CHAPTER 14: BOND PRICES AND YIELDS

1. a. Effective annual rate on 3-month T-bill:

\[
\frac{100,000}{97,645} \times \frac{1}{4} - 1 = 1.024124 - 1 = .10 \text{ or } 10\%
\]

b. Effective annual interest rate on coupon bond paying 5% semiannually:

\[(1.05)^2 - 1 = .1025 \text{ or } 10.25\%
\]

2. The effective annual yield on the semiannual coupon bonds is 8.16%. If the annual coupon bonds are to sell at par they must offer the same yield, which will require an annual coupon of 8.16%.

3. The bond callable at 105 should sell at a lower price because the call provision is more valuable to the firm. Therefore, its YTM should be higher.

4. Lower. As time passes, the bond price, which now must be above par value, will approach par.

5. We find the yield to maturity from our financial calculator using the following inputs:

\[n = 3, \text{ } FV = 1000, \text{ } PV = 953.10, \text{ } PMT = 80.\]

This results in

\[\text{YTM} = 9.88\%\]

**Realized compound yield:** First find the future value, FV, of reinvested coupons and principal:

\[FV = (80 \times 1.10 \times 1.12) + (80 \times 1.12) + 1080 = 1268.16\]

Then find the rate, y, that makes the FV of the purchase price equal to $1268.16.

\[953.10(1 + y)^3 = 1268.16\]

\[y = 9.99\% \text{ or approximately } 10\%\]
6. a. A sinking fund is a provision that calls for the mandatory early redemption of a bond issue. The provision may be for a specific number of bonds or a percentage of bonds over a specified time horizon. The sinking fund can retire all or a portion of an issue over its life.

b. (i) Compared to a bond without a sinking fund, the sinking fund will reduce the average life of the overall issue because some of the bonds are retired before stated maturity.

(ii) The company will make the same total principal payments over the life of the issue, although the timing of the payments will be affected. The total interest payments associated with the issue will be reduced given the early redemption of principal.

c. From the investor’s point of view, the key reason for demanding a sinking fund is to reduce credit risk. Default risk is reduced by the orderly retirement of the issue.

7. a. (i) Current yield = Coupon/Price = 70/960 = .0729 = 7.29%

(ii) YTM = 4% semiannually or 8% annual bond equivalent yield.

On your calculator, set n = 10 (semiannual payments)
PV = (–)960
FV = 1,000
PMT = 35

Compute the interest rate.

(iii) Realized compound yield is 4.166% (semiannually), or 8.33% annual bond equivalent yield. To obtain this value, first calculate the future value of reinvested coupons. There will be 6 payments of $35 each, reinvested semiannually at a per period rate of 3%: PV = 0; PMT = $35; n = 6; i = 3%.

Compute FV = $226.39.

The bond will be selling at par value of $1,000 in 3 years, since coupon is forecast to equal yield to maturity.

Therefore, total proceeds in 3 years will be $1,226.39. To find realized compound yield on a semiannual basis (i.e., for 6 half-year periods), we solve:

\[ 960 \times (1 + y)^6 = 1,226.39 \]

Which implies that \( y = 4.166\% \) (semiannual)
b. Shortcomings of each measure:

(i) Current yield does not account for capital gains or losses on bonds bought at prices other than par value. It also does not account for reinvestment income on coupon payments.

(ii) Yield to maturity assumes the bond is held until maturity and that all coupon income can be reinvested at a rate equal to the yield to maturity.

(iii) Realized compound yield is affected by the forecast of reinvestment rates, holding period, and yield of the bond at the end of the investor's holding period.

