# Homework 2 Answers

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Question 1. Specific Factors and Trade

*Finland is capital abundant relative to potential trading partners in the rest of the world. Telecommunications is a capital intensive industry relative to the business services industry. Businesses in Finland have made investments in both industries, creating stocks of capital that are devoted to either telecommunications or business services.*

*(a) Suppose that Finland is not trading. Draw the specific factors diagram for Finland, indicating how labor is divided between the two industries, and showing the prevailing wage $w_0$.*

$K_{BS}$ – capital: specific to business services
$K_T$ – capital: specific to telecommunications sector
$L$ – labor: mobile factor

$P_{BS}$ – price of business services
$P_T$ – price of telecommunications

$(K_F/L_F) > (K_{ROW}/L_{ROW})$ – Finland (F) is capital abundant relative to potential trading partners in the Rest of the World (ROW) therefore $(P_T/P_{BS})_F < (P_T/P_{BS})_{International} < (P_T/P_{BS})_{ROW}$

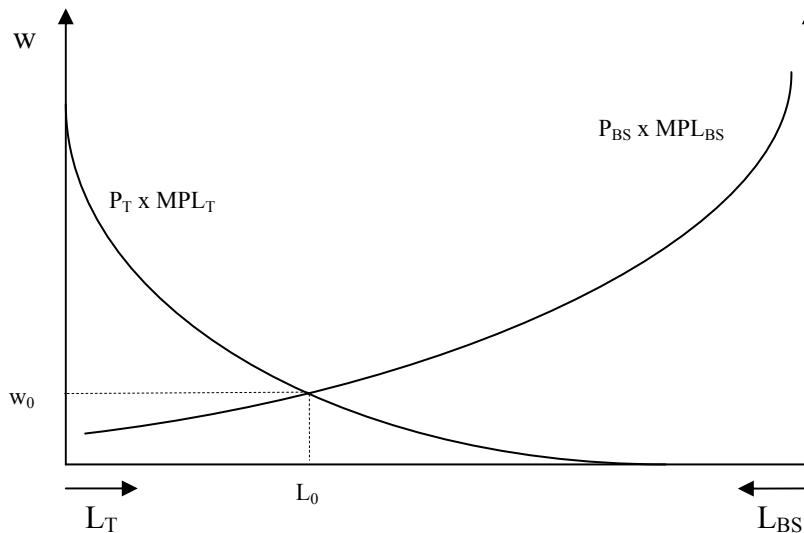$(K_T/L_T) > (K_{BS}/L_{BS})$ – Telecommunications is a capital intensive industry relative to the business services industry



**Figure 1**

In Figure 1, we see the initial allocation of labor $L_0$ and initial wage $w_0$. The Business services sector has relatively more labor employed because it is a labor intensive industry in comparison to Telecommunications which as stated in the problem is a capital intensive industry.
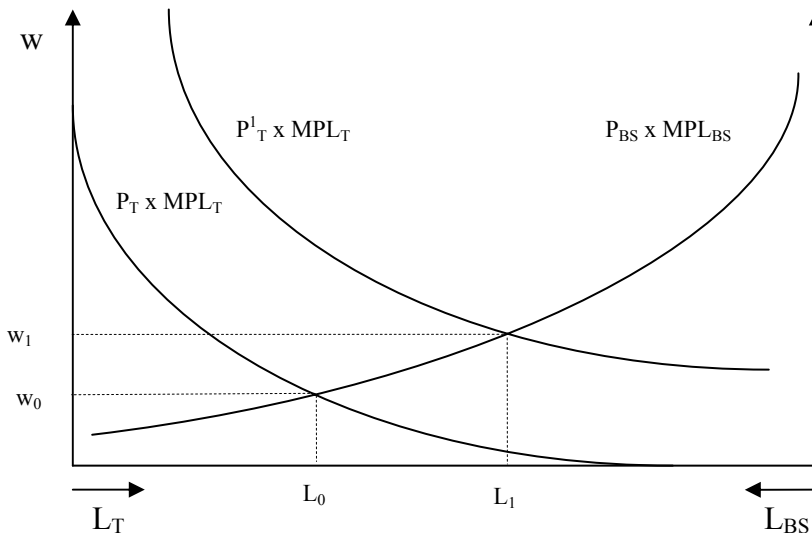
**Figure 2**

