ is essentially the same as the optimal agreement in Grossman and Helpman’s (1995) model. The optimal agreement in our model is the point that maximizes $\Psi(\bar{t}, x)$ along the ER curve. We need to consider two possibilities, depending on whether $t^\Psi(x)$ passes below or above the $(t^W, x^W)$ point. If it passes below, then the analysis is basically the same as in Section 2, and a result analogous to Proposition 5 obtains. In particular, when capital mobility is sufficiently high, the agreement entails free trade.

Consider now the case in which $t^\Psi(x)$ passes above the $(t^W, x^W)$ point, as illustrated in Figure 4. It is simpler to start with the case of full capital mobility, $z = 1$. By definition of $t^\Psi(x)$, $\Psi(\bar{t}, x^W)$ increases as $\bar{t}$ falls from $t^f(x^W)$ to $t^\Psi(x^W)$ but then decreases as $\bar{t}$ continues to fall to $t^W$. On the other hand, we already know that $\Psi(\bar{t}, x)$ increases as we move along the ER curve from the $(t^W, x^W)$ point towards free trade. Thus, there are two local maxima: $t^\Psi(x^W)$
and \( t = 0 \). Depending on parameters, either one may be the best agreement. In particular this depends on the height of the optimal terms-of-trade tariff \( t^W \). If \( t^W \) is low (which is the case when trade volume is low, which in turn happens when \( v \) is relatively high), then \( t^\Psi(x^W) \) is close to \( t^J(x^W) \) and the \((t^W, x^W)\) point is close to the free trade point. This implies that the decline in \( \Psi \) as \( \bar{t} \) falls from \( t^\Psi(x^W) \) to \( t^W \) exceeds the increase in welfare as we move from \((t^W, x^W)\) to the free trade point, hence \( t^\Psi(x^W) \) is better than free trade.

Now consider the general case of \( z \in [0, 1] \). It is useful to consider how the optimal agreement varies as \( z \) increases from zero. Since \( t^\Psi(x^W) > t^W \), then the optimal agreement remains at \( t^\Psi(x^W) \) for all \( z \) if this point is better than free trade. If not, then there will be a sufficiently high level \( z \) at which the best agreement switches discontinuously from \( t^\Psi(x^W) \) to \( t_{er}(x^W(1 - z)) \), following this curve until the free trade point as \( z \) increases further.

To summarize the main point of this section, if governments have strong bargaining power vis-à-vis their lobbies, it is still true that trade liberalization is (weakly) increasing in \( z \), but it may no longer be true that with sufficient capital mobility the trade agreement entails free trade.

### 4.2 Exact tariff commitments

Here we consider the case in which \( \delta \) is so high that Proposition 2 does not hold (recall that such a case can occur only if \( z < 1 \): if capital mobility is perfect, the result holds for any \( \delta \)). As explained in Section 2.5, in this case governments may prefer exact tariff commitments to tariff ceilings. Of course, in this case one of our main results – namely the optimality of tariff ceilings – no longer holds, but do the other results still hold? This is the question we address in this section.

Let us characterize the optimal exact-tariff agreement. The analysis is quite similar to that of agreements with tariff ceilings, so the presentation will be very schematic. First note that, since there are no contributions, the ex-ante joint surplus is simply:

\[
\Omega(t, x) \equiv aW(t, x) + aW^x(t, x) + \delta p(t, x)x + \delta(x - x)
\]

Let \( t^\Omega(x) \) be the level of \( t \) that maximizes \( \Omega(t, x) \). It is easy to show that

\[
t^\Omega(x) = \delta x/a
\]

Next let \( t_{er}^\Psi(x) \) be the tariff that equalizes returns across sectors when there are no contributions,
given \( x \). This is given implicitly by

\[
p(t, x) = 1
\]

Note that both \( t^\Omega(x) \) and \( t^\pi_N(x) \) are increasing functions of \( x \). The next proposition shows that a result analogous to the one in Proposition 5 holds:

**Proposition 8** The optimal exact-tariff agreement is given by

\[
\hat{t}_A^N = \begin{cases} 
\min(t^\pi_N(x_z), t^\Omega(x_z)) & \text{for } x_z \geq x^{ft} \\
0 & \text{for } x_z < x^{ft} 
\end{cases}
\]

The above proposition characterizes the optimal exact tariff as a function of \( z \). With perfect capital mobility (\( z = 1 \)) the agreement entails free trade, just as in the case of agreements with tariff ceilings. As capital mobility (\( z \)) decreases, the agreed-upon tariff increases, so the agreement entails less and less trade liberalization. To see this, note that \( t^\Omega(x) \) and \( t^\pi_N(x) \) are increasing functions of \( x \), so \( \min(t^\pi_N(x_z), t^\Omega(x_z)) \) is a decreasing function of \( z \).

