1. Solutions to PS 1:

1. a. (iv)
   b. (ii)  \[ \frac{6.75}{(1 - .34)} = 10.2 \]  
   c. (i) Writing a call entails unlimited potential losses as the stock price rises.

7. The bill has a maturity of one-half year, and an annualized discount of 9.18%. Therefore, its actual percentage discount from par value is \[ 9.18\% \times \frac{1}{2} = 4.59\% \]. The bill will sell for \[ $100,000 \times (1 - .0459) = $95,410 \].

15. If the after-tax yields are equal, then \[ 5.6\% = 8\% \times (1 - t) \]. This implies that \[ t = .30 \], so the correct choice is (a).

2. Solutions to Question in Chapter 5 (except those in PS 3)

1. Your holding period return for the next year on the money market fund depends on what 30 day interest rates will be each month when it is time to roll over maturing securities. The one-year savings deposit will offer a 7.5% holding period return for the year. If you forecast the rate on money market instruments to rise significantly above the current yield of 6%, then the money market fund might result in a higher HPR for the year. While the 20-year Treasury bond is offering a yield to maturity of 9% per year, which is 150 basis points higher than the rate on the one-year savings deposit at the bank, you could wind up with a one-year HPR of much less than 7.5% on the bond if long-term interest rates rise during the year. If Treasury bond yields rise above 9% during the year, then the price of the bond will fall, and the capital loss will wipe out some or all of the 9% return you would have received if bond yields had remained unchanged over the course of the year.

2. a. If businesses decrease their capital spending they are likely to decrease their demand for funds. This will shift the demand curve in Figure 5.1 to the left and reduce the equilibrium real rate of interest.
   
   b. Increased household saving will shift the supply of funds curve to the right and cause real interest rates to fall.
   
   c. An open market purchase of Treasury securities by the Fed is equivalent to an increase in the supply of funds (a shift of the supply curve to the right). The equilibrium real rate of interest will fall.

3. a. The Inflation-Plus CD is safer because it guarantees the purchasing power of the investment. Using the approximation that the real rate equals the nominal rate minus the inflation rate, the CD provides a real rate of 3.5% regardless of the inflation rate.
b. The expected return depends on the expected rate of inflation over the next year. If the rate of inflation is less than 3.5% then the conventional CD will offer a higher real return than the Inflation-Plus CD; if inflation is more than 3.5%, the opposite will be true.

c. If you expect the rate of inflation to be 3% over the next year, then the conventional CD offers you an expected real rate of return of 4%, which is 0.5% higher than the real rate on the inflation-protected CD. But unless you know that inflation will be 3% with certainty, the conventional CD is also riskier. The question of which is the better investment then depends on your attitude towards risk versus return. You might choose to diversify and invest part of your funds in each.

d. No. We cannot assume that the entire difference between the nominal risk-free rate (on conventional CDs) of 7% and the real risk-free rate (on inflation-protected CDs) of 3.5% is the expected rate of inflation. Part of the difference is probably a risk premium associated with the uncertainty surrounding the real rate of return on the conventional CDs. This implies that the expected rate of inflation is less than 3.5% per year.

5. Probability distribution of price and 1-year holding period return on 30-year Treasuries (which will have 29 years to maturity at year’s end):

<table>
<thead>
<tr>
<th>Economy</th>
<th>Probability</th>
<th>YTM</th>
<th>Price</th>
<th>Capital gain</th>
<th>Coupon</th>
<th>HPR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom</td>
<td>.20</td>
<td>11.0</td>
<td>$74.05</td>
<td>–$25.95</td>
<td>$8.00</td>
<td>–17.95%</td>
</tr>
<tr>
<td>Normal Growth</td>
<td>.50</td>
<td>8.0</td>
<td>100.00</td>
<td>0.00</td>
<td>8.00</td>
<td>8.00%</td>
</tr>
<tr>
<td>Recession</td>
<td>.30</td>
<td>7.0</td>
<td>112.28</td>
<td>12.28</td>
<td>8.00</td>
<td>20.28%</td>
</tr>
</tbody>
</table>

6. The average risk premium on stocks for the period 1926-1999 was 9.29% per year. Adding this to a risk-free rate of 6% gives an expected return of 15.29% per year for the S&P 500 portfolio.

