UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2013
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Problem Set 6
Due in lecture Tuesday, November 12

1. Consider an economy that lasts for two periods and that consists of equal numbers of two types of agents, Type A and Type B. The objective function of a representative agent of Type *i* is

$$C_1^i + \beta E\left[C_2^i - \frac{1}{2}a(C_2^i)^2\right], \quad a > 0.$$

where C_t^i is the consumption of an agent of Type *i* in period t. Assume that the C_2^i's are always in the range where marginal utility is positive.

Agents of Type *i* receive an endowment of W_1^i in period 1 and W_2^i in period 2. The W_1^i's are certain and the W_2^i's are uncertain.

Endowments cannot be stored or saved in any way. Thus equilibrium requires $C_1^A + C_1^B = W_1^A + W_1^B$ and $C_2^A + C_2^B = W_2^A + W_2^B$.

a. Suppose the only asset that can be traded is a riskless bond. Specifically, consider an asset that will pay 1 unit for sure in period 2.

i. Set up the problem of an agent of Type *i* choosing how much of the asset to buy. The agent takes P, the price of the asset in period 1 in units of period-1 endowment, as given. The amount bought can be positive or negative (that is, the agent can buy or sell the asset).

ii. Find the demand of an agent of Type *i* for the asset as a function of P and of any relevant parameters (for example, a, β, W_1^i, and the mean and variance of W_2^i).

iii. What is the equilibrium price of the asset? (Hint: What must the sum of the quantities of the asset demanded by the two types of agents be for the market to be in equilibrium?)

b. Suppose agents cannot trade a safe asset, but can trade two risky assets, A and B. The payoff to Asset *i* is W_2^i. Let P_i denote the period-1 price of Asset *i* in units of period-1 endowment. (Thus, if an agent of Type *i* buys Q_A^i of Asset A and Q_B^i of Asset B, his or her consumption is $W_1^i - P_A Q_A^i - P_B Q_B^i$ in period 1, and $W_2^i + Q_A^i W_2^A + Q_B^i W_2^B$ in period 2.)

i. Set up the problem of an agent of Type *i* choosing how much of each of the two assets to buy. The agent takes the prices of the assets in period 1 as given. (As in part (a), the amounts bought can be positive or negative.)

ii. Find the first-order conditions for the problem you set up in part (b)(i).

iii. Assume $W_1^A = W_1^B$, and that W_2^A and W_2^B have the same distribution as one another and are independent. If $P_A = P_B$, will a Type-A agent demand more of Asset A or of Asset B? (A good logical explanation is enough.)

iv. Continue to make the assumptions in part (b)(iii). Get as far as you can in describing the equilibrium quantities ($Q_A^A, Q_B^A, Q_A^B,$ and Q_B^B). (As in part (iii), a good logical argument is enough.)

(OVER)

2. Consider the continuous-time consumption problem discussed in lecture: an individual lives from 0 to T; has initial wealth A(0); and a path of labor income given by Y(t). The path of the instantaneous interest rate is given by r(t). There is no uncertainty.

Suppose the individual's instantaneous utility function is logarithmic. That is, lifetime utility is $\int_{t=0}^{T} e^{-\delta t} \ln[C(t)]\, dt.$ Derive an expression for C(t) as a function of things the individual takes as given.

3. Consider a consumer maximizing $U(C_1) + U(C_2)$, with $U'(\bullet) > 0$, $U''\bullet) < 0$, and $U'''(\bullet) > 0$, who can save or borrow at a real interest rate of zero. Then, letting $E[\bullet]$ denote expectations conditional on period-1 information, if the consumer is optimizing:

A. $E[C_2] < C_1$ and $E[U'(C_2)] < U'(C_1)$.
B. $E[C_2] < C_1$ and $E[U'(C_2)] = U'(C_1)$.
C. $E[C_2] < C_1$ and $E[U'(C_2)] > U'(C_1)$.
D. $E[C_2] = C_1$ and $E[U'(C_2)] < U'(C_1)$.
E. $E[C_2] = C_1$ and $E[U'(C_2)] = U'(C_1)$.
F. $E[C_2] = C_1$ and $E[U'(C_2)] > U'(C_1)$.
G. $E[C_2] > C_1$ and $E[U'(C_2)] < U'(C_1)$.
H. $E[C_2] > C_1$ and $E[U'(C_2)] = U'(C_1)$.
I. $E[C_2] > C_1$ and $E[U'(C_2)] > U'(C_1)$.
J. Because the consumer can borrow at a zero interest rate, he or she will make C_1 and C_2 arbitrarily large.

4. Romer, Problem 8.10.

EXTRA PROBLEMS (NOT TO BE HANDED IN/ONLY SKETCHES OF ANSWERS WILL BE PROVIDED)

5. (The risk-free rate puzzle.) There is considerable evidence that individuals are quite impatient and quite risk averse. In light of this, consider the standard Euler equation relating consumption in periods t and t+1 under certainty: $U'(C_t) = [(1 + r)/(1 + \rho)]U'(C_{t+1})$. Suppose that ρ is 5 percent, the coefficient of relative risk aversion is 4, and that the growth rate of consumption is 1.5 percent. What must r be for consumers to be satisfying their Euler equation?

6. An individual lives for 3 periods. In period 1, his or her objective function is $U(C_1) + \beta U(C_2) + \gamma U(C_3)$. In period 2, his or her objective function is $U(C_2) + \delta U(C_3)$. The individual's preferences are not time consistent if:

A. $\delta \neq \beta$.
B. $\delta \neq \gamma$.
C. $\delta \neq \beta/\gamma$.
D. $\delta \neq \gamma/\beta$.

7. Romer, Problem 8.14.

8. Romer, Problem 8.15.