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Forces limiting the extent to which sophisticated investors are willing to make trades that move asset prices back toward fundamentals

As described in lecture last week, researchers have identified 3 factors that limit the extent to which sophisticated investors are willing to buy assets that are undervalued relative to fundamentals, and sell (or sell short) assets that are overvalued relative to fundamentals: fundamental risk, noise-trader risk, and performance-based risk.[1] What follows is a simple model, based loosely on DeLong, Shleifer, Summers and Waldmann (1990) and Shleifer and Vishny (1997), that captures these ideas.

Assumptions

There are three periods, denoted 0, 1, and 2.

There are two assets:

The first is a safe asset in perfectly elastic supply. For simplicity, its rate of return is normalized to zero. Thus, one unit of the economy's single good in period 0 can be invested in a way that yields one unit of the good for sure in period 1; likewise, one unit of the good invested in this asset in period 1 yields one unit for sure in period 2.

The second is a risky asset. Its payoff, which is realized in period 2, is $1 + F_1 + F_2$, where F_t is distributed normally with mean 0 and variance V_t^F. F_1 is observed in period 1, and F_2 is observed in period 2.

There are two types of traders:

The first type are the source of shocks that potentially move asset prices away from their fundamental values. To capture this idea, we introduce some traders who buy or sell the risky asset for random reasons unrelated to anything else happening in the economy. Their actions convey no information about fundamental values, so if the forces pushing asset prices toward fundamentals were strong enough, the actions would not affect prices. These traders are referred to as "noise traders."

The noise traders demand quantity N_0 of the asset in period 0, and $N_0 + N_1$ in period 1, where N_t is distributed normally with mean 0 and variance V_t^N. F_1, F_2, N_0, and N_1 are independent.

The other traders are sophisticated traders. These traders maximize expected utility and have rational expectations. A_0 sophisticated traders are born in period 0, and A_1 are born in period 1, where A_0 and A_1 are exogenous and certain.

[1] A potential fourth factor is "model-based risk": sophisticated investors cannot be certain that their estimates of fundamental values are in fact the best estimates given the available information. Modeling model-based risk raises deep and hard issues, so we will not pursue it.

Sophisticated traders live for two periods (0 and 1 for the ones born in period 0; 1 and 2 for the ones born in period 1). They care only about consumption in the second period of their life, and have constant absolute risk aversion (CARA) utility, $U(C) = -e^{-2\gamma C}$, $\gamma > 0$. Each sophisticated investor has wealth W at birth. Sophisticated investors act as price-takers.

For simplicity, the supply of the risky asset is normalized to zero. Thus, letting Q_t denote each sophisticated investor's holdings of the risky asset in period t, equilibrium in the market for the risky asset requires $N_0 + Q_0A_0 = 0$ in period 0, and $N_0 + N_1 + Q_1A_1 = 0$ in period 1. (In period 2, the payoff to the asset is realized and the holders consume the proceeds; there are no trades.)

<u>Preliminaries</u>

1. What are fundamental values in this economy? If there were any risk-neutral agents who could buy and sell in unlimited quantities, the price of the risky asset would have to be 1 in period 0, and $1 + F_1$ in period 1. Call these prices P_0^* and P_1^*. If the price in period t were less than P_t^*, selling one unit of the safe asset and investing the proceeds in the risky asset would raise expected consumption; if the price were more than this, selling one unit of the risky asset and investing the proceeds in the safe asset would raise expected consumption.

In the absence of noise traders, the price of the risky asset would again be P_0^* and P_1^* in the two periods. At those prices, the expected rate of return on the risky asset would be the same as that on the safe asset. Thus, sophisticated investors would not want to hold either positive or negative quantities of the asset. Since the supply of the asset is zero, the market would clear.

Thus, fundamental values are P_0^* and P_1^*. Our question concerns departures of actual prices from these values.