A decomposition between the domestic commitment motive and the terms-of-trade motive can be shown to hold also in this setting just as in the case of agreements with tariff ceilings.

In summary, when \( \delta \) is high all the results of the previous sections continue to hold except that tariff ceilings might not be optimal.

## 5 Conclusion

In this paper we have presented a theory that gives a prominent role to "politics" in the determination of trade agreements. This stands in contrast to the standard theory, according to which even politically-motivated governments sign trade agreements only to deal with terms-of-trade externalities. We developed a model where trade agreements may be motivated both by terms of trade externalities and by domestic commitment considerations. There are two key elements of our model: first, capital mobility and its interaction with lobbying, which generates a problem of time inconsistency in trade policy; and second, a structural approach that distinguishes between ex-ante and ex-post lobbying, and that takes into account the effect of contributions on both the governments’ payoffs and the returns to capital. The resulting framework is rich in implications. In particular, it leads to the prediction that trade agreements result in deeper trade liberalization when governments are more politically motivated (provided capital mobility is sufficiently high), when lobbies have less influence on the negotiation of the
agreement and when capital can move more freely across sectors. In addition, it implies that under some conditions governments prefer to commit to tariff ceilings rather than to exact tariff levels, just as we observe in reality. Finally, the model predicts that trade liberalization occurs in two stages: an immediate slashing of tariffs relative to their noncooperative levels, and a subsequent gradual reduction of tariffs, where the instantaneous tariff cut is a reflection of the terms-of-trade motive for the agreement, while the domestic-commitment motive is reflected in the gradual phase of trade liberalization.

There is one assumption in our model that merits further discussion. We have assumed that international agreements are perfectly enforceable, while there are no domestic commitment mechanisms. An alternative approach would be to assume that there are no exogenous enforcement mechanisms, so that both domestic and international agreements have to be self-enforced through "punishment" strategies in a repeated game. The question that arises in this case is the following: if domestic punishments are not enough to solve the domestic commitment problem, is it the case that international punishments can help governments live up to their domestic commitments?

A more precise way to formulate the above question is the following. Consider an infinitely repeated version of our model, and compare two punishment strategies: (i) a purely domestic punishment strategy where, if a country’s tariff deviates from its equilibrium level, the capital allocation and the tariff in that country revert to their long-run noncooperative levels; (ii) an international punishment strategy where, if a country’s tariff deviates from its equilibrium level, the capital allocation and the tariff in both countries revert to their long-run noncooperative levels. Suppose the optimal international agreement with perfect enforcement entails tariff $t^A$, and the optimal domestic-commitment tariff in the absence of international agreements is $t^{DC} > t^A$. Now suppose that a purely domestic punishment strategy is not enough to sustain $t^{DC}$, so that the domestic commitment problem remains partially unsolved. We then ask: can an international punishment strategy sustain $t^A$? If this is the case, then we can say that the international punishment strategy helps governments fully solve their domestic commitment problems. We conjecture that there will be a region of parameters for which this is the case. However, a rigorous examination of self-enforcing trade agreements will have to await further research.

\[24\] One could consider more severe international punishment strategies, for example a reversion to autarky. This would only strengthen the argument we are making here.
Appendix

Proof of Proposition 1:

We start by deriving $\hat{x}$. To do this, simply plug $t^J(x)$ into the ER curve, to get:

$$v - \frac{1}{2}(x + 1 - t^J(x)) - c(t^J(x), x) = 1$$

or

$$v - \frac{1}{2}(x + 1 - (1/3)(1 - x + 2x/a)) - x/6a = 1$$

This is an equation in $x$ that yields a unique solution given by:

$$\hat{x} = 2a \left( \frac{3v - 4}{4a - 1} \right)$$

Note that the condition on $a$ assumed in the Proposition implies that $a > 1/4$ and hence the denominator of the previous expression for $\hat{x}$ is positive, and also the numerator is positive given that we have assumed that $v > 3/2$.

The equilibrium tariff is given by

$$\hat{t} = t^J(\hat{x}) = \frac{1}{3}[1 + \frac{4 - 2a}{4a - 1}(3v - 4)]$$

Differentiating with respect to $a$,

$$\frac{d\hat{t}}{da} = -\frac{7}{3} \cdot \frac{3v - 4}{(4a - 1)^2} < 0$$

which shows the last part of the claim.

