## Problem Set 9

## NOT TO BE HANDED IN/ONLY SKETCHES OF ANSWERS WILL BE PROVIDED

1. (This follows Jacklin, 1987.) Consider the Diamond-Dybvig model as presented in lecture. But suppose that instead of a bank, there is a firm. The firm obtains S units of the economy's endowment by selling S shares in period 0 (the price of a share in units of period 0 endowment is 1). The firm's business plan (to which it is committed) is to invest the S units; pay a dividend of  $D_1$  per share in period 1 (by liquidating fraction  $D_1$  of its investment); and then pay a dividend of  $D_2$  per share in period 2 that leaves it with no remaining assets.

a. Explain why  $D_2 = R(1 - D_1)$ .

b. Suppose all agents use their endowment to buy shares in the firm, and suppose there is a market for shares in the firm in period 1 after  $D_1$  has been paid. If all type 1 agents sell their shares and all type 2 agents use all of their period 1 dividends to buy shares, what will the price of shares, P, be as a function of  $D_1$  and  $\theta$ ?

c. Continue to assume that all type 1 agents sell their shares and all type 2 agents use all of their period 1 dividends to buy shares. What will be the consumption of type 1 agents in period 1? The consumption of type 2 agents in period 2?

d. Is there a value of  $D_1$  that yields the social optimum? Explain. (Recall that the social optimum is for the type 1 agents to consume  $1/[\theta + (1 - \theta)\rho]$  in period 1 and for the type 2's to consume  $\rho R/[\theta + (1 - \theta)\rho]$  in period 2.)

e. For what values of P will the type 1 agents want to sell their shares (or be indifferent)? For what values of P will the type 2 agents want to buy shares (or be indifferent)? With  $D_1$  equal to the value you found in part (d), are these conditions satisfied?

f. Is this equilibrium vulnerable to a run? Explain.

2. Consider an asset that has potentially stochastic payoffs in multiple periods,  $Z_1, Z_2, ...$  Suppose that its price is given by:

$$P_t = E_t \left[ \sum_{k=1}^{\infty} \frac{1}{(1+\rho)^k} \frac{U'(C_{t+k})}{U'(C_t)} Z_{t+k} \right] \quad \text{for all t.}$$

Show that this implies

$$P_t = E_t \left[ \frac{1}{1+\rho} \frac{U'(C_{t+1})}{U'(C_t)} (Z_{t+1} + P_{t+1}) \right].$$

(Note: As always, we assume that  $C_t$  is known as of time t.)

(OVER)

3. Consider a consumer who lives for two periods. The consumer's lifetime utility is  $U = u(c_1) + u(c_2)$ ,  $u'(\cdot) > 0$ ,  $u''(\cdot) < 0$ . The consumer's period-1 labor income,  $Y_1$ , is certain, and is equal to  $\overline{Y}$ . The consumer's period-2 labor income,  $Y_2$ , is uncertain, with mean  $\overline{Y}$  and variance  $\sigma_Y^2$ . The consumer's initial wealth is zero.

a. Assume the consumer can borrow and lend at an interest rate of r = 0, and that there are no other financial assets. What is the first-order condition for  $c_1$ ?

For parts (b)-(d), assume that in addition to the safe asset with a real return of zero, there is a second, risky asset whose return has a mean  $\bar{r} > 0$  and variance  $\sigma_r^2 > 0$ . The payoff to the risky asset is uncorrelated with Y<sub>2</sub>.

b. Without using any math, explain in a sentence or two whether the consumer will purchase a strictly positive amount, a strictly negative amount, or none of this asset.

c. Without using any math, explain in a sentence or two why the following argument is wrong: "Because the return on the risky asset is uncorrelated with  $Y_2$  and has a mean that is greater than that on the riskless asset, the consumer would like to borrow as much as possible at the riskless interest rate and purchase as much as possible of the risky asset."

d. Set up the consumer's maximization problem, and find the first-order conditions.

4. Suppose the return on Asset A is riskless and the return on Asset B is risky. Then in equilibrium:

A. The expected return on Asset B must exceed the expected return on Asset A.

B. Asset B will provide higher expected return than Asset A to individuals for whom its return covaries positively with their consumption growth, but lower expected return than Asset A to individuals for whom its return covaries negatively with their consumption growth.

C. Asset B will not be held in equilibrium.

D. None of the above.

5. A reduction in an entrepreneur's wealth is likely to increase the agency costs associated with obtaining financing for the entrepreneur's project because:

A. The entrepreneur is in a weaker bargaining position with respect to outside investors.

B. The entrepreneur will spend less on obtaining outside certification of the soundness of his or her project.

C. The entrepreneur's incentives to devote effort on dimensions that outside investors cannot monitor will be lower.

D. Because the entrepreneur needs more outside funding, he or she is likely to need funds from wealthier investors, who will face higher marginal tax rates and so require higher returns.

6. In the Diamond-Dybvig model, the key departure from Walrasian assumptions is:

A. Asymmetric information between entrepreneurs and outside investors

- B. The presence of noise traders.
- C. Preference shocks.

D. The lack of observability of agents' types.