2. Before proceeding, it useful to say more about the behavior of the sophisticated investors. We will see that because the underlying shocks are normally distributed, each sophisticated investor's consumption will be normally distributed. And recall the rule for the mean of a variable that is distributed lognormally: if x is distributed normally with mean μ and variance V, $E[e^x] = e^{\mu}e^{V/2}$. Thus if the investor's consumption is distributed normally with mean E[C] and variance Var(C), the expectation of $-e^{-2\gamma C}$ is $-e^{-2\gamma E[C]}e^{2\gamma^2 Var(C)}$. To maximize expected utility, the investor will therefore want to make $-2\gamma E[C] + 2\gamma^2 Var(C)$ as small as possible. Equivalently, he or she will maximize $E[C] - \gamma Var(C)$.

<u>Equilibrium in period 1 and "fundamental risk"</u>

Recall that the condition for equilibrium in period 1 is $N_0 + N_1 + Q_1A_1 = 0$. The sophisticated investors born in period 1 care about consumption in period 2. The representative investor's period-2 consumption is their holdings of the safe asset, which are $W - P_1Q_1$, plus the product of their holdings of the risky asset, Q_1, and the payoff of each unit of the asset, $1 + F_1 + F_2$: $C = W - P_1Q_1 + Q_1(1 + F_1 + F_2)$. In period 1, F_1 has already been realized (and investors can observe the price of the asset, P_1). Thus, the expectation of their consumption given period-1 information is $W - P_1Q_1 + Q_1(1 + F_1)$, and its variance is $V_2^F Q_1^2$. Thus, the problem of the representative sophisticated investor in period 1 is

$$\max_{Q_1} W - P_1 Q_1 + Q_1(1 + F_1) - \gamma V_2^F Q_1^2.$$

The first-order condition for the investor's choice of Q_1 is therefore

$$-P_1 + (1 + F_1) - 2\gamma V_2^F Q_1 = 0.$$

Market-clearing requires $Q_1 = -(N_0 + N_1)/A_1$. Substituting this into the first-order condition and rearranging gives us:

(*) $$P_1 - (1 + F_1) = \left(\frac{2\gamma}{A_1}\right) V_2^F (N_0 + N_1).$$

The left-hand side of this expression is the departure of the price of the asset from its fundamental value. Consider the three terms on the right-hand side of this expression:

$N_0 + N_1$ is noise traders' demand for the asset. In this model, if agents enter the market and demand some of the asset for reasons unrelated to economic fundamentals, the price of the asset rises: without risk-neutral investors, prices can deviate from fundamentals.

V_2^F is the variance of fundamentals in period 2. What deters the sophisticated investors from fully eliminating the mispricing is that the realized value of the asset may differ from its expected value – that is, there is "fundamental risk." That is, *fundamental risk prevents sophisticated investors from taking infinite positions (and thereby eliminating departures of prices from fundamentals).*

$A_1/(2\gamma)$ is the "depth" of the market: when there are more sophisticated investors or they are less risk averse, prices depart less from fundamentals.

<u>Equilibrium in period 0 and "noise-trader risk"</u>

The period-1 consumption of a representative sophisticated investor born in period 0 is $W - P_0Q_0 + P_1Q_0$. Note that it depends not on the ultimate realization of the value of the risky asset, but on its price in period 1. The investor's expected consumption is therefore $W - P_0Q_0 + E[P_1]Q_0$, and the variance of his or her consumption is $Q_0^2 Var(P_1)$. Proceeding along similar lines as before, one can show that the resulting first-order condition for the investor's choice of Q_0 is:

(**) $$-P_0 + E[P_1] - 2\gamma Q_0 Var(P_1) = 0.$$

We can then use expression (*) above to find the mean and variance of P_1 given the information available at time 0 (that is, to find $E[P_1]$ and $Var(P_1)$). Substituting those expressions into (**) and then into the market-clearing condition, $Q_0A_0 + N_0 = 0$, and then performing algebra gives us an expression for the departure of the period-0 price of the asset from fundamentals:

$$P_0 - 1 = \left\{\frac{V_2^F}{A_1} + \frac{1}{A_0}\left[\left(\frac{2\gamma V_2^F}{A_1}\right)^2 V_1^N + V_1^F\right]\right\} 2\gamma N_0.$$