From Figure 2 we can see the effects of Finland opening up to trade. In first place, the price of telecommunications ($P_T$) will increase given that Finland will now face a World demand for Telecommunications instead of only the domestic one. The increase in $P_T$ will cause the Value of Marginal Product of Labor in the Telecommunications sector to shift to the North-East (up and to the right). The shift in this curve will cause the allocation of labor in both sectors to change. The employment of labor in the Telecommunications sector will increase and employment in the Business services sector to decrease since there is a limited amount of total labor available.

(c) *How does the opening of trade affect capital owners in Finland's telecommunications industry? Describe and show in your graph.*

The shift of the Value of Marginal Product of Labor in Telecommunications, described in question b, will cause the following changes in the marginal productivities of capital in the Telecommunications and Business Services, respectively.

$MPK_T$ will increase (i.e. shift up since there are now more workers per unit capital)
$MPK_{BS}$ will decrease (i.e. shift down since there are now less workers per unit land)

Nominal returns to capital in the Telecommunications sector, $r_T$:

$r_T = P_T \times MPK_T$ (assuming perfectly competitive markets)
$P_T$ increases due to the opening up to trade, this provides the first shift of the curve, and then $MPK_T$ increases, a second shift of the curve, this only means that $r_T$ must increase.

Real returns to capital in Telecommunications sector:

$r_T/P_T = MPK_T$: increased since $MPK_T$ in addition we can say the increase in $r_T$ has been greater than the increase in $P_T$
$r_T/P_{BS}$ has increased because $r_T$ has increased and $P_{BS}$ has remained constant.

Since returns have increased in terms of both goods, real returns to capital in the Telecommunications sector have risen unambiguously.

Nominal returns to capital in business services, $r_{BS}$:

$r_{BS} = P_{BS} \times MPK_{BS}$ (assuming perfectly competitive markets)
Since $P_{BS}$ has remained constant and $MPK_{BS}$ has fallen, $r_{BS}$ must fall.

Real returns to capital in business services:
$r_{BS}/P_{BS} = MPK_{BS}$: decreased since $MPK_{BS}$ has decreased
$r_{BS}/P_T$ has decreased because $r_{BS}$ has decreased and $P_T$ has increased
Since returns have fallen relative to both goods, real returns to capital in the business services sector have fallen unambiguously.
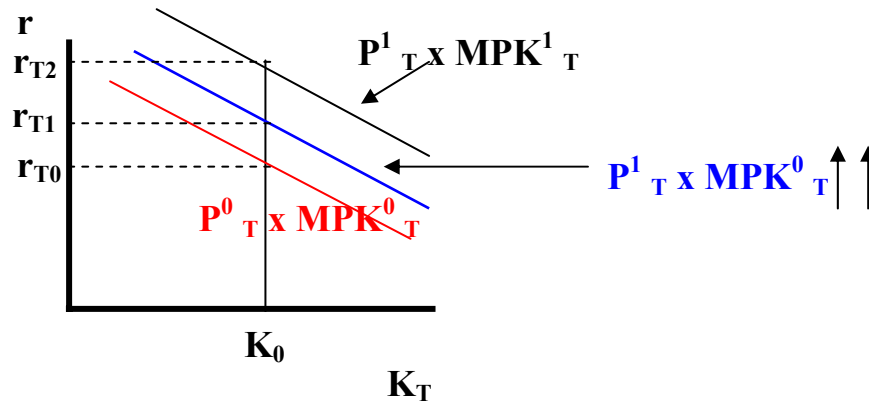