7. The average rate of return and standard deviation are quite different in the sub periods:

<table>
<thead>
<tr>
<th></th>
<th>STOCKS</th>
<th>BONDS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std. Dev.</td>
</tr>
<tr>
<td>1926-1999</td>
<td>13.11%</td>
<td>20.21%</td>
</tr>
<tr>
<td>1926-1941</td>
<td>6.39%</td>
<td>30.33</td>
</tr>
</tbody>
</table>

I would prefer to use the risk premiums and standard deviations estimated over the period 1970-1999, because it seems to have been a different economic regime. After 1955 the U.S. economy entered the Keynesian era, when the Federal government actively attempted to stabilize the economy and prevent extreme cycles of boom and bust. Note that the standard deviation of stocks has gone down in the later period while the standard
deviation of bonds has increased.

9. From Table 5.2, the average real rate on bills has been approximately 3.82% – 3.17% = .65%.
   a. T-bills: .65% real rate + 3% inflation = 3.65%
   b. Large stock return: 3.65% T-bill rate + 9.29% historical risk premium = 12.94%
   c. The risk premium on stocks remains unchanged. [A premium, the difference between two rates, is a real value, unaffected by inflation].

10. Real interest rates are expected to rise. The investment activity will shift the demand for funds curve to the right in Figure 5.1 and therefore increase the equilibrium real interest rate.

11. a [Expected dollar return on equity investment is $18,000 versus $5,000 return on T-bills]

13. d

14. c

15. b

16. Probability of neutral economy is .50, or 50%. Given a neutral economy, the stock will experience poor performance 30% of the time. The probability of both poor stock performance and a neutral economy is therefore .30 \times .50 = .15 = 15%. Choice (b) is correct.

17. b

18. a. Probability Distribution of HPR on the Stock Market and Put

<table>
<thead>
<tr>
<th>State of the Economy</th>
<th>Probability</th>
<th>Ending price + $4 dividend</th>
<th>HPR</th>
<th>Ending Value</th>
<th>HPR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom</td>
<td>.25</td>
<td>$144</td>
<td>44%</td>
<td>$0</td>
<td>-100%</td>
</tr>
<tr>
<td>Normal Growth</td>
<td>.50</td>
<td>114</td>
<td>14%</td>
<td>0</td>
<td>-100%</td>
</tr>
<tr>
<td>Recession</td>
<td>.25</td>
<td>84</td>
<td>-16%</td>
<td>30</td>
<td>150%</td>
</tr>
</tbody>
</table>

Remember that the cost of the stock is $100 per share, and that of the put is $12.

b. The cost of one share of stock plus a put is $112. The probability distribution of HPR on the stock market plus put is:
c. Buying the put option guarantees you a minimum HPR of 1.8% regardless of what happens to the stock's price. Thus, it offers insurance against a price decline.

19. The probability distribution of the dollar return on CD plus call option is:

<table>
<thead>
<tr>
<th>Economy</th>
<th>Probability</th>
<th>Ending Value CD</th>
<th>Ending Value Call</th>
<th>Combined Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom</td>
<td>.25</td>
<td>$114</td>
<td>$30</td>
<td>$144</td>
</tr>
<tr>
<td>Normal Growth</td>
<td>.50</td>
<td>114</td>
<td>0</td>
<td>114</td>
</tr>
<tr>
<td>Recession</td>
<td>.25</td>
<td>114</td>
<td>0</td>
<td>114</td>
</tr>
</tbody>
</table>

3. Solutions to Question in Chapter 6 (except those in PS 3)

3. Points on the curve are derived as follows:

\[ U = 5 = E(r) - .005\sigma^2 = E(r) - .015\sigma^2 \]

