Answers to 280C Problem Set 2

(a) We calculate utility as a function of $E$ and $W$ by maximizing $C_H^\gamma C_F^{1-\gamma}$ subject to the constraint $W = C_H + EC_F$. From the first-order condition

$$\frac{(1 - \gamma)C_H^\gamma C_F^{-\gamma}}{\gamma C_H^{\gamma-1}C_F^{1-\gamma}} = E,$$

implying that

$$\frac{1 - \gamma}{\gamma} \left( \frac{W - EC_F}{C_F} \right) = E$$

and thus that $C_F = (1 - \gamma)W/E$, we also derive (using the first-order condition above) that $C_H = \gamma W$. Accordingly utility can be expressed as

$$U = (1 - R)^{-1} \left[ \gamma^\gamma (1 - \gamma)^{1-\gamma} \frac{W}{E^{1-\gamma}} \right]^{1-R}.$$

(b) Straightforward.

(c) Substitution yields

$$EU = \gamma^\gamma (1 - \gamma)^{1-\gamma} (1 - R)^{-1} \mathbb{E} \left[ \frac{(1 + i)I + (E/E_0)(1 + i^*)(W_0 - I)}{E^{1-\gamma}} \right]^{1-R}.$$

Differentiation with respect to $I$ yields:

$$\frac{dEU}{dI} = \gamma^\gamma (1 - \gamma)^{1-\gamma} \mathbb{E} \left\{ \left( \frac{W}{E^{1-\gamma}} \right)^{-R} \cdot \frac{1}{E^{1-\gamma}} \cdot \left[ (1 + i) - \frac{E}{E_0} (1 + i^*) \right] \right\} = 0.$$

This reduces to the expression in equation (0.2).

(d) Since $E = \rho E_0 (1 + i)/(1 + i^*)$, we can pull out in front the factor $1 + i$ and express equation (0.2) as

$$(1 + i) \left[ \frac{E_0 (1 + i)}{(1 + i^*)} \right]^{-(1-\gamma)(1-R)} \mathbb{E} \{ W^{-R} \rho^{-(1-\gamma)(1-R)} (1 - \rho) \} = K \mathbb{E} \{ W^{-R} \rho^{-(1-\gamma)(1-R)} (1 - \rho) \} = 0.$$
(e) Let \( \tilde{K} \equiv \gamma(1 - \gamma)^{1-\gamma} K \). Calculate that, at the portfolio optimum,

\[
\frac{d}{d\gamma} \left( \frac{d \mathbb{E}U}{dI} \right) = \tilde{K} \cdot \frac{d}{d\gamma} \mathbb{E} \left\{ W^{-R} \rho^{-(1-\gamma)(1-R)} (1 - \rho) \right\} \\
+ \mathbb{E} \left\{ W^{-R} \rho^{-(1-\gamma)(1-R)} (1 - \rho) \right\} \cdot \frac{d\tilde{K}}{d\gamma} \\
= \tilde{K} \cdot \frac{d}{d\gamma} \mathbb{E} \left\{ W^{-R} \rho^{-(1-\gamma)(1-R)} (1 - \rho) \right\} + 0 \cdot \frac{d\tilde{K}}{d\gamma} \\
= \tilde{K} \cdot \mathbb{E} \left\{ (1 - \rho) W^{-R} \frac{d}{d\gamma} \left[ \rho^{-(1-\gamma)(1-R)} \right] \right\} \\
= \tilde{K} \cdot \mathbb{E} \left\{ (1 - \rho) W^{-R} (1 - R) \rho^{-(1-\gamma)(1-R)} \log(\rho) \right\}.
\]

Observe that above, \((1 - \rho) \log \rho\) always is nonpositive, so that the expected derivative above will have the same sign as \( R - 1 \). Thus,

\[
\frac{d^2 \mathbb{E}U}{d\gamma dI} > 0 \text{ iff } R > 1.
\]

Generally speaking, economists view \( R > 1 \) as more plausible than \( R < 1 \), so that greater preference for home goods would, in the preceding framework, increase the demand for home bonds. In more detail: If \( R > 1 \), this means that if one is initially at a portfolio optimum with \( d\mathbb{E}U/dI = 0 \) and then \( \gamma \) rises (higher weight on domestic goods in expenditure), the incentive to raise investment in the home bond rises from zero to a positive number.