**Figure 3**

Figure 3 above presents the outcome for capital owners in the Telecommunications sector in Finland.
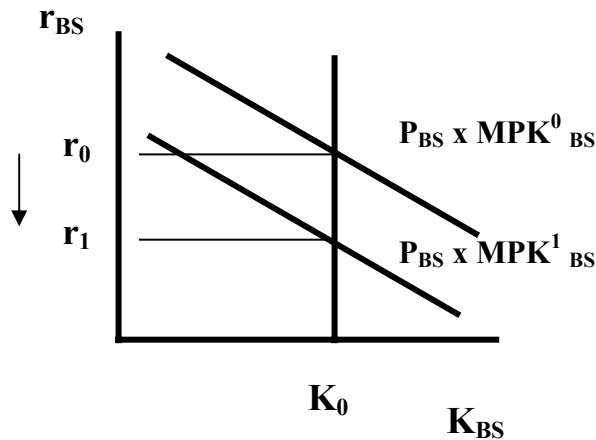


**Figure 4**

Figure 4 above presents the outcome for the capital owners in the business services sector.

(d)   Are Finnish workers likely to benefit or lose from the opening of trade?


From Figure 2 we can see:
Since $L_{BS}$ decreased, equilibrium $MPL_{BS}$ increases.
Since $L_T$ increased, equilibrium $MPL_T$ decreases.

Therefore:
$w/P_{BS} = MPL_{BS}$ has increased
$w/P_T = MPL_T$ has decreased

The returns have fallen relative to Telecommunications and risen relative to business services. The overall impact on real returns to labor is ambiguous.


(e)   Suppose Finnish workers consumer as much telecommunications as they can, while they buy very few business services. How does this affect the magnitude of worker gains or losses?

If we assume that most of labor's consumption is on Telecommunications, then it is likely that real returns to labor decrease, although it is still ambiguous.


*Question 2. The HO Framework*

(a)   *Mexico produces two goods, GM cars and corn.  There are two factors of production, labor and capital.  Total available hours of labor and capital are 100 labor and 100 capital.  The unit labor requirements for the production of each good are given below.  What does the production possibility frontier look like?*


|  | *Labor Requirement (Hours)* | *Capital Requirement (Hours)* |
|---|---|---|
| *GM Cars* | *5* | *20* |
| *Corn* | *10* | *5* |

Since we are given unit requirements (fixed) rather than a function specifying marginal productivities (variable), we know that the PPF will be linear with a kink. The boundaries of the PPF will be determined by the line showing full employment of labor (the labor constraint) and the line showing full employment of capital (the capital constraint).

Labor constraint:
Intercept car axis where all labor in car production → cars = $L/a_{LC}$ = 100/5 = 20
Intercept corn (food) axis where all labor in corn production → corn = $L/a_{LF}$ = 100/10 = 10