We need to show that imports are positive at the equilibrium we just found. This requires $1 - \hat{x} > \hat{t}$. Plugging in the values for $\hat{x}$ and $\hat{t}$ we find the condition

$$a > (6v - 7)/6(2 - v)$$

which is the condition assumed in the proposition.

For future reference we also show that the $t^{er}(x)$ curve is upward sloping. Differentiation shows

$$\frac{dt^{er}(x)}{dx} = \begin{cases} 1/2+(a/12x)(3t^{er}(x)-\Delta x)-c(t^{er}(x), x)/x & \text{if } x > x^W \\ 1/2-(a/4x)(3t^{er}(x)-\Delta x) & \text{if } x \leq x^W \\ \end{cases}$$

(12)
\( t^{tr}(x) \) is clearly upward sloping if \( x \leq x^W \), so let’s now focus on the case \( x > x^W \). It is easy to show that the denominator is positive as long as \( t < t^J(x) \), while it is zero if \( t = t^J(x) \) and negative otherwise. Moreover, a sufficient condition for the numerator to be positive is that

\[
1/2 - c(t^J(x), x)/x > 0
\]

But

\[
c(t^J(x), x)/x = 1/6a
\]

Hence, a sufficient condition for \( t^{tr} \) to slope upwards below the \( t^J \) curve is that \( a > 1/3 \). But note that this implied by the assumption of the proposition. Q.E.D.

**Proof of Proposition 2:** The reader should read Section 4.2, where the optimal exact-tariff agreement is characterized, before reading this proof.

To prove the first claim, we show that it holds for \( \delta = 1 \). We will show that the best exact tariff is dominated by a tariff ceiling set at the same level. We know that the best exact tariff agreement entails \( t^A_N \leq t^{tr}(x_z) \). We now argue that a tariff ceiling at \( t = t^A_N \) delivers a joint welfare that is at least as high as the joint welfare obtained under the exact tariff. To see this, note that if \( t^A_N \leq t^W(x_z) \) then the joint surplus in the two cases is the same, since there are no contributions. If, on the other hand, \( t^A_N > t^W(x_z) \), then there are contributions, and hence:

\[
\Psi(t^A_N, x_z) = a[W(t^A_N, x_z) + W^*(t^A_N, x_z)] + (1 - \delta)c(t^A_N, x_z)x_z + \delta[xp(t^A_N, x_z) + (\bar{x} - x_z)]
\]

\[
= \Omega(t^A_N, x_z) + (1 - \delta)c(t^A_N, x_z)x_z
\]

\[
\geq \Omega(t^A_N, x_z)
\]

where the last inequality is ensured by \( \delta \leq 1 \).

To prove the second claim, note that with perfect capital mobility the net return to capital is equal to one, so the joint surplus can be written as \( \Psi = a(W + W^*) + C + [\cdot] \). A similar argument as above can then be applied, noting that it is valid for any \( \delta \). Q.E.D.

**Proof of Proposition 3:** We will argue that free trade achieves a higher value of the objective than any \( t > 0 \). Using the notation \( t^W(\bar{t}) \) as shorthand for \( t^W(x^{tr}(\bar{t})) \), the previous statement is clearly true for \( \bar{t} < t^W(\bar{t}) \). Let us then focus on \( \bar{t} > t^W(\bar{t}) \). We need to show that

\[
W(t^W(\bar{t}), x^{tr}(\bar{t})) + W^*(\bar{t}, x^{tr}(\bar{t})) < W(0, x^{ft}) + W^*(0, x^{ft})
\]
Since $W^*$ is decreasing in $t$,

$$W(tW(\bar{t}, x^{er}(\bar{t}))) + W^*(\bar{t}, x^{er}(\bar{t})) < W(tW(\bar{t}, x^{er}(\bar{t}))) + W^*(tW(\bar{t}), x^{er}(\bar{t}))$$

Since $W + W^*$ is decreasing in $t$,

$$W(tW(\bar{t}, x^{er}(\bar{t}))) + W^*(tW(\bar{t}), x^{er}(\bar{t})) < W(0, x^{er}(\bar{t})) + W^*(0, x^{er}(\bar{t}))$$