8. Zero coupon 8% coupon 10% coupon
   a. Current prices $463.19 $1000.00 $1134.20
   b. Price 1 year from now $500.25 $1000.00 $1124.94
      Price increase $37.06 $0.00 – $9.26
      Coupon income $0.00 $80.00 $100.00
      Pre-tax income $37.06 $80.00 $90.74
      Pre-tax rate of return (%) 8.00 8.00 8.00
      Taxes* $11.12 $24.00 $28.15
      After-tax income $25.94 $56.00 $62.59
      After-tax rate of return (%) 5.60 5.60 5.52
   c. Price 1 year from now $543.93 $1065.15 $1195.46
      Price increase $80.74 $65.15 $61.26
      Coupon income $0.00 $80 $100.00
      Pre-tax income $80.74 $145.15 $161.26
      Pre-tax rate of return (%) 17.43 14.52 14.22
      Taxes $24.22 $37.03 $42.25
      After-tax income $56.52 $108.12 $119.01
      After-tax rate of return (%) 12.20 10.81 10.49

* In computing taxes, we assume that the 10% coupon bond was issued at par and that the drop in price when the bond is sold at year end is treated as a capital loss and not as an offset to ordinary income.

9. a. Use the following inputs: n = 40, FV = 1000, PV = (–)950, PMT = 40. You will find that the yield to maturity on a semi-annual basis is 4.26%. This implies a bond equivalent yield to maturity of $4.26\% \times 2 = 8.52\%$.

   Effective annual yield to maturity = \((1.0426)^2 - 1 = .0870 = 8.70\%\)
b. Since the bond is selling at par, the yield to maturity on a semi-annual basis is the same as the semi-annual coupon, 4%. The bond equivalent yield to maturity is 8%.

Effective annual yield to maturity = \((1.04)^2 - 1 = .0816 = 8.16\%\)

c. Keeping other inputs unchanged but setting PV = \((-)1050\), we find a bond equivalent yield to maturity of 7.52%, or 3.76% on a semi-annual basis.

Effective annual yield to maturity = \((1.0376)^2 - 1 = .0766 = 7.66\%\)

10. Since the bond now makes annual payments instead of semi-annual payments, the bond equivalent yield to maturity will be the same as the effective annual yield to maturity. The inputs are: \(n = 20\), \(FV = 1000\), \(PV = (-)price\), \(PMT = 80\). The resulting yields for the three bonds are:

<table>
<thead>
<tr>
<th>Bond Price</th>
<th>Bond equivalent yield = Effective annual yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>$950</td>
<td>8.53%</td>
</tr>
<tr>
<td>$1000</td>
<td>8.00%</td>
</tr>
<tr>
<td>$1050</td>
<td>7.51%</td>
</tr>
</tbody>
</table>

The yields computed in this case are lower than the yields calculated when the coupon payments were semi-annual. All else equal, annual payments make the bonds less attractive to the investor, since more time elapses before payments are received. If the bond price is no lower when the payments are made annually, the bond's yield to maturity must be lower.

11. Assuming semi-annual coupon periods:

<table>
<thead>
<tr>
<th>Price</th>
<th>Maturity (years)</th>
<th>Maturity (half-years)</th>
<th>Semi annual YTM</th>
<th>Bond equivalent yield to maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$400</td>
<td>20</td>
<td>40</td>
<td>2.317%</td>
<td>4.634%</td>
</tr>
<tr>
<td>$500</td>
<td>20</td>
<td>40</td>
<td>1.748%</td>
<td>3.496%</td>
</tr>
<tr>
<td>$500</td>
<td>10</td>
<td>20</td>
<td>3.526%</td>
<td>7.052%</td>
</tr>
<tr>
<td>$376.89</td>
<td>10</td>
<td>20</td>
<td>5.000%</td>
<td>10.000%</td>
</tr>
<tr>
<td>$456.39</td>
<td>10</td>
<td>20</td>
<td>4.000%</td>
<td>8.000%</td>
</tr>
<tr>
<td>$400</td>
<td>11.68</td>
<td>23.36</td>
<td>4.000%</td>
<td>8.000%</td>
</tr>
</tbody>
</table>

12. a. The bond pays $50 every 6 months

Current price = $50 \times \text{Annuity factor}(4\%, 6) + $1000 \times \text{PV factor}(4\%, 6) = $1052.42