The necessary value of \( E(r) \), given the value of \( \sigma^2 \), is therefore:

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>( \sigma^2 )</th>
<th>( E(r) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>0</td>
<td>5.0%</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>5.375</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>6.5</td>
</tr>
<tr>
<td>15</td>
<td>225</td>
<td>8.375</td>
</tr>
<tr>
<td>20</td>
<td>400</td>
<td>11.0</td>
</tr>
<tr>
<td>25</td>
<td>625</td>
<td>14.375</td>
</tr>
</tbody>
</table>

The indifference curve is depicted by the bold line in the following graph (labeled Q3, for Question 3).
4. Repeating the analysis in Problem 3, utility is:

\[ U = E(r) - .005A \sigma^2 = E(r) - .02 \sigma^2 = 4 \]

leading to the equal-utility combinations of expected return and standard deviation presented in the table below. The indifference curve is the upward sloping line appearing in the graph of Problem 3, labeled Q4 (for Question 4).

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>( \sigma^2 )</th>
<th>( E(r) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>0</td>
<td>4.00%</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>4.50</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>6.00</td>
</tr>
<tr>
<td>15</td>
<td>225</td>
<td>8.50</td>
</tr>
<tr>
<td>20</td>
<td>400</td>
<td>12.00</td>
</tr>
<tr>
<td>25</td>
<td>625</td>
<td>16.50</td>
</tr>
</tbody>
</table>

The indifference curve in Problem 4 differs from that in Problem 3 in both slope and intercept. When \( A \) increases from 3 to 4, the higher risk aversion results in a greater slope for the indifference curve since more expected return is needed to compensate for additional \( \sigma \). The lower level of utility assumed for Problem 4 (4% rather than 5%), shifts the vertical intercept down by 1%.

5. The coefficient of risk aversion of a risk neutral investor is zero. The corresponding utility is simply equal to the portfolio's expected return. The corresponding indifference curve in the expected return-standard deviation plane is a horizontal line, drawn in the graph of Problem 3, and labeled Q5.
6. A risk lover, rather than penalizing portfolio utility to account for risk, derives greater utility as variance increases. This amounts to a negative coefficient of risk aversion. The corresponding indifference curve is downward sloping, as drawn in the graph of Problem 3, and labeled Q6.

7. c [Utility for each portfolio = \( E(r) – 0.005 \times 4 \times \sigma^2 \). We choose the portfolio with the highest utility value.]

8. d [When investors are risk neutral, \( A = 0 \), and the portfolio with the highest utility is the one with the highest expected return.]

9. b

10. The column labeled \( U(A = 5) \) in the table above is computed from \( U = E(r) – 0.005 \times A \times \sigma^2 = E(r) – 0.025 \sigma^2 \) (since \( A = 5 \)). It shows that the more risk averse investors will prefer the position with 40% in the market index portfolio, rather than the 80% market weight preferred by investors with \( A = 3 \).

13. SugarKane is now less of a hedge, and the entire probability distribution is:

<table>
<thead>
<tr>
<th></th>
<th>Normal Sugar Crop</th>
<th>Sugar Crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bullish Stock Market</td>
<td>.5</td>
<td>.3</td>
</tr>
<tr>
<td>Bearish Stock Market</td>
<td>.5</td>
<td>.3</td>
</tr>
<tr>
<td>Probability</td>
<td>.5</td>
<td>.3</td>
</tr>
<tr>
<td>Best Candy</td>
<td>25%</td>
<td>10%</td>
</tr>
<tr>
<td>SugarKane</td>
<td>10</td>
<td>-5</td>
</tr>
<tr>
<td>Humanex's Portfolio</td>
<td>17.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Using the portfolio rate of return distribution, its expected return and standard deviation can be calculated as follows:

\[
E(r_p) = 0.5 \times 17.5 + 0.3 \times 2.5 + 0.3 \times (-2.5) = 9\%
\]

\[
\sigma_p = [0.5(17.5 - 9)^2 + 0.3(2.5 - 9)^2 + 0.2(-2.5 - 9)^2]^{1/2} = 8.67\%
\]