But clearly

$$W(0, x^{er}(\bar{t}))) + W^*(0, x^{er}(\bar{t})) < W(0, x^{ft}) + W^*(0, x^{ft})$$

which proves the claim. QED

**Proof of Proposition 4:** See the proof of the more general Proposition 6.

**Proof of Proposition 5:** The key is to show that $\Psi(\bar{t}, x^{er}(\bar{t}))$ is decreasing in $\bar{t}$. Focus first on the case $\bar{t} < tW(\bar{t})$. Here there are no contributions, so $x^{er}(\bar{t})$ is the standard supply response to a tariff and hence the objective is the standard world welfare with mobile $x$, which is decreasing in $\bar{t} = 0$. What about the case $\bar{t} \geq tW(\bar{t})$? We want to show that $F(t) \equiv W(tW(\bar{t}, x^{er}(\bar{t}))) + W^*(\bar{t}, x^{er}(\bar{t}))$ is decreasing in $\bar{t}$ for $\bar{t} > tW(\bar{t})$. Applying the Envelope Theorem, then:

$$F'(t) = W_x(tW(\bar{t}, x^{er}(\bar{t})))dx^{er}(\bar{t})/d\bar{t} + W_x^*(\bar{t}, x^{er}(\bar{t}))dx^{er}(\bar{t})/d\bar{t}$$

Since $W_x^* < 0$ and $dx^{er}(\bar{t})/d\bar{t} > 0$, it suffices to show that $W_x(tW(\bar{t}, x^{er}(\bar{t})))$ and $W_x^*(\bar{t}, x^{er}(\bar{t}))$ are both negative. The second part is obvious given that the only effect of $x$ on $W^*$ is through terms of trade, and an increase in $x$ worsens Foreign’s terms of trade. As to the first part, we now show that $W_x(tW(x), x) < 0$ for $x > xW$. Simple derivation and some manipulation reveals that:

$$W_x(tW(x), x) = p(tW(x), x) - 1$$  \hspace{1cm} (13)

Given the definition of $xW$ (i.e., $p(tW(xW), xW) = 1$) then the result follows immediately.

Thus, if $t^\Psi(x_z) < t^{er}(x_z)$ then - by definition of $t^\Psi(x_z)$ - the point $(t^\Psi(x_z), x_z)$ is superior to the point $(t^{er}(x_z), x_z)$ which in turn is superior to all the other points on the curve $\bar{x^{er}}(\bar{t})$ for $\bar{t} > t^{er}(x_z)$. On the other hand, if $t^{er}(x_z) < t^\Psi(x)$ then all points on the vertical segment of
curve $\tilde{x}^{er}(\tilde{t})$ are dominated by the point $(t^{er}(x_z), x_z)$. Hence, in this case, the optimal tariff binding is simply $t^{er}(x_z)$. Of course this is true only as long as $x_z \geq x^f t$, otherwise the optimum is free trade.

To prove that $\hat{t} - \tilde{t}^A$ is (weakly) increasing in $z$, note that (a) $t^{er}(x)$ is increasing in the relevant range; and (b) the cross derivative $\Psi_{tx}(\tilde{t}, x)$ is positive for all $\tilde{t}$ and $x$. This implies that $\Psi(\tilde{t}, x)$ is supermodular in $\tilde{t}$ and $x$, which in turn implies that $t^{\Psi}(x)$ is increasing. As a consequence, $\min(t^{er}(x_z), t^{\Psi}(x_z))$ is increasing in $x_z$, and hence decreasing in $z$, which implies the claim.

Point (iii) follows from the observations we made previously, that (a) $\hat{t} - \tilde{t}^A$ is increasing in $a$ for $z = 0$ and (b) $\hat{t} - \tilde{t}^A$ is decreasing in $a$ for $z = 1$. Since the problem is continuous in $z$, the claim follows.

To prove point (iv), note first that $\hat{t}$ and $t^{er}(x_z)$ are independent of $\delta$, so all we need to show is that $t^{\Psi}(x_z)$ is weakly increasing in $\delta$. It is direct to check that $\Psi$ is supermodular in $\tilde{t}$ and $\delta$, which implies the claim. Q.E.D.

**Proof of Proposition 6:** Consider first the case $z = 1$. The DC agreement maximizes

$$F(\tilde{t}) \equiv J(\tilde{t}, x^{er}(\tilde{t})) = \begin{cases} aW(t^W(\tilde{t}), x^{er}(\tilde{t})) + \delta \hat{x} & \text{for } \tilde{t} \geq t^W \\ aW(\tilde{t}, x^{er}(\tilde{t})) + \delta \hat{x} & \text{for } \tilde{t} < t^W \end{cases}$$