Assuming the market interest rate remains 4% per half year:
Price 6 months from now =

\[ \$50 \times \text{Annuity factor}(4\%, 5) + \$1000 \times \text{PV factor}(4\%, 5) = \$1044.52 \]

b. Rate of return = \[ \frac{\$50 + (\$1044.52 - \$1052.42)}{\$1052.42} \]
\[ = \frac{\$50 - \$7.90}{\$1052.42} = \frac{\$42.10}{\$1052.42} = .04 \text{ or } 4\% \text{ per six months} \]

13. a. Initial price, \( P_0 = 705.46 \) \([n = 20; \text{PMT} = 50; \text{FV} = 1000; \text{i} = 8]\)

Next year's price, \( P_1 = 793.29 \) \([n = 19; \text{PMT} = 50; \text{FV} = 1000; \text{i} = 7]\)

HPR = \[ \frac{50 + (793.29 - 705.46)}{705.46} = .1954 = 19.54\% \]

b. Using OID tax rules, the price path of the bond under the constant yield method is obtained by discounting at an 8% yield, simply reducing maturity by one year at a time:

Constant yield prices
\( P_0 = 705.46 \)
\( P_1 = 711.89 \) implies implicit interest over first year = \$6.43
\( P_2 = 718.84 \) implies implicit interest over second year = \$6.95

Tax on explicit plus implicit interest in first year = \(.40 \times (\$50 + \$6.43) = \$22.57\)

Capital gain in first year = Actual price – constant yield price
\[ = 793.29 - 711.89 = \$81.40 \]

Tax on capital gain = \(.30 \times \$81.40 = \$24.42\)

Total taxes = \$22.57 + \$24.42 = \$46.99

c. After tax HPR = \[ \frac{50 + (793.29 - 705.46) - 46.99}{705.46} = .1288 = 12.88\% \]

d. Value of bond after 2 years equals \$798.82 \([\text{using } n = 18; \text{i} = 7]\)

Reinvested coupon income from the two coupons equals \$50 \times 1.03 + \$50 = \$101.50

Total funds after two years equals \$798.82 + \$101.50 = \$900.32.

Therefore, the \$705.46 investment grows to \$900.32 after 2 years.

\[ 705.46 (1 + r)^2 = 900.32 \text{ which implies that } r = .1297 = 12.97\% \]
e. Coupon received in first year: $50.00
   Tax on coupon @ 40% – 20.00
   Tax on imputed interest (.40 × $6.43) – 2.57
   Net cash flow in first year $27.43

   If you invest the year-1 CF at an after-tax rate of 3% × (1 – .40) = 1.8% it will grow by year 2 to $27.43 × (1.018) = $27.92.

   You sell the bond in the second year for $798.82 [n = 18; i = 7%]
   Tax on imputed interest in second year – 2.78 [.40 × $6.95]
   Coupon received in second year net of tax + 30.00 [$50 × (1 – .40)]
   Capital gains tax on sales price – constant yield value – 23.99 [.30 × (798.82 – 718.84)]
   CF from first year's coupon (reinvested) + 27.92 [from above]
   TOTAL $829.97

   705.46 (1 + r)² = 829.97
   r = .0847 = 8.47%

14. The reported bond price is 100 2/32 percent of par, which equals $1,000.625. However, 15 days have passed since the last semiannual coupon was paid, so accrued interest equals $35 × (15/182) = $2.885. The invoice price is the reported price plus accrued interest, or $1003.51.

15. If the yield to maturity is greater than the current yield, the bond must offer the prospect of price appreciation as it approaches its maturity date. Therefore, the bond must be selling below par value.

16. The coupon rate must be below 9%. If coupon divided by price equals 9%, and price is less than par, then price divided by par must be less than 9%.

17.

<table>
<thead>
<tr>
<th>Time</th>
<th>Inflation in year just ended</th>
<th>Par value</th>
<th>Coupon payment</th>
<th>Principal repayment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>$1,000.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2%</td>
<td>$1,020.00</td>
<td>$40.80</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>3%</td>
<td>$1,050.60</td>
<td>$42.02</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1%</td>
<td>$1,061.11</td>
<td>$42.44</td>
<td>$1,104.55</td>
</tr>
</tbody>
</table>

   The nominal rate of return on the bond in each year is:
Nominal rate of return = \( \frac{\text{interest} + \text{price appreciation}}{\text{initial price}} \).

