1. Consider the long-run employment decision of a perfectly competitive firm, and imagine this firm facing a monthly salary of $2,550 per worker and monthly capital costs of $1,700 per unit. Suppose the cost-minimizing level of employment is $E^* = 120$ workers, and that this cost-minimizing level is attained at a cost outlay of $C_0 = 663,000$ per month.

(a) What is the optimal level of units of capital $K^*$ associated with $E^*$?

(b) Which condition has to hold at the cost-minimizing allocation of capital and labor? Give an interpretation of this condition.

(c) Sketch the solution in a capital/labor diagram. Then suppose that the monthly salary decreases to $2,040 per worker. What can we say about the new cost-minimizing point of $K$ and $E$? Specifically, what happens to the isocost line? Will the firm increase or decrease its initial cost outlay $C_0$? Why?

(a) A firm's production costs are given by the function $C = wE + rK$. If we know the input prices $w$ and $r$, the cost outlay $C_0$, and the optimal level of employment $E^*$, we can rewrite this function and solve for the optimal level of $K^*$ associated with $E^*$:

$$K^* = \frac{C_0}{r} \frac{w}{r} E^* = \frac{663,000}{1,700} - \frac{2,550}{1,700} \times 120 = 210.$$

The cost-minimizing level of capital is $K^* = 210$ units.

(b) The firm minimizes its costs using the capital-labor-combination where the isocost is tangent to the isoquant, i.e. cost minimization requires that the marginal rate of substitution equals the ratio of prices: $\frac{MP_E}{MP_K} = \frac{w}{r}$.

Rewriting this condition shows that the output yield of the last dollar spent on capital ($=MP_K/r$) must equal the output yield of the last dollar spent on labor ($=MP_E/w$).

(c) Our initial cost-optimum is given by the isocost line $K = 390 - 1.5 \times E$ and some isoquant (capturing production level $q_0$) with the tangency point at $E^* = 120$ and $K^* = 210$. This optimum is point P in the following graph:
If the wage decreases to $2,050, the isocost becomes flatter, since its slope is given by the negative of the ratio of the input prices \(-w/r\). As the wage falls, the firm increases \(E\) and moves to new cost-minimizing \(R\). This move can be decomposed into two effects, as illustrated by the graph:

The move from \(P\) to \(Q\) shows the scale effect, i.e. the effect on the demand for firm's inputs as the firm expands production, taking advantage of the lower price of labor. This is illustrated using a hypothetical budget line \(DD\) tangent to the new isoquant (= expanded production level \(q_1\)) and parallel to the isocost that the firm faced before the wage reduction. Assuming that capital and labor are "normal inputs", the scale effect increases both the firm's employment and the capital stock.

The move from \(Q\) to \(R\) is the substitution effect, i.e. the effect on the firm's employment as the wage changes, holding output constant. It implies that firms will always substitute towards the input that has become relatively cheaper, i.e., holding output constant at \(q_1\), the firm moves down the isoquant and in this case adopts a more labor-intensive input mix.

Both substitution and scale effects induce the firm to increase employment as the wage falls, i.e. \(Q\) must lie to the right of \(P\), and \(R\) must lie to the right of \(Q\). We also know that the substitution effect must decrease the firm's demand for capital, i.e. \(R\) must lie below \(Q\). However, we do not know whether the new optimum \(R\) will lie above (as drawn in the figure) or below \(P\), i.e. whether the firm hires more or less capital as the wage falls. The latter is the case when the substitution effect dominates the scale effect, and the former when the scale effect dominates the substitution effect – again, this is the scenario depicted in the graph. And in this scenario (scale>substitution, and firm uses less capital) the intercept of the new isocost \(C_1/r\) will lie above the initial intercept \(C_0/r\), indicating – because \(r\) is constant – that the firm increased its cost outlay. If the substitution effect dominated the scale effect, this would imply a reduction in the cost outlay.
2. Consider a perfectly competitive firm with a production function
\[ q = f(x_1, x_2, x_3), \]
where \( x_1 \) denotes units of capital, \( x_2 \) denotes unskilled workers, and \( x_3 \) denotes skilled workers. The firm maximizes profits using an input mix of \( x_1 = 200 \), \( x_2 = 70 \), and \( x_3 = 50 \). The initial price of capital is $80 per unit. Suppose the firm faces a change in the price of capital to $96 per unit. As a consequence, it increases its employment of unskilled workers to 77, while at the same time laying off 5 skilled workers.