We now show that $\tilde{t}^{DC} = t^W$. Consider first the case $\tilde{t} < t^W$. Differentiation yields:

$$F'(\tilde{t}) = aW_t(\tilde{t}, x^{er}(\tilde{t})) + aW_x(\tilde{t}, x^{er}(\tilde{t}))$$

where we have used the fact that $dx^{er}/d\tilde{t} = 1$ for $\tilde{t} < t^W$. Since $\tilde{t} < t^W$ implies $\tilde{t} < t^W(\tilde{t})$, and hence $W_t(\tilde{t}, x^{er}(\tilde{t})) > 0$, whereas $W_t(t^W, x^{er}(t^W)) = 0$. Coming now to the second term above, differentiation yields

$$W_x(\tilde{t}, x^{er}(\tilde{t})) = (1/2)(1 - (v - 1) - 2\tilde{t})$$

It is easy to show that this is equal to zero for $\tilde{t} = t^W$, and hence is positive for $\tilde{t} < t^W$. Thus, $F'(\tilde{t}) > 0$ for $\tilde{t} < t^W$. The previous arguments establish also that $F'(t^W) = 0$.

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25To see this, first note the following: if $\delta > 1/3$ then $\Psi$ is concave in $t$; if $\delta < 1/3$, then $\Psi$ is concave in $t$ for $t < t^W(x)$ and convex in $t$ for $t \in [t^W(x), t^I(x)]$. But since $\Psi_x(t^I(x), x) < 0$, then the maximum is attained for $t \leq t^W(x)$.

The statement in the text is clearly true if $\Psi(t, x)$ is concave in $t$. If it is not, then as argued above the maximum is attained for $t \leq t^W(x)$, and since $\Psi(t, x)$ is concave in this interval, then it must be that $\Psi(t, x)$ is increasing for $t < t^\Psi(x)$. 

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Now consider the case $\tilde{t} > t^W$. In this case, differentiation yields (using the Envelope Theorem):

$$F'(\tilde{t}) = [aW_x(t^W(\tilde{t}), x_{er}(\tilde{t}))]d_{x_{er}}/d\tilde{t}$$

We have already established that $d_{x_{er}}/d\tilde{t} > 0$. Hence, it is sufficient to show that $W_x(t^W(\tilde{t}), x_{er}(\tilde{t})) < 0$ for $\tilde{t} > t^W$. This is equivalent to $W_x(t^W(x), x) < 0$ for $x > x^W$. But given (13), it is clear that $W_x(t^W(x), x) = 0$ for $x = x^W$ and negative for $x > x^W$, hence the result follows immediately.

Next consider the case $z < 1$. We showed in the text that $J(t, x)$ is maximized by $t^I(x)$. Applying the same logic as in the proof of Proposition 5 one can show that

$$t^{DC}(z) = \begin{cases} \min(t^r(x), t^I(x)) & \text{for } x \geq x^W \\ t^W & \text{for } x < x^W \end{cases}$$

But $\min(t^r(x), t^I(x)) = t^r(x)$, hence the claim. Q.E.D.

**Proof of Proposition 7:** We will prove the result under a weaker condition than the one stated in the text ($a \geq \frac{6\delta(3v-4)+1}{6(2-v)}$). Here we will assume that *either* of the following two conditions is satisfied: (i) $a \geq \frac{6\delta(3v-4)+1}{6(2-v)}$; (ii) $a \geq \max\left\{\frac{\delta(3v-4)}{2-v}, \frac{\delta}{23-1}\right\}$ and $\delta > \frac{1}{2}$. Note that also this weaker restriction requires that, for given $\delta$ and $v$, the parameter $a$ should be higher than some critical level.

Using $\beta(s)$ as the Kuhn-Tucker multiplier of the constraint on the control, $u(s) + zx(s) \geq 0$, and $\phi(s)$ as the multiplier function of the pure state constraint, $y(s) \leq 0$, then we have the following Hamiltonian:

$$H = e^{-\rho s}\Psi(t, x) + \beta [u + zx] - \phi y + \lambda_x u + \lambda_y [(\rho + z)y - g(t, x)]$$

Necessary conditions for optimality are $H_t = H_u = 0$, plus the Euler equations, $\dot{\lambda}_x = -H_x$ and $\dot{\lambda}_y = -H_y$, plus the constraints $u + zx \geq 0$, $y \leq 0$, and the complementary slackness (CS) conditions:

$$\beta \geq 0, \beta(u + zx) = 0, \text{ and } \phi \geq 0, \phi y = 0$$

$H_t = 0$ implies:

$$e^{-\rho s}\Psi_t - \lambda_y g_t = 0 \quad (14)$$

while $H_u = 0$ implies $\beta + \lambda_x = 0$, or

$$\beta = -\lambda_x \quad (15)$$
The Euler equation $\dot{\lambda}_x = -H_x$ yields:
\[
\dot{\lambda}_x = -e^{-\rho s}\Psi_x - \beta z + \lambda_y g_x
\]  \hspace{1cm} (16)
while $\dot{\lambda}_y = -H_y$ yields:
\[
\dot{\lambda}_y = \phi - \lambda_y (\rho + z)
\]  \hspace{1cm} (17)