Real rate of return = \( \frac{1 + \text{nominal return}}{1 + \text{inflation}} - 1 \)

<table>
<thead>
<tr>
<th>Second year</th>
<th>Third year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal return = ( \frac{42.02 + 30.60}{1020} = .071196 )</td>
<td>Nominal return = ( \frac{42.44 + 10.51}{1050.60} = .05040 )</td>
</tr>
</tbody>
</table>

Real return = \( \frac{1.071196}{1.03} - 1 = .04, \text{ or } 4\% \) \( \frac{1.05040}{1.01} - 1 = .04, \text{ or } 4\% \)

The real rate of return in each year is precisely the 4\% real yield on the bond.

18. The price schedule is as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Remaining Maturity, T</th>
<th>Constant yield value ( \frac{1000}{(1.08)^T} )</th>
<th>Imputed interest (Increase in constant yield value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (now)</td>
<td>20 years</td>
<td>214.55</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>19</td>
<td>231.71</td>
<td>17.16</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>250.25</td>
<td>18.54</td>
</tr>
<tr>
<td>19</td>
<td>1</td>
<td>925.93</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0</td>
<td>1000.00</td>
<td>74.07</td>
</tr>
</tbody>
</table>

19. The bond is issued at a price of $800. Therefore, its yield to maturity is 6.824%. Using the constant yield method, we can compute that its price in one year (when maturity falls to 9 years) will be (at an unchanged yield) $814.62, representing an increase of $14.62. Total taxable income is $40 + $14.62 = $54.62.

20. a. The bond sells for $1,124.72 based on the 3.5\% yield to maturity. \( [n = 60; i = 3.5\%; FV = 1,000; PMT = 40] \)

Therefore, yield to call is 3.368\% semiannually. \( [n = 10 \text{ semiannual periods}; PV = (\text{\textdollar})1,124.72 ; FV = 1,100; PMT = 40] \)

b. If the call price were $1,050, we would set \( FV = 1,050 \) and redo part (a) to find that yield to call is 2.976\%. With a lower call price, the yield to call is lower.
c. Yield to call is 3.031% semiannually.  \[ n = 4; (-)PV = 1,124.72 ; FV = 1,100; PMT = 40 \]

21. Using annual interest payments, the yield to maturity based on promised payments equals 16.075%. \[ n = 10; PV = (-)900; FV = 1,000; PMT = 140 \]

Based on expected coupon payments of $70 annually, the expected yield to maturity would be only 8.526%.

22. The bond is selling at par value. Its yield to maturity equals the coupon rate, 10%. If the first-year coupon is reinvested at an interest rate of r percent, then total proceeds at the end of the second year will be \( 100 \times (1 + r) + 1100 \). Therefore, realized compound yield to maturity will be a function of r as given in the following table:

<table>
<thead>
<tr>
<th>r</th>
<th>Total proceeds</th>
<th>Realized YTM = ( \sqrt{\frac{\text{Proceeds}}{1000}} - 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8%</td>
<td>$1208</td>
<td>( \sqrt{\frac{1208}{1000}} - 1 = .0991 = 9.91% )</td>
</tr>
<tr>
<td>10%</td>
<td>$1210</td>
<td>( \sqrt{\frac{1210}{1000}} - 1 = .1000 = 10.00% )</td>
</tr>
<tr>
<td>12%</td>
<td>$1212</td>
<td>( \sqrt{\frac{1212}{1000}} - 1 = .1009 = 10.09% )</td>
</tr>
</tbody>
</table>

23. Zero coupon bonds provide no coupons to be reinvested. Therefore, the investor's proceeds from the bond are independent of the rate at which coupons could be reinvested (if they were paid). There is no reinvestment rate uncertainty with zeros.