(a) What is the cross-elasticity of demand for unskilled workers with respect to capital?

(b) What is the cross-elasticity of demand for skilled workers with respect to capital?

(c) Are the two inputs "unskilled labor" and "capital" substitutes or complements? Why?

(d) Are the two inputs "skilled labor" and "capital" substitutes or complements? Why?

(e) Are the results from (a) through (d) consistent with the so-called "capital-skill complementarity hypothesis"? Why (not)?

We use the cross-elasticity of factor demand to measure the demand sensitivity for a specific input to the price of some other input. Denote US=Unskilled and S=Skilled.

(a) \[ \eta_{US,K} = \frac{(77 - 70)}{70} \div \frac{(96 - 80)}{80} = 0.5 \text{, i.e. a 10% increase in the price of capital increases demand for unskilled labor by 5\%.} \]

(b) \[ \eta_{S,K} = \frac{(45 - 50)}{50} \div \frac{(96 - 80)}{80} = -0.5 \text{, i.e. a 10% increase in the price of capital reduces demand for skilled labor by 5\%.} \]

(c) Since the cross-elasticity of demand for unskilled labor with respect to the price of capital is positive, the two are substitutes. As the price for capital rises (and thus the demand for capital falls), the demand for unskilled labor increases, i.e. the firm reduces its use of the now more expensive input (capital) and replaces it with the relatively cheaper input, unskilled labor.

(d) Since the cross-elasticity of demand for skilled labor with respect to the price of capital is negative, the two are complements. The demand for skilled labor falls as a result of an increase in the price of capital, i.e. the two inputs "go together" and respond in a same way to the change in the price of capital.

(e) The results from (a) through (d) are entirely consistent with the capital-skill complementarity-hypothesis, which says that unskilled labor and capital are substitutes, and that skilled labor and capital are complements.
3. In a particular industry, labor supply is \( ES = 10 + w \) while labor demand is \( ED = 40 - 4w \), where \( E \) is the level of employment and \( w \) is the hourly wage.

(a) What is the equilibrium wage and employment if the labor market is competitive? What is the unemployment rate?

In equilibrium, the quantity of labor supplied equals the quantity of labor demanded, so that \( ES = ED \). This implies that \( 10 + w = 40 - 4w \). The wage rate that equates supply and demand is $6. When the wage is $6, 16 persons are employed. There is no unemployment because the number of persons looking for work equals the number of persons employers are willing to hire.

(b) Suppose the government sets a minimum hourly wage of $8. How many workers would lose their jobs? How many additional workers would want a job at the minimum wage? What is the unemployment rate?

If employers must pay a wage of $8, employers would only want to hire \( ED = 40 - 4(8) = 8 \) workers, while \( ES = 10 + 8 = 18 \) persons would like to work. Thus, 8 workers lose their job following the minimum wage and 2 additional people enter the labor force. Under the minimum wage, the unemployment rate would be 10/18, or 55.6 percent.

(c) What can you say about the efficiency of the employment outcome once the minimum wage is imposed? Compare producer and worker surplus with the competitive equilibrium. Is there a deadweight loss?

