Economics 202A, Problem Set 1

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1. *Hicks Meets Solow.* Consider the Solow model, but now with the assumption that
$$Y = AF(K, L),$$
where
$$\frac{\dot{A}}{A} = g.$$
(a) Show what happens if we try to derive a balanced growth path like the one derived in class. (The Solow model features labor-augmenting technical change or Harrod-neutral technical change; the form of technical change shown above is called Hicks neutral.) (b) What can you say in the special case $F(K, L) = K^{\alpha}L^{1-\alpha}$?

2. *Investment Rates in the Transition.* In the Solow model, imagine the economy starts out at some initial capital intensity ratio k_0 that is very close to 0. (a) Show how the investment rate (relative to effective labor supply growth) will change over time (by graphing $\dot{k}$ against time) as $k \to \bar{k}$. (b) At what level of the capital stock is $\dot{k}$ maximized?

3. *The Golden Rule.* (a) Find the level of steady-state capital intensity $\bar{k}$ at which consumption per capita is maximized. (b) What saving rate s^* leads to this golden-rule balanced growth path? (c) Explain this result intuitively.

4. *Solow is So Slow....* Assume a discrete-time Solow model in which $L_{t+1} = (1 + n)L_t$ and $A_{t+1} = (1 + g)A_t$. Define z by
$$(1 + z) \equiv (1 + n)(1 + g).$$
(a) Show that with a Cobb-Douglas production function, the Solow model is summarized by the dynamic equation:
$$k_{t+1} - k_t = \frac{sk_t^{\alpha} - (z + \delta)k_t}{1 + z}.$$

(a) Calculate the value of steady-state capital intensity $\overline{k}$. (b) Define the deviation from the steady-state as $\tilde{k}_t \equiv k_t - \overline{k}$. Show that a first-order Taylor approximation to the preceding equation, valid near $k = \overline{k}$, is

$$\tilde{k}_{t+1} - \tilde{k}_t = \frac{s\alpha\overline{k}^{\alpha-1}\tilde{k}_t - (z+\delta)\tilde{k}_t}{1+z}.$$

(c) Show that another way to write this expression is as the difference equation in $\tilde{k}_t$:

$$\tilde{k}_{t+1} = \left[1 + \frac{(\alpha-1)(z+\delta)}{1+z}\right]\tilde{k}_t.$$

(d) For a given initial value $\tilde{k}_0 = k_0 - \overline{k}$ that is not too big, solve for the approximate value of k_t, capital intensity at time t. (e) Assuming that $\alpha = \frac{1}{3}$ and that n, g, and δ are measured at annual rates as $n = 0.01$, $g = 0.02$, and $\delta = 0.3$, compute the *half-life* (in years) of the distance from steady-state capital intensity – the number of years it takes the distance from the steady state to fall by half.