Problem Set 7
Due in lecture, Tuesday, November 20

1. ***The saddle-path of the q-theory model.*** (Special ground rules for this problem: (1) You must try it first under exam conditions – no readings, no notes, no help. (2) You may then look at lecture notes and readings, but not consult with others. (3) You may then consult with others. (4) As always, you may not look at last year's problem set solutions (or at least year's midterm solutions).)

Consider the two equations of the q-theory model,

$$\dot{q}(t) = rq(t) - \pi(K(t)), \qquad\qquad \dot{K}(t) = f(q(t)).$$

a. Define the steady state of the model, $(\bar{q}, \bar{K})$. Show that the model's linear (Taylor) approximation in the neighborhood of the steady state takes the form:

$$[\dot{q}, \dot{K}]' \approx G[q - \bar{q}, K - \bar{K}]',$$

where G =

A	B
C	0

Be sure to show how A, B, and C depend on exogenous parameters, the steady-state values of q and K, and/or the properties of $\pi(\cdot)$ and $f(\cdot)$.

b. Show that the characteristic roots of the preceding 2x2 matrix are:

$$\lambda_1, \lambda_2 = \frac{r \mp \sqrt{r^2 - 4f'(1)\pi'(\bar{K})}}{2},$$

where $\lambda_1 > 0$ and $\lambda_2 < 0$. Please indicate why the second condition holds.

c. Show that the eigenvectors of the matrix G are proportional to the matrix

X =

$\lambda_1/f'(1)$	$\lambda_2/f'(1)$
1	1

d. Define $[\tilde{q}, \tilde{K}]' \equiv X^{-1}[q - \bar{q}, K - \bar{K}]'$, and note that this implies that $[\dot{\tilde{q}}, \dot{\tilde{K}}]' = X^{-1}[\dot{q}, \dot{K}]'$. Explain how that change of variables enables us to write the solution of our differential equation system in the form $[\tilde{q}(t), \tilde{K}(t)]' = [\tilde{q}(0)e^{\lambda_1 t}, \tilde{K}(0)e^{\lambda_2 t}]'$ for arbitrary initial conditions $\tilde{q}(0)$ and $\tilde{K}(0)$.

e. From this last relationship deduce that:

$$q(t) - \bar{q} = \tilde{q}(0)(\lambda_1/f'(1))e^{\lambda_1 t} + \tilde{K}(0)(\lambda_2/f'(1))e^{\lambda_2 t},$$

$$K(t) - \bar{K} = \tilde{q}(0)e^{\lambda_1 t} + \tilde{K}(0)e^{\lambda_2 t}.$$

f. Recalling that $\lambda_1 > 0$ and $\lambda_2 < 0$, identify the initial condition that will ensure the economy is on the

convergent saddle-path in the usual phase diagram with K on the horizontal axis and q on the vertical axis.

  g. For our linear approximation above, express the (linear) equation for the saddle-path in the form

$$q(t) - \bar{q} = \Omega[K(t) - \bar{K}]$$

for an appropriate constant slope $\Omega < 0$. Be sure to show how Ω depends on the model parameters, the steady-state values of q and K, and/or the properties of $\pi(\cdot)$ and $f(\cdot)$.

  h. Recall that

$$\lambda_2 = \frac{r - \sqrt{r^2 - 4f'(1)\pi(\bar{K})}}{2}.$$

Discuss which parameters and steady state values affect the slope of the saddle-path. How do they impact the slope? Why?

2. Romer, Problem 9.6.

3. In the firm optimization problem in the q-theory model, the transversality condition rules out:
        A. Paths where the firm is violating its budget constraint by going further and further into debt.
        B. Paths where investment does not satisfy $1 + C'(I(t)) = q(t)$.
        C. Paths where the firm is constantly increasing its investment even though the profitability of capital is constantly falling.
        D. Paths where investment approaches zero.

4. Consider the basic q-theory model of investment. As adjustment costs approach infinity, the $\dot{q} = 0$ locus:
        A. Is unaffected.   B. Becomes horizontal.
        C. Becomes vertical.   D. Collapses to a single point.

5. Suppose a household has objective function $U(C_1) + \beta U(C_2)$. The household can save at an interest rate of r^s and borrow at an interest rate of r^b, $r^b > r^s$. There is no uncertainty. Then a necessary condition for optimization is:
        A. $U'(C_1) = (1 + r^s)\beta U'(C_2)$.
        B. $U'(C_1) = (1 + r^b)\beta U'(C_2)$.
        C. $U'(C_1) = (1 + r^s)\beta U'(C_2)$ if $C_1 < C_2$ and $U'(C_1) = (1 + r^b)\beta U'(C_2)$ if $C_1 > C_2$.
        D. $1 + r^s < U'(C_1)/[\beta U'(C_2)] < 1 + r^b$.
        E. None of the above.

6. An individual lives for 3 periods. In period 1, his or her objective function is $U(C_1) + \beta U(C_2) + \gamma U(C_3)$. In period 2, his or her objective function is $U(C_2) + \delta U(C_3)$. The individual's preferences are not time consistent if:
        A. $\delta \neq \beta$.   B. $\delta \neq \gamma$.
        C. $\delta \neq \beta/\gamma$.   D. $\delta \neq \gamma/\beta$.

EXTRA PROBLEMS (NOT TO BE HANDED IN/ONLY SKETCHES OF ANSWERS WILL BE PROVIDED)

Romer, Problems 8.8, 8.9, 8.11, 9.7.