While the expected return has even improved slightly, the standard deviation is significantly greater and only marginally better than investing half in T-bills.
14. The expected return of Best is 10.5% and its standard deviation 18.9%. The mean and standard deviation of SugarKane are now:

\[ E(r_{SK}) = .5 \times 10 + .3 \times (-5) + .2 \times 20 = 7.5\% \]

\[ \sigma_{SK} = \left[ .5(10 - 7.5)^2 - .3(-5 - 7.5)^2 + .2(20 - 7.5)^2 \right]^{1/2} = 9.01\% \]

and its covariance with Best is

\[ \text{Cov} = .5 \times 10 \times 7.5 + .3 \times -5 \times -5 + .2 \times 20 \times -25 = -68.75 \]

15. From the calculations in (14), the portfolio expected rate of return is

\[ E(r_p) = .5 \times 10.5 + .5 \times 7.5 = 9\% \]

Using the portfolio weights \( w_B = w_{SK} = .5 \) and the covariance between the stocks, we can compute the portfolio standard deviation from rule 5.

\[ \sigma_p = \left[ w_B^2 \sigma_B^2 + w_{SK}^2 \sigma_{SK}^2 + 2w_Bw_{SK}\text{Cov}(r_B,r_{SK}) \right]^{1/2} \]

\[ = \left[ .5^2 \times 18.9^2 + .5^2 \times 9.01^2 + 2 \times .5 \times .5 \times (-68.75) \right]^{1/2} = 8.67\% \]

4. Solutions to Question in Chapter 7 (except those in PS 3)

1. Expected return = .3 \times 8\% + .7 \times 18\% = 15\% per year.

   Standard deviation = .7 \times 28\% = 19.6\%

2. Investment proportions:
   \[ .7 \times 25\% = 17.5\% \text{ in stock A} \]
   \[ .7 \times 32\% = 22.4\% \text{ in stock B} \]
   \[ .7 \times 43\% = 30.1\% \text{ in stock C} \]

3. Your reward-to-variability ratio = \( \frac{18 - 8}{28} = .3571 \)

   Client's reward-to-variability ratio = \( \frac{15 - 8}{19.6} = .3571 \)
5. a. \[ E(r_C) = r_f + E[(r_p) - r_f] \ y = 8 + 10y \]

If the expected return of the portfolio is equal to 16%, then solving for \( y \) we get:

\[ 16 = 8 + 10 \ y, \quad \text{and} \quad y = \frac{16 - 8}{10} = .8 \]

Therefore, to get an expected return of 16% the client must invest 80% of total funds in the risky portfolio and 20% in T-bills.

b. **Investment proportions of the client's funds:**

- 20% in T-bills,
- \(.8 \times 25\% = 20.0\% \) in stock A
- \(.8 \times 32\% = 25.6\% \) in stock B
- \(.8 \times 43\% = 34.4\% \) in stock C

c. \[ \sigma_c = .8 \times \sigma_p = .8 \times 28\% = 22.4\% \text{ per year} \]

6. a. \[ \sigma_c = y \times 28\%. \text{ If your client wants a standard deviation of at most 18\%, then} \]

\[ y = \frac{18}{28} = .6429 = 64.29\% \text{ in the risky portfolio.} \]

b. \[ E(r_C) = 8 + 10y = 8 + .6429 \times 10 = 8 + 6.429 = 14.429\% \]

7. a. \[ y^* = \frac{E(r_p) - r_f}{.01 \times A \sigma_p^2} = \frac{18 - 8}{.01 \times 3.5 \times 28^2} = \frac{10}{27.44} = .3644 \]

So the client's optimal proportions are 36.44% in the risky portfolio and 63.56% in T-bills.

b. \[ E(r_C) = 8 + 10y^* = 8 + .3644 \times 10 = 11.644\% \]
\[ \sigma_c = .3644 \times 28 = 10.20\% \]

8. a. **Slope of the CML** = \[ \frac{13 - 8}{25} = .20 \]

The diagram follows.
b. My fund allows an investor to achieve a higher mean for any given standard deviation than would a passive strategy, i.e., a higher expected return for any given level of risk.