Our methodology is to guess that the solution is the one stated in the Proposition and verify that it satisfies necessary and sufficient conditions for an optimum. Our conjectured solution entails three phases: the first phase goes from $s = 0$ to $s = \tilde{s}$, and has $t = t^\Psi(x^\tilde{s}(s))$; the second phase goes from $s = \tilde{s}$ to $s = s^{ft}$, where $s^{ft}$ is defined by the time at which $e^{-z}\hat{x}$ reaches $x^{ft}$, and entails $t = t^{er}(x^\tilde{s}(s))$; finally, the third stage starts at $s = s^{ft}$ and involves a steady state with $t = 0$ and $x = x^{ft}$.

To check this conjecture, we move backwards, checking first that free trade can be a steady state. From 14 we get:
\[
\lambda_y(s) = e^{-\rho s}\delta x^{ft}
\]  \hspace{1cm} (18)
This implies $\dot{\lambda}_y = -\rho \lambda_y$. Plugging in 17 yields $\phi(s) = z\lambda_y(s)$. Hence, using 18 we get:
\[
\phi(s) = e^{-\rho s}z\delta x^{ft}
\]  \hspace{1cm} (19)
Note that this clearly satisfies the condition $\phi(s) \geq 0$.

Now, since at the free trade steady state we have $u = 0$, then $u + oz = oz > 0$, and hence the CS conditions imply $\beta = 0$. Equation 15 then implies that
\[
\lambda_x(s) = 0
\]  \hspace{1cm} (20)
Plugging this (and $\dot{\lambda}_x = 0$) into the Euler equation 16 yields $e^{-\rho s}\Psi_x = \lambda_y g_x$, which can be shown to hold by noting that at free trade we have $\Psi_x = \delta x^{ft}g_x$ (since $W_x + W^*_x = 0$ at free trade) whereas from 18 we see that $\lambda_y g_x = e^{-\rho s}\delta x^{ft}g_x$.

We can now move backwards to the second phase, $s \in [\tilde{s}, s^{ft}]$, and solve for $\lambda_x$ and $\lambda_y$, and check that $\beta, \phi$ are positive (as required by the CS conditions).

Condition 14 can be used to solve for $\lambda_y$:
\[
\lambda_y = e^{-\rho s}\Psi_t/g_t
\]  \hspace{1cm} (21)
Plugging this result plus $\beta = -\lambda_x$ into 16, and using $dt^{er}/dx = g_x/g_y$, yields:
\[
\dot{\lambda}_x = \lambda_x z - e^{-\rho s} \frac{d\Psi}{dx} \bigg|_{g=0}
\]
Now, since $e^{-\rho s} \left. \frac{d \Psi}{dx} \right|_{y=0}$ given $x = x^z(s)$ is merely a function of time, we can denote it as $\mu(s)$, and hence we have a simple differential equation, which can be solved imposing $\lambda_x(s^{ft}) = 0$. This yields:

$$\lambda_x(s) = \int_s^{s^{ft}} \mu(v) e^{-z(v-s)} dv \quad (22)$$

Note that since $\left. \frac{d \Psi}{dx} \right|_{y=0} < 0$ (this holds for $s < s^{ft}$ since $x^z(s) < x^{ft}$, see proof of Proposition 5) then $\lambda_x(s) < 0$, implying that $\beta(s) > 0$.

The only condition left to check for the second phase is that $\phi(s) \geq 0$ for $s \in [\bar{s}, s^{ft}]$. But from the Euler equation 17 we can see that this is true as long as

$$\phi(s) = \lambda'_y(s) + (\rho + z) \lambda_y(s) \geq 0$$

To verify this inequality, note that from 14 we get $\lambda_y = e^{-\rho s} \Psi_t/g_t$ evaluated at $t = t^{er}(s)$ (where, to simplify notation, we write $t^{er}(s)$ for $t^{er}(x^z(s))$). Letting

$$f(s) \equiv \frac{\Psi_t(t^{er}(s), x^z(s))}{g_t(t^{er}(s), x^z(s))}$$

then

$$\phi(s) = z \lambda_y(s) + e^{-\rho s} f'(s) \quad (23)$$

It can be shown that the assumption we made on $a$ implies $t^{er}(x) < t^W(x)$ in the second phase of adjustment, thus there are no contributions. Differentiation shows that $f(s) = \delta x^z(s) - at^{er}(s)$. Using $dt^{er}/dx = 1$, we find:

$$f'(s) = az x^z(s) [1 - \delta/a]$$

Given that $\delta < a$ (this is ensured by our assumption on $a$), we conclude that $\phi(s) \geq 0$ for $s \in [\bar{s}, s^{ft}]$.