Capital constraint:
Intercept car axis where all capital in car production → cars = $L/a_{KC}$ = 100/20 = 5
Intercept corn (food) axis where all capital in corn production → corn = $L/a_{KF}$ = 100/5 = 20
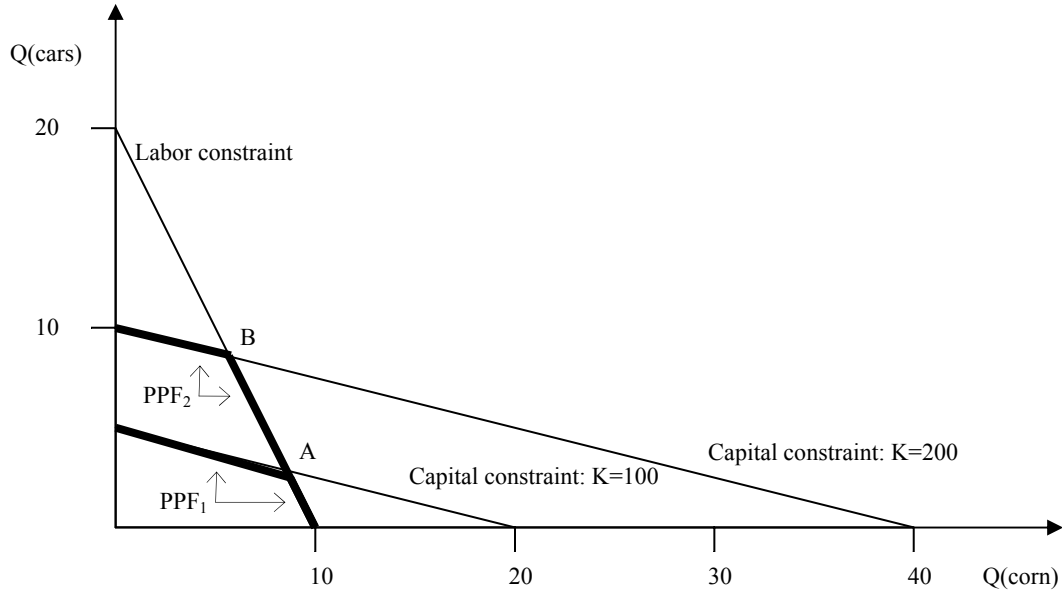
**Figure 5**

The PPF looks like PPF$_1$ in Figure 5.

(b) *Following NAFTA, Mexico receives an influx of foreign direct investment from the US, raising the available pool of capital from 100 to 200. What happens to the relative production of GM cars and corn? Please show graphically.*

The capital constraint will shift out. Production will move from point A to point B on Figure 5. The production of cars will increase and the production of corn will decrease. Therefore, the relative production of cars will increase.

(c) *What is likely to happen to the pattern of exports in Mexico (of cars versus corn)? How is this related to the Heckscher-Ohlin Theorem?*

We do not know which way trade flows go unless we know the endowments of a trading partner. Depending on how relatively well endowed with capital Mexico's trading partners are, different things may happen. However, it is likely that relative to at least some of its trading partners, Mexico will now become relatively well endowed with capital. This means that Mexico may switch from exporting corn to exporting cars to these partners. This is an example of the Heckscher-Ohlin Theorem because Mexico will export the good (cars) that uses its newly relatively more abundant factor (capital) more intensively. Note that we can see that cars use capital relatively more intensively because $(K/L)_{cars} = 20/5 = 4 > (K/L)_{corn} = 5/10 = \frac{1}{2}$.

(d) *If the United States continues to lose capital to Mexico, what will happen to its pattern of exports?*

The pattern of exports for the United States will be such that the country will export the good that uses the factor that is relatively more abundant. If the United States keeps losing capital to Mexico so that it becomes a labor abundant country then it is highly likely that it will export corn and import GM cars.

*Assume Mexico has 100 labor and 200 capital. Using your knowledge of Stolper-Samuelson, what will happen to the returns to wages and capital in Mexico after trade liberalization? What if the endowment of labor and capital is reversed?*

Again, if we are assuming that Mexico is relatively well endowed with capital when compared to its trading partner, then the Stopler-Samuelson theorem says that real wages will decrease and the real return to capital will increase with trade.

*Question 3. More on the Specific-Sector Model and Trade*

*Home can produce machinery and flowers in bundles of 1000. The production of the two industries are given by $Qm=K^{1/2}L_m^{1/2}$ and $Qf=T^{1/2}L_f^{1/2}$. K is capital, T is land, and L is labor, Prices of machinery Pm and of food Pf are both equal to 1. Factor supply is Lm + Lf = 100 and T=K=100.*

(a) *Derive the marginal products of labor MPLm and MPLf for the two industries.*