![Graph showing CML and CAL]

CAL: Slope = .3571
CML: Slope = .20

9. a. With 70% of his money in my fund's portfolio the client gets a mean return of 15% per year and a standard deviation of 19.6% per year. If he shifts that money to the passive portfolio (which has an expected return of 13% and standard deviation of 25%), his overall expected return and standard deviation become:

\[ E(r_C) = r_f + .7[E(r_M) - r_f] \]

In this case, \( r_f = 8\% \) and \( E(r_M) = 13\% \). Therefore,

\[ E(r_C) = 8 + .7 \times (13 - 8) = 11.5\% \]

The standard deviation of the complete portfolio using the passive portfolio would be:

\[ \sigma_C = .7 \times \sigma_M = .7 \times 25\% = 17.5\% \]

Therefore, the shift entails a decline in the mean from 14\% to 11.5\% and a decline in the standard deviation from 19.6\% to 17.5\%. Since both mean return and standard deviation fall, it is not yet clear whether the move is beneficial or harmful. The disadvantage of the shift is that if my client is willing to accept a mean return on his total portfolio of 11.5\%, he can achieve it with a lower standard deviation using my fund portfolio, rather than the
passive portfolio. To achieve a target mean of 11.5%, we first write the mean of the complete portfolio as a function of the proportions invested in my fund portfolio, \( y \):

\[
E(r_C) = 8 + y(18 - 8) = 8 + 10y
\]

Because our target is: \( E(r_C) = 11.5\% \), the proportion that must be invested in my fund is determined as follows:

\[
11.5 = 8 + 10y, \quad y = \frac{11.5 - 8}{10} = .35
\]

The standard deviation of the portfolio would be: \( \sigma_C = y \times 28\% = .35 \times 28\% = 9.8\% \). Thus, by using my portfolio, the same 11.5% expected return can be achieved with a standard deviation of only 9.8% as opposed to the standard deviation of 17.5% using the passive portfolio.

b. The fee would reduce the reward-to-variability ratio, i.e., the slope of the CAL. Clients will be indifferent between my fund and the passive portfolio if the slope of the after-fee CAL and the CML are equal. Let \( f \) denote the fee.

Slope of CAL with fee = \[
\frac{18 - 8 - f}{28} = \frac{10 - f}{28}
\]

Slope of CML (which requires no fee) = \[
\frac{13 - 8}{25} = .20
\] Setting these slopes equal we get:

\[
\frac{10 - f}{28} = .20
\]

\[
10 - f = 28 \times .20 = 5.6
\]

\[
f = 10 - 5.6 = 4.4\% \text{ per year}
\]

10. a. The formula for the optimal proportion to invest in the passive portfolio is:

\[
y^* = \frac{E(r_M) - r_f}{.01 \times A\sigma_M^2}
\]

With \( E(r_M) = 13\%; \ r_f = 8\%; \sigma_M = 25\%; \ A = 3.5 \), we get
\[
y^* = \frac{13 - 8}{.01 \times 3.5 \times 25^2} = .2286
\]

b. The answer here is the same as in 9b. The fee that you can charge a client is the same regardless of the asset allocation mix of your client's portfolio. You can charge a fee that will equalize the reward-to-variability ratio of your portfolio with that of your competition.