In the competitive equilibrium the total gain from trade is defined by the sum of the producer surplus, the area of triangle in the graph, and the worker surplus, the area of triangle \( Q \). Because the competitive equilibrium provides an efficient allocation of labor the total gain is maximal. In the equilibrium outcome with minimum wage the producer surplus shrinks to the area of triangle \( P' \), and the worker surplus is given by triangle \( Q' \). Note that the worker surplus may either be larger or smaller, depending on the labor supply elasticity and the minimum wage level. The total gain of trade, however, is smaller than in the competitive equilibrium, and the deadweight loss is given by triangle \( DW \). This implies that the equilibrium outcome with minimum wage is not an efficient allocation of labor.
4. Suppose the supply curve in a particular industry is given by \( w = 10 + 1.2E \) and the demand curve in that industry is given by \( w = 50 - 0.8E \), where \( E \) = employment and \( w \) = hourly wage.

(a) Calculate the equilibrium wage and employment level in that industry. Illustrate your solution in a graph.

(b) Now suppose the government imposes a payroll tax of $8 on the firm for every employee-hour it hires. What happens to the initial equilibrium in that industry? In your explanation, also calculate the new equilibrium wage and employment level, and illustrate your solution in a graph [you can include the solution in the graph you already drew in part (a)].

(c) In the new equilibrium, what is the total cost of hiring an employee-hour of work? Therefore, what share of the $8 tax will the firm pay, and how much will be shifted to the worker? In what respect does the fraction of the payroll tax that workers pay depend on the elasticity of the supply curve?

(d) Does it matter whether the $8 payroll tax is imposed on the firm or on the worker? Why (not)?

(a) The initial equilibrium is given by the point where supply equals demand:
\[
10 + 1.2E = 50 - 0.8E
\]
\[
\Rightarrow E_0 = 20, \quad w_0 = 34
\]
(b) Because firms are only willing to pay a total of $w_0=34$ for each employee-hour to hire an employment level of $E_0=20$ workers, they will now only want to pay $w_0 - 8 = 34 - 8 = 26$ for each employee-hour due to the tax. This shifts the initial demand curve $D_0$ down to $D_1$: $w = 42 - 0.8E$, as illustrated in the graph. $D_1$ reflects the wedge that exists between (i) the total amount firms must pay ($w_0$) and (ii) the amount workers actually receive ($w_0 - 8$). The new equilibrium is given by the point where the initial supply curve intersects the new demand curve:

$$10 + 1.2E = 42 - 0.8E$$

$$\Rightarrow E_1 = 16, \quad w_1 = 29.2$$

i.e. $w_1=29.2$ is the wage rate actually received by workers.

(c) The total cost of hiring an employee-hour of work is given by $w_1 + 8 = 29.2 + 8 = 37.2$.

The firm's share of the tax is given by $|w_1 + 8 - w_0| = 37.2 - 34 = 3.2$ (illustrated by $\Delta_1$ in the graph).

The worker's share of the tax is given by $|w_0 - w_1| = 34 - 29.2 = 4.8$ (illustrated by $\Delta_2$ in the graph).

In this case, the firm shifts a fraction of 4.8/8=60% of the tax to the worker. In general, the more inelastic the supply curve, the greater the fraction of the payroll tax that workers end up paying.

(d) In a competitive market, it does not matter whether the payroll tax is imposed on the firm or on the worker. Even though a payroll tax assessed on the firm shifts down the demand curve, it has the same labor market impacts as a revenue-equivalent payroll tax assessed on workers, which shifts up the supply curve.

Illustration + calculations [not required in solution]:

The new equilibrium is given by

$$18 + 1.2E = 50 - 0.8E$$

$$\Rightarrow E_1 = 16, \quad w_1 = 37.2$$

where the firm's total cost of hiring is given by $w_1 = 37.2$ (compare with same result above!)

and the worker's actual after-tax wage is $w_1 - 8 = 29.2$ (ditto).

The firm's share of the tax is $|w_1 - w_0| = 37.2 - 34 = 3.2$ ($\Delta_1$)

and the worker's share is $|w_0 - (w_1 - 8)| = 34 - 29.2 = 4.8$ ($\Delta_2$).