The Production Function

• A production function shows how businesses transform factors of production into output of goods and services through the applications of technology.

Factors of production:
- Capital ($K$)
- Labor ($N$)
- Other (raw materials, land, energy, etc.)

The productivity of factors depends on technology and management ($A$).
The Production Function

- The economy’s production function is:

\[ Y = AF(K, N) \]

- Shows how much output \( Y \) can be produced from a given amount of capital \( K \) and labor \( N \) and a given level of technology \( A \).

- The parameter \( A \) is “total factor productivity” or the effectiveness with which \( K \) and \( N \) are used.

The Production Function

- A more specific production function that works well in macroeconomics is the Cobb-Douglas production function.

\[ Y = AK^\alpha N^{1-\alpha} \]

- For the U.S. economy it would be:

\[ Y = AK^{0.3}N^{0.7} \]
The Production Function

- The Production Function: Output and Capital
  
  ➢ Two main properties of this production function:
    
    • Exhibits increasing returns to capital.
      - Slopes upward because more K produces more Y.
    
    • Exhibit diminishing marginal product of capital.
      - Slope becomes flatter because each additional increment of K produces less additional Y.

The Production Function

- The Production Function: Output and Capital
  
  ➢ Marginal product of capital, \( MPK = \frac{\Delta Y}{\Delta K} \).
    
    • Equals the slope of production function graph (\( Y \) vs. \( K \)).
    
    • \( MPK \) is always positive.
    
    • \( MPK \) declines as \( K \) increases.

The Marginal Product of Capital

\[ Y = A_F(K, N) \]

The Marginal Product of Capital

\[ MPK \]

\[ K \]
The Production Function

• The Production Function: Output and Capital

  ➢ What happens if $N$ or $A$ changes?

The Production Function: Output & Capital

\[ Y = A_0 F(K, N_0) \]

The Production Function

• The Production Function: Output and Labor

  ➢ Shows how $Y$ depends on $N$ for a given $K$ and $A.$
The Production Function

- The Production Function: Output and Labor

  ➢ Two main properties of this production function:
    
    - Exhibits increasing returns to labor.
      - Slopes upward because more N produces more Y.
    
    - Exhibit diminishing marginal product of labor.
      - Slope becomes flatter because each additional increment of N produces less additional Y.

The Production Function

- The Production Function: Output and Labor

  ➢ Marginal product of labor, $MPN = \Delta Y/\Delta N$.
    
    - Equals the slope of production function graph ($Y$ vs. $N$).
    
    - $MPN$ is always positive.
    
    - $MPN$ declines as $N$ increases.

The Marginal Product of Labor

\[ Y = A_0F(K_0, N) \]

The Marginal Product of Labor

\[ MPN \]
The Production Function

• The Production Function: Output and Labor
  ➢ What happens if $K$ or $A$ changes?

Y = A_0 F(K_0, N)

The Production Function

• Productivity is calculated as a residual:

$$A = \frac{Y}{K^{0.3} N^{0.7}}$$

• Productivity growth is calculated as:

$$\% \Delta A = \Delta A / A \times 100$$

The Production Function: Output & Labor

• Observations about productivity growth:
  ➢ Productivity moves sharply from year to year.
  ➢ Productivity grew strongly from the mid-1950’s through 1973, very slowly from 1973 through 1995, and more quickly again since 1995.
The Production Function

• Supply shocks:
  - Supply shocks affect the amount of output that can be produced for a given amount of inputs.
  - Also called productivity shocks.

The Production Function

• Supply shocks:
  - Supply shocks shift the production function.
    - **Negative or adverse shock:** A decline in \( A \) usually causes the slope of production function to decrease at each level of input.
    - **Positive or beneficial shock:** An increase in \( A \) usually causes the slope of production function to increase at each level of input.

Effects of an Adverse Supply Shock

\[ Y = A_f(K, N) \]

The Demand for Labor

• The demand for labor is determined by individual business firms.
  - The **aggregate demand for labor** is the sum of all the business firms’ demand for labor.

• The demand for labor depends on the costs and benefits of hiring additional workers.
The Demand for Labor

• How much labor do firms want to use?

➢ Assumptions:
  • The capital stock fixed, i.e., a short-run analysis.
  • Workers are homogeneous.
  • The labor market is competitive.
  • Firms maximize profits.

The Demand for Labor

• What is the cost of hiring one more worker?

➢ The marginal cost of hiring one more worker is the cost of that worker to the firm, i.e., the nominal wage:

\[ W \]

The Demand for Labor

• What is the benefit of hiring one more worker?

➢ The benefit of hiring one more worker is the additional income that the worker generates, i.e., the marginal revenue product of labor:

\[ MRPN = P \times MPN \]

The Demand for Labor

• How much labor do firms want to use?

➢ A profit-maximizing firm will hire additional workers up to the point where the marginal revenue product of labor equals the nominal wage:

\[ W = MRPN = P \times MPN \]
The Demand for Labor

• How much labor do firms want to use?
  ➢ This equilibrium condition:
  \[ W = MRPN = P \times MPN \]
  ➢ can be re-written as:
  \[ w = MPN \]
  • because \( w = \frac{W}{P} \) and \( MRPN = P \times MPN \).

Marginal Cost of Hiring an Extra Worker

Marginal Benefit of Hiring an Extra Worker

The Determination of Labor Demand
The Demand for Labor

• How much labor do firms want to use?
  ➢ Costs and benefits of hiring one extra worker.
    • If \( w > MPN \), profits rise if number of workers declines.
    • If \( w < MPN \), profits rise if number of workers increases.
    • When \( w = MPN \), profits are maximized.

The Demand for Labor

• How much labor do firms want to use?
  ➢ The labor demand curve shows the relationship between the real wage rate (\( w \)) and the quantity of labor demanded (\( N \)).

Determination of the Labor Demand Curve

The Demand for Labor

• The Labor Demand Curve, \( N_D \).
  ➢ Changing the real wage rate:
    • An increase in the real wage rate means \( w > MPN \) unless \( N \) is reduced so the \( MPN \) increases.
    • A decrease in the real wage rate means \( w < MPN \) unless \( N \) is increased so the \( MPN \) decreases.
The Demand for Labor

- The Labor Demand Curve, $N_D$.
  - The labor demand curve is downward sloping.
    - The higher the real wage, the less labor firms will hire.
  - Because $w = MPN$ in equilibrium (regardless of what $w$ is), the $N_D$ curve is the same as the $MPN$ curve.

The (Aggregate) Labor Demand Curve

- Factors that shift the labor demand curve:
  - Changes in the capital stock, $\Delta K$.
    - Increases in $K$ raise $MPN$ and shift the labor demand curve to the right.
  - Supply shocks, $\Delta A$.
    - Beneficial supply shocks raise $MPN$ and shift the labor demand curve to the right.

Effect of an Increase in $K$ or $A$
Key Diagram #1: The Production Function

\[ Y = A_0 F(K_0, N) \]

Key Diagram #2a: Demand for Labor

Key Diagrams #1 & #2a.

- Factors that Shift the Production Function and the Demand for Labor:
  - Increases in the capital stock, \( K \), shift the production function higher, increase the MPN and the demand for labor.
  - Increases in productivity, \( A \), shift the production function higher, increase the MPN and the demand for labor.