Econ 100B: Macroeconomic Analysis  
Fall 2008  

Problem Set #9 ANSWERS  
(Due September 29 - 30, 2008)

A. Solow Growth Model:

1. Clearly and accurately draw and label a diagram of the Solow Growth Model.

```
  | Y/N, S/N, Ib/N
  | Y/N

  | S/N, Ib/N
  | S/N = s * Y/N
  | “A”

  | K/N

  | K/N

Y/N = A₀ * f(K/N)

Ib/N = (n + d) * K/N

S/N = s * Y/N
```

2. Provide an economic explanation of the shape of the curve(s) in your diagram in #1.

The per-worker production function shows a **positive relationship** between income-per-worker, Y/N, and the capital-to-labor ratio, K/N, for a given level of productivity, A.

The per-worker production function slopes upward because there are **increasing returns to a higher capital-to-labor ratio**, i.e., as the capital-to-labor ratio increases, income-per-worker also increases.

The slope of the per-worker production function decreases as the capital-to-labor ratio increases because of the **diminishing marginal product of capital**, i.e., as the capital-to-labor ratio increases, income-per-worker also increase but by progressively smaller and smaller amounts.

The per-worker saving function also shows a **positive relationship** between saving-per-worker, S/N, and the capital-to-labor ratio, K/N. Because of the assumption of a constant saving rate, the per-worker saving function is simply proportional to the per-worker production function, the proportion dependent on the saving rate.
The per-worker balanced investment function also shows a positive relationship between balanced-investment-per-worker, \( \text{Ib}/N \), and the capital-to-labor ratio, \( K/N \). Balanced-investment-per-worker, \( \text{Ib}/N \), is proportional to the capital-to-labor ratio, \( K/N \), with the degree of proportionality dependent upon the labor force growth rate, \( n \), and the depreciation rate, \( d \).

3. List the endogenous and exogenous variables in this model.

**Endogenous variables:** Income-per-worker, \( Y/N \), saving-per-worker, \( S/N \), which is also actual investment-per-worker, \( I_a/N \), balanced-investment-per-worker, \( \text{Ib}/N \), and the capital-to-labor ratio, \( K/N \).

**Exogenous variables:** Productivity or technology, \( A \), the saving rate, \( s \), the growth rate of the labor force, \( n \), and the depreciation rate, \( d \).

4. List the variables (and the direction of their change) that would shift the production function higher. Also provide an economic explanation for why each of these variables would shift the production function.

*Increases in productivity, \( A \), shift the per-worker production function higher. At every capital-to-labor ratio, an increase in productivity allows the economy to produce a greater level of output-per-worker.*

5. List the variables (and the direction of their change) that would shift the saving function higher. Also provide an economic explanation for why each of these variables would shift the saving function.

*Increases in productivity, \( A \), shift the per-worker production function higher. At every capital-to-labor ratio, an increase in productivity allows the economy to produce a greater level of output-per-worker. A higher level of output-per-worker with a constant saving rate generates a higher level of saving-per-worker, represented by an upward shift of the per-worker saving function.*

*Increases in the saving rate, \( s \), shift the per-worker saving function higher. At every capital-to-labor ratio, the per-worker production function indicates the level of income-per-worker than is generated. A higher saving rate would increase the level of saving-per-worker at every income-per-worker, represented by an upward shift of the per-worker saving function.*

6. List the variables (and the direction of their change) that would rotate the balanced investment function higher. Also provide an economic explanation for why each of these variables would shift the balanced investment function.

*An increase in the labor force growth rate, \( n \), would rotate the per-worker balanced investment function higher. If the labor force begins to grow more quickly, then in order to maintain a constant capital-to-labor ratio, actual investment per worker would have to increase in order to equip each new worker with the same amount of capital that each member of the existing labor force initially had.*

*An increase in the depreciation rate, \( d \), rotates the per-worker balanced investment function higher. If the capital stock (per worker) begins to wear out at a faster rate, then in order to maintain a constant capital-to-labor ratio, actual investment per worker would have to increase in order to replace the additional capital stock that is wearing out.*
7. Assume that the economy starts in equilibrium. Suppose now that there is an increase in technology. Describe the adjustment process that moves the economy from its initial equilibrium to its final equilibrium.

An increase in technology or productivity will shift both the per-worker production function and the per-worker saving function higher. At the initial capital-to-labor ratio, saving-per-worker and actual investment-per-worker have now increased and are greater than balanced-investment-per-worker, i.e., actual investment per worker is greater than is necessary in order to keep the capital-to-labor ratio constant.

Because actual-investment-per-worker is greater than balanced-investment-per-worker at the initial capital-to-labor ratio, the capital-to-labor ratio will begin to increase. As the capital-to-labor ratio increases, income-per-worker will increase along the new, higher per-worker production function. In addition, saving-per-worker and actual-investment-per-worker will also increase along the new, higher per-worker saving function. Finally, as the capital-to-labor ratio increases, balanced-investment-per-worker will also increase along the original per-worker balanced investment function.

However, balanced-investment-per-worker increases faster than actual-investment-per-worker (or saving-per-worker) because increases in saving-per-worker is subject to diminishing returns to the capital-to-labor ratio while balanced-investment-per-worker is not.

The capital-to-labor ratio will continue to increase until saving per-worker is once again equal to balanced investment per-worker. At the new, higher steady state, the capital-to-labor ratio will have increased. Income-per-worker will also have increased, both because of the increase in the capital-to-labor ratio and also because of the increase in productivity. Saving-per-worker will also have increased, both because of the increase in the capital-to-labor ratio and also because of the increase in productivity. The higher capital-to-labor ratio also increases balanced-investment-per-worker.