UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2012
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Problem Set 5
Due in lecture Tuesday, November 6

1. According to the permanent-income hypothesis, if a consumer learns in period t that his or her income will be temporarily lower in period t+1, his or her consumption:

A. Falls permanently in period t.

B. Falls permanently in period t+1.

C. Falls in period t, remains at that lower level in period t+1, rises from that level in period t+2, and does not change after that.

D. Does not change.

E. None of the above.

2. (Consumption with state-contingent goods.) Consider a consumer whose labor income (which he or she takes as exogenous) is uncertain. Specifically, the consumer's labor income in state s in period t is Y_{st}. The probability that the state in period t is s is π_{st}. Thus, for each t, $\sum_s \pi_{st} = 1$. The realization of the state each period is independent of the realization in all other periods.

The consumer seeks to maximize $E\left[\sum_t \frac{1}{(1+\delta)^t} U(C_t)\right], U'(\cdot) > 0, U"(\cdot) < 0$. The consumer can purchase state-contingent goods and sell his or her state-contingent income. The price of consumption in period t in state s is p_{st}. Thus, we can write the consumer's objective function as $\sum_t \sum_s \pi_{st} \frac{1}{(1+\delta)^t} U(C_{st})$, and his or her budget constraint as $\sum_t \sum_s p_{st} C_{st} \leq \sum_t \sum_s p_{st} Y_{st}$.

a. Set up the consumer's maximization problem, and find the first-order condition for C_{st}.

b. Consider two states in some period t, s' and s''. Under what conditions is consumption the same in the two states? (That is, under what conditions is $C_{s't} = C_{s''t}$?)

c. Consider state s' in period t' and state s'' in period t''. Under what conditions is $C_{s't'} = C_{s''t''}$?

d. Consider 2 consumers who differ only in their Y_{st}'s. Show or provide a counterexample to the following claim: If Consumer 1's consumption in one period is greater than Consumer 2's consumption in that period, Consumer 1's consumption in each period is greater than Consumer 2's consumption in the same period.

e. Suppose that both consumers have constant relative risk aversion utility, with the same coefficient of relative risk aversion. What, if anything, can one say about how the ratio of Consumer 1's consumption to Consumer 2's consumption behaves over time?

f. In practice, we often see consumption reversals (that is, one consumer initially having consumption higher than another, but later having lower consumption). List 2 or 3 ways the assumptions of this problem could fail that could make such reversals possible; explain each possibility in no more than a sentence.

g. Suppose that in some period, the realization of s is the one that has the highest value of $p_{st}Y_{st}$ for that period for the consumer. How, if at all, will that affect the consumer's consumption in later periods?

3. Romer, Problem 8.5. (OVER)

4. Consider a conventional problem of a finitely-lived household choosing its path of consumption to maximize lifetime utility, with one change: the household's discount rate is not constant. Specifically, the household maximizes:

$$\int_{t=0}^{T} e^{-D(t)} u(C(t))dt, \;\; u'(\bullet) > 0, \;\; u''(\bullet) < 0,$$

where $D(t) = \int_{\tau=0}^{t} \rho(\tau)d\tau$. The household has initial wealth of $A(0)$, and its wealth evolves according to $\dot{A}(t) = r(t)A(t) + Y(t) - C(t)$, where $r(t)$ is the interest rate at t and $Y(t)$ is non-labor income (both of which the household takes as given). The household can borrow and lend, but it cannot die in debt. That is, its budget constraint is $A(T) \geq 0$.

This problem asks you to use optimal control to derive the household's Euler equation. Specifically:

a. Set up the present value Hamiltonian.

b. Find the conditions for optimality.

c. Use your results to find an expression for $\dot{C}(t)/C(t)$.

EXTRA PROBLEMS (NOT TO BE HANDED IN/ONLY SKETCHES OF ANSWERS WILL BE PROVIDED)

5. A consumer facing income uncertainty whose objective function is $E_0[\sum_{t=0}^{\infty} \beta^t U(C_t)]$ and who can borrow and lend at the risk-free interest rate r will satisfy:

A. $U'(C_t) = (1 + r)\beta U'(C_{t+1})$.
B. $U'(C_t) = (1 + r)\beta E_0[U'(C_{t+1})]$.
C. $E_0[U'(C_t)] = (1 + r)\beta E_0[U'(C_{t+1})]$.
D. $U'(C_t) = (1 + r)\beta E_t[U'(C_{t+1})]$.
E. (A) and (C).
F. (B) and (D).
G. (C) and (D).
H. All of the above.

6. Consider a conventional problem of an infinitely-lived household choosing its path of consumption to maximize lifetime utility. The household has an endowment of E and no subsequent labor income. The interest rate is constant and equal to the household's discount rate. Thus, the household maximizes:

$$\int_{t=0}^{T} e^{-\rho t} u(C(t))dt, \quad u'(\bullet) > 0, \quad u''(\bullet) < 0.$$

The flow budget constraint is $\dot{A}(t) = rA(t) - C(t)$, where $r = \rho$ by assumption. The other constraints are that $A(0) = E$ and that the present discounted value of the household's consumption cannot exceed E (which is the present discounted value of the household's lifetime resources).

a. *Without formally setting up and solving the maximization problem*, describe the household's optimal consumption path, and explain your answer.

b. Consider the same problem as before, except that the interest rate is equal to $r^H > \rho$ from $t = 0$ to $t = t_1$ (where $t_1 > 0$), then equal to ρ. Again *without formally setting up and solving the maximization problem*, sketch the household's optimal consumption path, and compare it with the optimal path in part (a). Explain your answer.

7. Romer, Problem 2.2.

8. Romer, Problem 8.12.

9. Romer, Problem 8.6.