The Marginal Product of Labor is the contribution to output of the last worker hired. In mathematical terms we define the Marginal Product of Labor as the derivative of the production function with respect to labor. In this case the Marginal Product of Labor for machinery and flowers are the following, respectively:

The Marginal Product of Labor in the machinery sector is the following: $\dfrac{\partial Q_m}{\partial L_m}=\dfrac{1}{2}L_m^{-\frac{1}{2}}K^{\frac{1}{2}}$ and the

Marginal Product of Labor in the flower sector is the following: $\dfrac{\partial Q_f}{\partial L_f}=\dfrac{1}{2}L_f^{-\frac{1}{2}}T^{\frac{1}{2}}$

(b) *Autarky wage: graph the labor demand curves in the machinery and flower sectors, and calculate the equilibrium wage rate in autarky.*
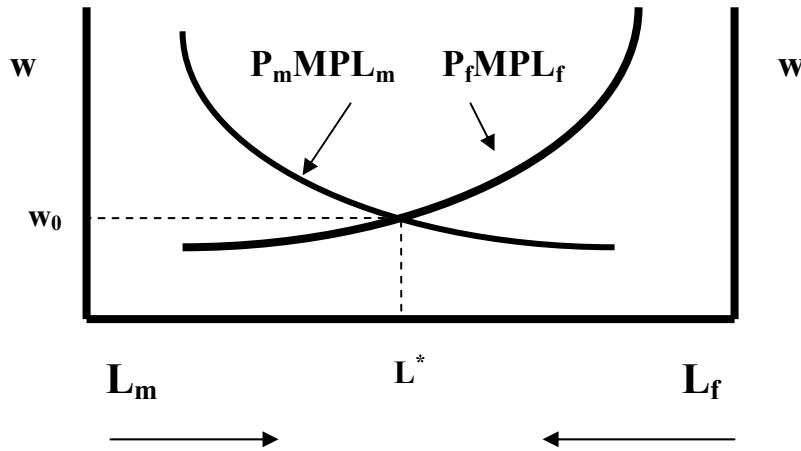


**Figure 6**

We are given the following information to calculate the initial wage level in autarky and the labor allocations: *Lm + Lf = 100; T=K=100.* We know that nominal wages have to equal the Value of the Marginal Product of Labor in machinery and also equal the Value of the Marginal Product of Labor in flowers. Building those conditions.

$$w=\dfrac{1}{2}L_m^{-\frac{1}{2}}K^{\frac{1}{2}} \text{ and } w=\dfrac{1}{2}L_f^{-\frac{1}{2}}T^{\frac{1}{2}}$$

Set them equal to each other and solve for $L_m$ or $L_f$.

$$\frac{1}{2}L_m^{-\frac{1}{2}}K^{\frac{1}{2}} = \frac{1}{2}L_f^{-\frac{1}{2}}T^{\frac{1}{2}}$$

After some algebra we are able to determine the following relationship:

$$\frac{L_f}{L_m} = \frac{T}{K}$$

Substitute in the values of $T$ and $K$ and to obtain $Lm=50$ and $Lf=50$. Substitute these values into the Value of Marginal Product of Labor in machinery or into the Value of Marginal Product of Labor of flowers to obtain wages.

$$w = \frac{\sqrt{2}}{2}$$

(c) The pattern of trade: after opening up to free trade, home faces a relative price of $Pm/Pf = 2$. How do the allocation of labor and wages change?