24. April 15 is midway through the semiannual coupon period. Therefore, the invoice price will be higher than the stated ask price by an amount equal to one-half of the semiannual coupon. The ask price is 101.125 percent of par, so the invoice price is:

\[ $1011.25 + \frac{1}{2} \times $50 = $1036.25 \]

25. Factors which might make the ABC debt more attractive to investors, therefore justifying a lower coupon rate and yield to maturity, are:

a. The ABC debt is a larger issue and therefore may sell with more liquidity.

b. An option to extend the term from 10 years to 20 years is favorable if interest rates in 10 years are lower than today’s. In contrast, if interest rates are rising, the investor can present the bond for payment and reinvest the money for better returns.

c. In the event of trouble, the ABC debt is a more senior claim. It has more underlying security in the form of a first claim against real property.
d. The call feature on the XYZ bonds makes the ABC bonds relatively more attractive since ABC bonds cannot be called from the investor.

e. The XYZ bond has a sinking fund requiring XYZ to retire part of the issue each year. Since most sinking funds give the firm the option to retire this amount at the lower of par or market value, the sinking fund can work to the detriment of bondholders.

26. a. The floating rate note pays a coupon that adjusts to market levels. Therefore, it will not experience dramatic price changes as market yields fluctuate. The fixed rate note therefore will have a greater price range.

b. Floating rate notes may not sell at par for any of several reasons:

The yield spread between 1-year Treasury bills and other money market instruments of comparable maturity could be wider (or narrower) than when the bond was issued.

The credit standing of the firm may have eroded (or improved) relative to Treasury securities which have no credit risk. Therefore, the 2% premium would become insufficient to sustain the issue at par.

The coupon increases are implemented with a lag, i.e., once every year. During a period of changing interest rates, even this brief lag will be reflected in the price of the security.

c. The risk of call is low. Because the bond will almost surely not sell for much above par value (given its adjustable coupon rate), it is unlikely that the bond will ever be called.

d. The fixed-rate note currently sells at only 88% of the call price. Call risk is currently low, since yields would need to fall substantially for the firm to use its option to call the bond.

e. The 9% coupon notes currently have a remaining maturity of 15 years and sell at a yield to maturity of 9.9%. This is the coupon rate that would be needed for a newly-issued 15-year maturity bond to sell at par.

f. Because the floating rate note consists of a variable stream of interest payments to maturity, the effective maturity for comparative purposes with other debt securities is closer to next coupon reset date than the final maturity date. Therefore, yield-to-maturity is an indeterminable calculation for a floating rate note, with “yield-to-recoupon date” a more meaningful measure of return.

27. a. The maturity of the bonds is 10 years, and we will assume that coupons are paid semiannually. The current yields to maturity on each bond equals the respective coupon rate, since both bonds are selling at par value.
If the yield declines by 1% to 5% (2.5% semiannual yield), the Sentinal bond will increase in value to 107.79 \[n=20; i = 2.5\%; FV = 100; PMT = 3\].

The price of the Colina bond will increase, but only to the call price of 102. The present value of scheduled payments is more than 102, but the call price puts a ceiling on the actual bond price.

b. If rates are expected to fall, the Sentinal bond is more attractive: since it is not subject to being called, its potential capital gains are higher.

If rates are expected to rise, Colina is a relatively better investment. Its higher coupon (which presumably is compensation to investors for the call feature of the bond) will provide a higher rate of return than the Sentinal bond.

c. An increase in the volatility of rates will increase the value of the firm’s option to call back the Colina bond. [If rates go down, the firm can call the bond, which puts a cap on possible capital gains. So higher volatility makes the option to call back the bond more valuable to the issuer.] This makes the bond less attractive to the investor in the Colina bond.

28. a. The yield on the par bond equals its coupon rate, 8.75%. All else equal, the 4% coupon bond would be more attractive because its coupon rate is far below current market yields, and its price is far below the call price. Therefore, if yields do fall, capital gains on the bond will not be limited by the call price. In contrast, the 8 ¾% coupon bond can increase in value to at most $1050, offering a maximum possible gain of only 0.5%. The disadvantage of the 8 ¾% coupon bond in terms of vulnerability to being called shows up in its higher promised yield to maturity.

b. If an investor expects yields to fall substantially, the 4% bond will offer a greater expected return.

c. Implicit call protection is offered in the sense that any likely fall in yields would not be nearly enough to make the firm consider calling the bond. In this sense, the call feature is almost irrelevant.