As before, fundamental risk causes the sophisticated investors to not fully undo the impact of the noise traders' actions on price, and so allows the price to depart from its fundamental value. The key new result, however, involves the V_1^N term. This term shows that the response of P_0 to the period-0 noise traders is larger when V_1^N is larger. Intuitively, the sophisticated investors in period 0 risk losses not only from the fact that the fundamental value of the risky asset is likely to change by the time they need to sell, but also from the fact that the difference between the actual value and the fundamental value is also likely to change. This makes them more reluctant to made trades to correct departures of the price from its fundamental value. That is, *the risk created by the possibility of future departures of prices from fundamentals magnifies those departures today – "noise traders create their own space."*

Adding a variation on performance-based risk – momentum traders

Shleifer and Vishny (1997) argue that the fact that most sophisticated investors use not just their own funds, but funds from less sophisticated investors, adds another reason that sophisticated investors are reluctant to bet against departures of prices from fundamentals. They argue that if the sophisticated investors' trades fail to be profitable in the short run, the outside funders are likely to withdraw their funds. Thus if a departure of prices from fundamentals gets larger in the short run – and so expected profits from wise investment strategies are especially high – sophisticated investors may be forced to take losses. This fear further tempers their willingness to made trades that would mitigate mispricings.

Adding such performance-based risk to the model in a tractable way turns out to be difficult.[2] Let us therefore consider a variant: adding some "return-chasers" or "momentum investors." These are investors who buy in period 1 if the price of the risky asset has risen, and sell if it falls. They therefore exacerbate the effects of further departures from fundamentals, and so, like performance-based risk, make sophisticated investors more wary of trading against departures from fundamentals.

Concretely, assume an additional component of the demand for the risky asset in period 1 that takes the form $M_1 = m[P_1 - E_0[P_1]]$. (Assuming that the momentum traders respond to $P_1 - E_0[P_1]$ rather than to $P_1 - P_0$ makes the algebra easier.)

With this change, expression (*) for P_1 becomes

$$P_1 - (1 + F_1) = \left(\frac{2\gamma}{A_1}\right) V_2^F (N_0 + N_1 + M_1).$$

Since $E_0[M_1] = 0$, $E_0[P_1] = (2\gamma/A_1)V_2^F N_0$. Thus,

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[2] The problem is that with the natural approaches to adding outside investors who follow performance-based rules, the consumption of the sophisticated investors is no longer normally distributed. As a result, the model can no longer be solved analytically.

$$P_1 - E_0[P_1] = F_1 + \frac{2\gamma}{A_1} V_2^F (N_1 + M_1)$$

$$= F_1 + \frac{2\gamma}{A_1} V_2^F N_1 + \frac{2\gamma}{A_1} V_2^F m[P_1 - E_0[P_1]\,].$$

Solving this expression for $P_1 - E_0[P_1]$ yields:

$$P_1 - E_0[P_1] = \frac{1}{1 - \frac{2\gamma V_2^F}{A_1} m} \left[F_1 + \frac{2\gamma}{A_1} V_2^F N_1 \right].$$

Note that the presence of the momentum traders makes the price of the asset more responsive to both F_1 and N_1. Thus, it increases the variance of P_1.

Using this expression to find the variance of P_1, solving the maximization problem of the period-0 sophisticated investors, and then substituting into the expression for market-clearing in period 0 leads (after lots of algebra!) to:

$$P_0 - 1 = \left\{ \frac{V_2^F}{A_1} + \frac{1}{1 - \frac{2\gamma V_2^F}{A_1} m} \left[\left(\frac{2\gamma V_2^F}{A_1} \right)^2 V_1^N + V_1^F \right] \right\} 2\gamma N_0.$$

The point of all this algebra is that the expression in curly brackets is larger when m is larger – that is, the presence of the momentum traders in period 1 increases the impact of the noise traders in period 0 on the price of the asset. Thus, *the presence of forces (performance-based evaluation or momentum traders) that magnify future departures of prices from fundamentals magnifies price departures today*.