When the ratio of relative prices changes to $Pm/Pf = 2$ labor allocations change because producers realize the production of machinery is more profitable than the production of flowers. There is a need for re-optimization, i.e. set the new value of marginal products equal to each other and solve for $L_m$ and $L_f$. Proceeding in such manner and after some algebra we obtain the following relationship:

$$\frac{4L_f}{L_m} = \frac{T}{K}$$

Substitute the values for T and K and solve for $L_m$ and $L_f$. Therefore, $Lm=80$ and $Lf=20$. We obtain wages by substituting the new values for the labor allocations into the Value of Marginal Product for machinery or into the Value of Marginal Product for flowers. The new level of wages is equals to the following:

$$w = \frac{\sqrt{5}}{2}$$

(d) Using the general demand relationships for the two sectors, show that the production possibilities frontier is given by $-MPLf/MPLm = -Pm/Pf$

There are several ways to approach this question. One way to show the relationship posed above is to equate the Value of Marginal Products and play with those expression to obtain $-MPLf/MPLm = -Pm/Pf$. In this case, we would go about in the following way:

$$w = P_m MPL_m$$
$$w = P_f MPL_f$$
$$P_m MPL_m = P_f MPL_f$$
$$\frac{P_m}{P_f} = \frac{MPL_f}{MPL_m}$$

It is also possible to maximize a Value function of the following type:

*Max*

$$V = P_m Q_m + P_f Q_f$$

$$Q_f = \frac{V}{P_f} - \frac{P_m}{P_f} Q_m$$

From the above expression we see that the slope at the optimal value is equal to $\frac{P_m}{P_f}$

(e) *Draw the production possibilities frontier. How does the change in relative prices after trade affect production? Depict the gains from trade.*
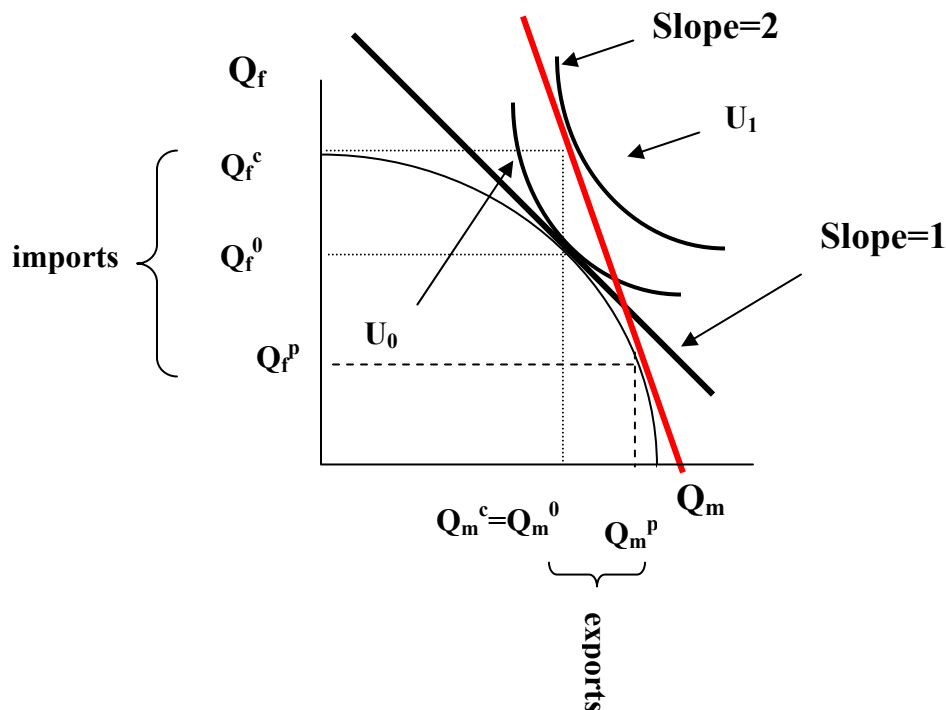