11. a. If 1926 - 1999 is assumed to be representative of future expected performance, \( A = 4 \), \( E(r_M) - r_f = 9.3\% \), and \( \sigma_M = 20.6\% \) (we use the standard deviation of the risk premium from the last column of Table 7.4), then \( y^* \) is given by:

\[
y^* = \frac{E(r_M) - r_f}{.01 \times A \sigma_M^2} = \frac{9.3}{.01 \times 4 \times 20.6^2} = .5479
\]

That is, 54.79\% should be allocated to equity and 45.21\% to bills.

b. If 1980 - 1999 is assumed to be representative of future expected performance, \( A = 4 \), \( E(r_M) - r_f = 11.6\% \); and \( \sigma_M = 13.8\% \), then \( y^* \) is given by:

\[
y^* = \frac{11.6}{.01 \times 4 \times 13.8^2} = 1.523
\]

Therefore, 152.3\% of the complete portfolio is allocated to equity. This is accomplished by borrowing 52.3\% of wealth and investing the entire investment portfolio (including the borrowed funds) in equity.

c. In (b) the market risk premium and the market risk are both expected to be at a lower level than in (a). The fact that the reward-to-variability ratio is expected to be higher explains the greater proportion invested in equity.

12. Assuming no change in tastes, that is, an unchanged risk aversion coefficient, \( A \), the denominator of the equation for the optimal investment in the risky portfolio will be higher. The proportion invested in the risky portfolio will depend on the relative change in the expected risk premium (the numerator) compared to the change in the perceived market risk. Investors perceiving higher risk will demand a higher risk premium to hold the same portfolio they held before. If we assume that the risk-free rate is unaffected, the increase in the risk premium would require a higher expected rate of return in the equity market.

14. Data: \( r_f = 5\% \), \( E(r_M) = 13\% \), and \( \sigma_M = 25\% \). In addition, \( r_f = 9\% \). Therefore, the CML and indifference curves are as follows:
For $y$ to be less than 1.0 (so that the investor is a lender), risk aversion must be large enough that:

$$y = \frac{E(r_M) - r_f}{.01 \times A \sigma_M^2} < 1$$

$$A > \frac{13 - 5}{.01 \times 25^2} = 1.28$$

For $y$ to be greater than 1.0 (so that the investor is a borrower), risk aversion must be small enough that:

$$y = \frac{E(r_M) - r_f^B}{.01 \times A \sigma_M^2} > 1$$

$$A < \frac{13 - 9}{.01 \times 625} = .64$$

For values of risk aversion within this range, the investor neither borrows nor lends, but instead holds a complete portfolio comprised only of the optimal risky portfolio:
y = 1 for \(0.64 \leq A \leq 1.28\)

16. a. The graph of problem 14 has to be redrawn here with \(E(r_p) = 11\%\) and \(\sigma_p = 15\%\)

b. For a lending position, \(A > \frac{11 - 5}{0.01 \times 225} = 2.67\)

For a borrowing position, \(A < \frac{11 - 9}{0.01 \times 225} = 0.89\)

In between, \(y = 1\) for \(0.89 \leq A \leq 2.67\)

17. The maximum feasible fee, denoted \(f\), depends on the reward-to-variability ratio.

For \(y < 1\), the lending rate, 5\%, is viewed as the relevant risk-free rate, and we solve for \(f\) from:

\[
\frac{11 - 5 - f}{15} = \frac{13 - 5}{25}
\]
\[ f = 6 - \frac{15 \times 8}{25} = 1.2\% \]

For \( y > 1 \), the borrowing rate, 9%, is the relevant risk-free rate. Then we notice that even without a fee, the active fund is inferior to the passive fund because:

\[ \frac{11 - 9}{15} = .13 < \frac{13 - 9}{25} = .16 \]

More risk tolerant investors (who are more inclined to borrow) therefore will not be clients of the fund even without a fee. (If you solved for the fee that would make investors who borrow indifferent between the active and passive portfolio, as we did above for lending investors, you would find that \( f \) is negative: that is, you would need to pay them to choose your active fund.) The reason is that these investors desire higher risk-higher return complete portfolios and thus are in the borrowing range of the relevant CAL. In this range the reward to variability ratio of the index (the passive fund) is better than that of the managed fund.

18. b
19. a
21. b
23. (a) is the correct choice:

\[
\text{Reward to variability ratio} = \frac{\text{Risk premium}}{\text{Standard deviation}} = \frac{10}{14} = .71.
\]