29. Market conversion value = value if converted into stock
   \[= 20.83 \times 28 = $583.24\]

   Conversion premium = Bond price – market conversion value
   \[= $775 – $583.24 = $191.76\]

30. a. The call provision requires the firm to offer a higher coupon (or higher promised yield to maturity) on the bond to compensate the investor for the firm's option to
call back the bond at a specified price if interest rate falls sufficiently. Investors are willing to grant this valuable option to the issuer, but only for a price that reflects the possibility that the bond will be called. That price is the higher promised yield at which they are willing to buy the bond.

b. The call option will reduce the expected life of the bond. If interest rates fall substantially and the likelihood of call increases, investors will begin to treat the bond as if it will "mature" and be paid off at the call date, not at the stated maturity date. On the other hand if rates rise, the bond must be paid off at the maturity date, not later. This asymmetry means that the expected life of the bond will be less than the stated maturity.

c. The advantage of a callable bond is the higher coupon (and a higher promised yield to maturity) when the bond is issued. If the bond turns out not to be called, then one will earn a higher realized compound yield on a callable bond issued at par than a non-callable bond issued at par on the same date. The disadvantage of the callable bond is the risk of call. If rates fall and the bond is called, the investor will receive the call price and will have to reinvest the proceeds at now-lower interest rates than the yield to maturity at which the bond originally was issued. In this event, the firm's savings in interest payments is the investor's loss.

31. a. (iv) The Euless, Texas, General Obligation Bond, which has been refunded and secured by U.S. Government bonds held in escrow, has as good a credit quality as the U.S. bonds backing it. Euless, Texas has issued new bond to refund this issue, and with the proceeds purchased U.S. Government bonds. They did this rather than simply retire the old bonds because the old bonds are not callable yet and because Euless gets to earn the rate on T-bonds while paying a lower rate on its own bonds.

The University of Kansas Medical Center Bonds are insured by a body which is not backed by the taxing power of the U.S. Treasury and therefore do not have as high a credit quality as the Euless bonds.

The other two bonds have indeterminate quality. Since both are bonds of small local governments they may be subject to significant risk. The Sumter, South Carolina, Water and Sewer Revenue Bond probably is less likely to default because the revenues from such essential services are more reliable than the general taxing power of Riley County, Kansas.

b. (ii) Economic uncertainty increases the chances of default on the BAA corporate bonds and therefore widens the spread between Treasury and BAA corporate bond yields.

c. (iii)

d. (iii) The yield on the callable bond must compensate the investor for the risk of call.
Choice (i) is wrong because, although the owner of a callable bond receives a premium plus the principal in the event of a call, the interest rate at which he can reinvest will be low. The low interest rate which makes it profitable for the issuer to call the bond makes it a bad deal for the bond’s holder.

Choice (ii) is wrong because a bond is more apt to be called when interest rates are low. Only if rates are low will there be an interest saving for the issuer.

e. (ii) is the only correct choice.

(i) is wrong because the YTM exceeds the coupon rate when a bond sells at a discount and is less than the coupon rate when the bond sells at a premium.

(iii) is wrong because adding the average annual capital gain rate to the current yield does not give the yield to maturity. For example, assume a 10-year bond with a 6% coupon rate paying interest annually and a YTM of 8% per year. Its price is $865.80. The average annual capital gain is equal to ($1000 – 865.80)/10 years = $13.42 per year. Using this number results in an average capital gains rate per year of $13.42/$865.80 = 1.55%. The current coupon yield is $60/$865.80 = .0693 per year or 6.93%. Therefore, the “total yield” is 8.48% (=1.55% + 6.93%) which is greater than the YTM.

(iv) is wrong because YTM is based on the assumption that any payments received are reinvested at the YTM and not at the coupon rate.

f. \((1+.12/4)^4 = 1.1255.\) The effective annual YTM is 12.55%. Choice (iii) is correct.

(g) (iii)

h. (ii)

i. (iii)

j. (iii)

k. (iv)

l. (iii)