Moving now to the first phase (i.e., $s < \bar{s}$), our conjecture $y < 0$ implies by the CS conditions that

$$\phi(s) = 0 \quad (24)$$

Moreover, $t(s) = t^\Psi(x^z(s))$ implies $\Psi_t = 0$, and hence from 14 we get $\lambda_y = 0$ and hence $\lambda_y(\bar{s}) = 0$. The second Euler equation (17) is trivially satisfied with

$$\lambda_y(s) = 0 \quad (25)$$
and $\phi = 0$. To check the first Euler equation (16) we use $\beta = -\lambda_x$ and $\lambda_y = 0$ to obtain:

$$\dot{\lambda}_x = -e^{-\rho s}\Psi_x + \lambda_x z$$

Since $\Psi_x$ is evaluated at $x = x^z(s)$ and $t^{\Psi}(x^z(s))$ then $\Psi_x$ is just a function of time, $s$, hence we can write $\Psi_x(s)$. Solving the above differential equation subject to some $\lambda_x(\tilde{s})$ yields:

$$\lambda_x(s) = \lambda_x(\tilde{s})e^{-z(\tilde{s}-s)} + \int_{\tilde{s}}^{s} \Psi_x(v)e^{-z(v-s)-\rho u} dv \quad (26)$$

We must now check that $\lambda_x(s) \leq 0$, so that $\beta(s) \geq 0$. We know from 22 that $\lambda_x(\tilde{s}) \leq 0$. Thus, it is sufficient to establish that $\Psi_x(s) \leq 0$ for all $s \in [0, \tilde{s}]$. We need to do this for the case of positive contributions ($t^{\Psi}(x) > t^W(x)$) and the case of zero contributions ($t^{\Psi}(x) \leq t^W(x)$).

If there are no contributions, (using $W_x + W^*_x = (x^{ft} - x)/2$) then

$$\Psi_x = -(a/2)(x - x^{ft}) + \delta(p - 1 - x/2)$$

Since $x > x^{ft}$ then the first term is negative. To show that the second term is also negative, note that there are two cases: (1) $t^\Psi(x) \leq t^{er}(x) \leq t^W(x)$ and (2) $t^\Psi(x) \leq t^W(x) \leq t^{er}(x)$. In case (1) $p(t^{er}(x), x) = 1$, which implies that $p(t^\Psi(x), x) < 1$. In case (2) we would have $x > x^W$, which implies that $p(t^W(x), x) < 1$ and hence $p(t^\Psi(x), x) < 1$. Thus, the second term is negative.

If there are positive contributions, then

$$\Psi_x = a(W_x(t^W, x) + W^*_x(t, x)) + \delta(p - c - 1 + x(p_x - c_x))$$

Given that $t^{er}(x) > t^\Psi(x) > t^W(x)$, then $x > x^W$ and consequently $W_x(t^W, x) < 0$, as we showed in the proof of Proposition 5. $W^*_x$ is always negative. $p - c - 1$ is zero at $t^{er}(x)$, hence it must be negative at $t^\Psi(x)$ given that $t^\Psi(x) < t^{er}(x)$. Hence, it suffices to show that $p_x - c_x < 0$ when evaluated at $t^\Psi(x)$. But $p_x - c_x = -1/2 - (1/b)[C_x - C/C]$. Since $C_x = (a/4)(t^\Psi - t^W) > 0$, then it is sufficient to establish that $1/2 - c(t^\Psi(x), x)/x > 0$. But in the proof of Proposition 1 we already established that $1/2 - c(t^j(x), x)/x > 0$. Given that $c(t^j(x), x) > c(t^\Psi(x), x)$, then this last inequality implies the previous one.

We have established that the conjectured solution $x = x^z(s)$ and

$$t = \begin{cases} 
   t^\Psi(x^z(s)) & \text{for } s \leq \tilde{s} \\
   t^{er}(x^z(s)) & \text{for } s > \tilde{s} 
\end{cases}$$
together with the implied state variable $y$ and costate variables $\lambda_x$ and $\lambda_y$ given by 26, 22, 20 and 25, 21, 18 in phases 1, 2, 3, respectively, and Kuhn-Tucker multipliers $\beta = -\lambda_x$ and $\phi$ given by 24, 23, and 19 in phases 1, 2, and 3, respectively, satisfy all the necessary conditions for an optimum. We now show that the conditions for sufficiency are also satisfied.

