The Production Function

- A production function shows how businesses transform factors of production into output of goods and services through the applications of technology.

Factors of production:
- Capital (K)
- Labor (N)
- Other (raw materials, land, energy, etc.)

- The productivity of factors depends on technology and management (A).
The Production Function

• The economy’s production function is:

\[ Y = AF(K, N) \]

➢ Shows how much output (Y) can be produced from a given amount of capital (K) and labor (N) and a given level of technology (A).

➢ The parameter \( A \) is “total factor productivity” or the effectiveness with which \( K \) and \( N \) are used.

The Production Function

• A more specific production function that works well in macroeconomics is the Cobb-Douglas production function.

\[ Y = AK^\alpha N^{(1-\alpha)} \]

• For the U.S. economy it would be:

\[ Y = AK^{0.3}N^{0.7} \]
The Production Function

- The Production Function: Output and Capital

  ➢ Two main properties of this production function:
    • Exhibits increasing returns to capital.
      – Slope upward because more K produces more Y.
    • Exhibit diminishing marginal product of capital.
      – Slope becomes flatter because each additional increment of K produces less additional Y.

  Marginal product of capital, \( MPK = \frac{\Delta Y}{\Delta K} \).
  • Equals the slope of production function graph (\( Y \) vs. \( K \)).
  • \( MPK \) is always positive.
  • \( MPK \) declines as \( K \) increases.

The Marginal Product of Capital

\[ Y = A_0F(K, N_0) \]

The Marginal Product of Capital

\[ MPK \]

\[ K \]
The Production Function

• The Production Function: Output and Capital
  ➢ What happens if \( N \) or \( A \) changes?

\[
Y = A_0 F(K, N_0)
\]

The Production Function: Output & Capital

The Production Function

• The Production Function: Output and Labor
  ➢ Shows how \( Y \) depends on \( N \) for a given \( K \) and \( A \).
The Production Function

- The Production Function: Output and Labor
  - Two main properties of this production function:
    - Exhibits increasing returns to labor.
      - Slopes upward because more N produces more Y.
    - Exhibit diminishing marginal product of labor.
      - Slope becomes flatter because each additional increment of N produces less additional Y.

Marginal product of capital, \( MPN = \frac{\Delta Y}{\Delta N} \).
- Equals the slope of production function graph (Y vs. N).
- \( MPN \) is always positive.
- \( MPN \) declines as N increases.

The Marginal Product of Labor

\[ Y = A_0 F(K_0, N) \]
The Production Function

- The Production Function: Output and Labor
  - What happens if $K$ or $A$ changes?

The Production Function: Output & Labor

\[ Y = A_0 F(K_0, N) \]

The Production Function

- Productivity is calculated as a residual:
  \[ A = \frac{Y}{K^{0.3} N^{0.7}} \]

- Productivity growth is calculated as:
  \[ \% \Delta A = \frac{\Delta A}{A} \times 100 \]

The Production Function

- Observations about productivity growth:
  - Productivity moves sharply from year to year.
  - Productivity grew strongly from the mid-1950’s through 1973, very slowly from 1973 through 1995, and more quickly again since 1995.
The Production Function

- Supply shocks:
  - Supply shocks affect the amount of output that can be produced for a given amount of inputs.
  - Also called productivity shocks.
  - Negative or adverse shock: A decline in $A$ usually causes the slope of production function to decrease at each level of input.
  - Positive or beneficial shock: An increase in $A$ usually causes the slope of production function to increase at each level of input.

The Demand for Labor

- The demand for labor is determined by individual business firms.
  - The aggregate demand for labor is the sum of all the business firms’ demand for labor.
  - The demand for labor depends on the costs and benefits of hiring additional workers.
The Demand for Labor

• How much labor do firms want to use?

  ➢ Assumptions:
    • The capital stock fixed, i.e., a short-run analysis.
    • Workers are homogeneous.
    • The labor market is competitive.
    • Firms maximize profits.

The Demand for Labor

• What is the cost of hiring one more worker?

  ➢ The marginal cost of hiring one more worker is the cost of that worker to the firm, i.e., the nominal wage:

  \[ W \]

The Demand for Labor

• What is the benefit of hiring one more worker?

  ➢ The benefit of hiring one more worker is the additional income that the worker generates, i.e., the marginal revenue product of labor:

  \[ MRPN = P \times MPN \]

The Demand for Labor

• How much labor do firms want to use?

  ➢ A profit-maximizing firm will hire additional workers up to the point where the marginal revenue product of labor equals the nominal wage:

  \[ W = MRPN = P \times MPN \]
The Demand for Labor

• How much labor do firms want to use?
  ➢ This equilibrium condition:
    \[ W = MRPN = P \times MPN \]
  ➢ can be re-written as:
    \[ w = MPN \]
  • because \( w = \frac{W}{P} \) and \( MRPN = P \times MPN \).

Marginal Cost of Hiring an Extra Worker

Marginal Benefit of Hiring an Extra Worker

The Determination of Labor Demand
The Demand for Labor

• How much labor do firms want to use?

➢ Costs and benefits of hiring one extra worker.
  • If \( w > MPN \), profits rise if number of workers decreases.
  • If \( w < MPN \), profits rise if number of workers increases.
  • When \( w = MPN \), profits are maximized.

The Demand for Labor

• How much labor do firms want to use?

➢ The labor demand curve shows the relationship between the real wage rate (\( w \)) and the quantity of labor demanded (\( N \)).

Determination of the Labor Demand Curve

• The Labor Demand Curve, \( N_D \).

➢ Changing the real wage rate:
  • An increase in the real wage rate means \( w > MPN \) unless \( N \) is reduced so the \( MPN \) increases.
  • A decrease in the real wage rate means \( w < MPN \) unless \( N \) is increased so the \( MPN \) decreases.
The Demand for Labor

- The Labor Demand Curve, $N_D$.
  - The labor demand curve is downward sloping.
    - The higher the real wage, the less labor firms will hire.
  - Because $w = MPN$ in equilibrium (regardless of what $w$ is), the $N_D$ curve is the same as the $MPN$ curve.

Effect of an Increase in K or A

- Factors that shift the labor demand curve:
  - Changes in the capital stock, $\Delta K$.
    - Increases in $K$ raise $MPN$ and shift the labor demand curve to the right.
  - Supply shocks, $\Delta A$.
    - Beneficial supply shocks raise $MPN$ and shift the labor demand curve to the right.
Key Diagram #1: The Production Function

\[ Y = A_0 F(K_0, N) \]

Key Diagram #2a: Demand for Labor

Factors that Shift the Production Function and the Demand for Labor:

- Increases in the capital stock, \( K \), shift the production function higher, increase the MPN and the demand for labor.
- Increases in productivity, \( A \), shift the production function higher, increase the MPN and the demand for labor.

Next Time

- Productivity, Output, and Employment, Part 2
  - The Supply of Labor
  - Labor Market Equilibrium
  - Unemployment
  - Okun’s Law