**Figure 7**

Figure 7 presents the Production Possibilities Frontier and the initial autarky point as $Q_m^0$ and $Q_f^0$ where production and consumption in the closed economy occur. We see that the ratio of relative prices is equal to 1 at that point and the economy has a utility level of $U_0$. Once the economy opens up to trade, the ratio of relative prices increases to 2. Therefore, we observe a shift in the slope of the budget line, i.e. it becomes steeper to denote the increase in the ratio of relative prices. Once the economy starts trading, observe that production and consumption decisions become disjoint, i.e. the production point is different from the consumption point. The production point when the economy is trading is denoted $Q_m^p$ and $Q_f^p$. The economy now produces more machinery and less flowers. This is a result of the increase in the ratio of relative prices. Producers realize that machinery is more profitable and therefore want to increase production of that good. This implies a reduction in the production of flowers. On the other hand, the consumption point is denoted by $Q_m^c$ and $Q_f^c$. We observe that at the consumption point the economy attains a utility level of $U_1$ which is greater than $U_0$. Therefore, this country is better off. Finally, we see this country is exporting machinery. The amount of exports is denoted by the difference between $Q_m^p$ and $Q_m^c$. Likewise, the amount of imports of flowers is given by the difference between $Q_f^c$ and $Q_f^p$.

*Question 4. The HO Framework.*

*We are given the following input requirements to produce food and computers:*

| | Labor To produce One Unit | Capital to Produce One Unit |
|---|---|---|
| *Food* | $a_{LF} = 6$ | $a_{KF} = 3$ |
| *Computers* | $a_{LC} = 4$ | $a_{KC} = 10$ |

*Chile is well endowed with labor, and consequently has a comparative advantage in labor-intensive goods. Chile is considering joining a free trade area with the United States. Prior to joining the free trade area, the price of food (Pf) is 15 and the price of computers (Pc) is 42. After joining the free trade area, the price of food is 21 and the price of computers in Chile is 30.*

(a) *What happens to the returns to labor and capital in Chile after it joins the free trade area? Calculate exactly using equations please, not just diagrams or qualitative answers.*

The relevant equations here are the zero profit conditions for before and after trade. They are:

$$P_c = w \times a_{LC} + r \times a_{KC} \qquad (1)$$
$$P_F = w \times a_{LF} + r \times a_{KF} \qquad (2)$$

Substituting the numbers into equations (1) and (2) before trade we get:

| | | | |
|---|---|---|---|
| $15 = 6w + 3r$ | $\rightarrow$ | $r = 5\text{-}2w$ | from (1) |
| $42 = 4w + 10r$ | | | from (2) |

Which we solve to give $r = 4$ and $w = 1/2$

Substituting the numbers into equations (1) and (2) after trade we get:

| | | | |
|---|---|---|---|
| $21 = 6w + 3r$ | $\rightarrow$ | $r = 7\text{-}2w$ | from (1) |
| $30 = 4w + 10r$ | | | from (2) |

Which we solve to give $r = 2$ and $w = 5/2$

Therefore nominal returns to labor (w) increase, and to capital (r) decrease.

Real returns to labor:
$w/P_C$ increases because w increases and $P_C$ decreases.
$w/P_F$ ? $w^2/P^2_F > w^1/P^1_F$ if and only if $w^2/w^1 > P^2_F /P^1_F$
substituting our before and after trade values we see that $w^2/w^1 = 5 > 7/5 = P^2_F /P^1_F$
So after trade, w has risen relatively more than the price of food, thus real returns have increased in terms of food as well. **So real returns to labor have increased unambiguously.**

Real returns to capital:
$r/P_F$ decreases because r decreases and $P_F$ increases
$r/P_c$ ? $r^2/P^2_c < r^1/P^1_c$ if and only if $r^2/r^1 < P^2_c /P^1_c$
substituting our before and after trade values we see that $r^2/r^1 = 1/2 < 5/7 = P^1_c /P^2_c$
So after trade, r has fallen relatively more than the price of computers, thus real returns have decreased in terms of computers as well. **So real returns to capital have decreased unambiguously.**

(b) *Which theorem is consistent with this result? Why or why not?*

This result is consistent with the Stolper –Samuelson Theorem because opening up to trade leads to an increase in the return to Chile's abundant factor (labor) and a fall in the return to its scarce factor (capital).