We need to show that the maximized Hamiltonian is concave in $(x, y)$. The maximized Hamiltonian is:

$$H^0(u(x, y), t(x, y), x, y, \lambda_x, \lambda_y, s) = e^{-\rho \phi} \Psi(t(x, y), x) - \lambda_x zx + \lambda_y [(\rho^L + z)y - g(t(x, y), x)]$$

Clearly, it is sufficient to show that $d^2H^0/dx^2 < 0$.

Let’s first analyze this in the first phase, where $t(x, y) = t^\Psi(x)$, $\Psi_t = 0$ and $\lambda_y = 0$. Differentiating and using $dt^\Psi/dx = -\Psi_{xt}/\Psi_{tt}$ yields

$$d^2H^0/dx^2 = (e^{-\rho \phi}/\Psi_{tt}) \left( \Psi_{xx}\Psi_{tt} - \Psi_{xt}^2 \right)$$

The SOC for $t^\Psi(x)$ requires that $\Psi_{tt} < 0$. Hence $d^2H^0/dx^2 < 0$ if and only if $\Psi_{xx}\Psi_{tt} - \Psi_{xt}^2 > 0$, which is a condition for $\Psi$ to be concave in $(x, t)$ at $(x^\psi(s), t^\Psi(x^\psi(s)))$.

We have to consider separately the cases in which there are positive and zero contributions. For the case with no contributions, we have $\Psi_{tt} = -a/2 < 0$ and

$$\Psi_{xx}\Psi_{tt} - \Psi_{xt}^2 = (1/4) \left[ a^2 + 2\delta a - \delta^2 \right]$$

Since $\delta < a$ then $\delta a > \delta^2$, hence the above expression is positive.

Now let’s consider the case with positive contributions. This necessarily implies that $\delta > 1/2$ and $a > \delta/(2\delta - 1)$. Differentiation yields

$$\Psi_{tt} = -a/2 + 3a/4 - 3\delta a/4 = (a/2)(3/2 - 1) - 3\delta a/4 = (a/4)(1 - 3\delta)$$

which is negative given that $\delta > 1/2$, and

$$\Psi_{xx}\Psi_{tt} - \Psi_{xt}^2 = (a/48)(5a + 12\delta + \delta a)(3\delta - 1) - [\delta/2 + (1 - \delta)(a/4)]^2$$

Some algebra shows that this expression is equal to

$$(a^2/12)(5\delta - 2) + (\delta/4)(2\delta - 1)(a + (a - \delta/(2\delta - 1)))$$
which is positive given $\delta > 1/2$ and $a > \delta/(2\delta - 1)$ (again, this last inequality must hold when there are positive contributions, given our assumptions).

Now let’s move to the second and third phases, where $t = t^{er}(x^z(s))$ and $g = 0$. We have:

$$d^2 H_0/dx^2 = e^{-\rho\delta} [\Psi_{tx} dt^{er}/dx + \Psi_t d^2 t^{er}/dx^2 + \Psi_{xx}]$$

We then need to show that the expression in the square parenthesis is negative. Since there are no contributions in the second and third phases, then $dt^{er}/dx = 1$, and hence

$$d^2 H_0/dx^2 = -e^{-\rho\delta} [a/2 + \delta/2]$$

which is clearly negative. Q.E.D.

**Proof of Proposition 8:** First notice that $\Omega(t^{er}_N(x), x)$ is decreasing. This follows simply from noting that:

$$\Omega(t^{er}_N(x), x) = a W(t^{er}_N(x), x) + a W^*(t^{er}_N(x), x) + \delta \hat{t}$$

which is obviously decreasing in $x$. This implies that the point $(t^{er}_N(x_z), x_z)$ is preferable to any point with $x > x_z$ along the ER curve with no contributions. Thus, if $t^{\Omega}(x_z) > t^{er}_N(x_z)$ then the optimal exact tariff is $t = t^{er}_N(x_z)$. But if $t^{\Omega}(x_z) < t^{er}_N(x_z)$ then by definition $\Omega(t, x)$ increases as we go down the vertical line at $x_z$ from $t^{er}_N(x_z)$ to $t^{\Omega}(x_z)$. Q.E.D.
References


Figure 1: The Long Run Non-Cooperative Equilibrium
Figure 2: The Trade Agreement with Perfect Capital Mobility

\[
W(x) \times t_j^W(0; x) \times \xi^W \times \xi^A \times \text{er}(t)
\]
Figure 3: The Trade Agreement with Imperfect Capital Mobility
Figure 4: The Trade Agreement when the Government has Full Bargaining Power