DO YOU KNOW THAT I KNOW THAT YOU KNOW…? HIGHER-ORDER BELIEFS IN SURVEY DATA

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Abstract: We implement a new survey of firms, focusing on their higher-order macroeconomic expectations. The survey provides a novel set of stylized facts regarding the relationship between first-order and higher-order expectations of economic agents, including how they adjust their beliefs in response to a variety of information treatments. We show how these facts can be used to calibrate key parameters of noisy-information models with infinite regress as well as to test predictions made by this class of models. We also consider a range of extensions to the basic noisy-information model that can potentially better reconcile theory and empirics. While some extensions like level-*k* thinking are unsuccessful, incorporating heterogeneous long-run priors can address the empirical shortcomings of the basic noisy-information model.

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I. Introduction

Keynes (1936) famously observed that a successful investment is one that anticipates what other investors will think is a successful investment. But if those other investors are following the same logic, successful investing then requires anticipating what others will anticipate you anticipate about their anticipation. This iteration over increasingly higher-order beliefs (i.e., what I think you think I think…) is central to many economic scenarios and has become increasingly emphasized and studied in the macroeconomic context. Woodford (2003), for example, shows how noisy private information can lead firms to change their prices very gradually due to the slow-moving higher-order beliefs about the actions of other firms. More recent work has emphasized how different assumptions about higher-order beliefs can alleviate the "forward guidance" puzzle[1] (e.g. Angeletos and Lian 2018, Gabaix 2020, Fahri and Werning 2019). But a major stumbling block to this literature has been the complete absence of any empirical evidence, other than that of the experimental literature, on the higher-order beliefs of economic agents, especially when it comes to their expectations of macroeconomic variables.[2]

This paper takes a first step at filling this gap by studying the higher-order macroeconomic expectations of firm managers using a novel survey of firms in New Zealand. This survey asks managers not only about their own expectations over macroeconomic variables (first-order expectations) as in Coibion, Gorodnichenko and Kumar (2018; henceforth CGK), but also about *what they think other managers expect* for inflation, i.e. their higher-order beliefs. This allows us to provide a unique set of facts relating first-order and higher-order macroeconomic beliefs of firms. We also provide new evidence on the learning process through a variety of randomized information treatments that characterize how agents' first- and higher-order beliefs respond to different kinds of information about the economy. Jointly, these data provide a novel set of empirical facts that can be used to discipline and test models of higher-order thinking. We document a number of dimensions along which noisy-information models with higher-order expectations are (and are not) consistent with the facts from the survey.

The survey builds on earlier surveys of firms in New Zealand described in CGK and Kumar et al. (2015). Relative to this earlier work, we rely on two new waves of the survey run in 2017Q4-2018Q2 using a fresh draw of firms. We highlight several features of this unique data set. First, the average higher-order forecast of inflation across firms is almost identical to the average first-order forecast of inflation. Second, the cross-sectional standard deviation of higher-order beliefs (disagreement) is significantly smaller than

[1] Standard New Keynesian models that abstract from information frictions imply that announcements about monetary policy in the distant future can have unrealistically large contemporaneous economic effects.
[2] The experimental literature has documented a number of striking features about higher-order expectations, and we explicitly build on this previous work. The main differences are that we study *actual* macroeconomic expectations of *actual* firm managers, rather than much more narrowly-defined expectations of undergraduates in typical experiments.

the dispersion in first-order beliefs about inflation. Third, the average uncertainty around firms' higher-order beliefs about inflation is significantly lower than their uncertainty around their own forecasts. Fourth, we find in the cross-section that firms with larger forecasts of inflation also tend to have larger higher-order inflation forecasts, but the correlation is not perfect. Fifth, the amount of disagreement across agents in both first-order and higher-order beliefs is greater than the average amount of uncertainty in both first-order and higher-order beliefs. Sixth, providing firms with information about the higher-order beliefs of other firms moves both first-order and higher-order beliefs significantly more than providing firms with information about the first-order beliefs of other firms.

These stylized facts provide a unique and novel way to test and calibrate noisy-information models with infinite regress (i.e., when agents can correctly infer what others think about what they think about what others think…). For example, we show that a basic noisy-information model in which agents receive both public and private signals about the fundamental is consistent with both the lower disagreement and uncertainty around higher-order forecasts than first-order forecasts. As the relative levels of each are determined by the same underlying parameter in the model, the empirical moments also provide an overidentification test that is not rejected by the data. However, we also document that the basic noisy-information model is inconsistent with several of the stylized facts coming from our survey. One such counterfactual prediction of the model is that the cross-sectional correlation between individuals' first-order and second-order beliefs should be exactly one, whereas that correlation is positive but less than one in the data. A second counterfactual prediction of the model is that the uncertainty in either first- or higher-order inflation forecasts should be equal to the cross-sectional disagreement in those same forecasts, whereas in the data uncertainty is systematically and significantly lower than the amount of disagreement. Finally, the model is inconsistent with the fact that information treatments regarding higher-order beliefs of firms have stronger estimated effects on expectations than information treatments regarding first-order beliefs. A basic noisy-information model instead implies that, in updating their beliefs in response to these signals, managers should place more weight on their prior beliefs when told about the average higher-order beliefs of other firms than when told about the average forecast of other firms, because they tend to be more confident about the former than the latter. Our empirical evidence goes in precisely the opposite direction: the estimated weight on prior beliefs is actually much smaller when firms are told about the higher-order beliefs of other firms than about their first-order beliefs.

Given the inability of the basic noisy-information model to explain these empirical facts, we consider whether variations on the basic model help match the data. Most help only along some dimensions. For example, if agents receive both private and semi-public (rather than public) signals, this can potentially explain the imperfect cross-sectional correlation between first-order and higher-order beliefs found in the survey. However, this alternative formulation of the model does not help in reconciling theory and empirics along the other margins: the cross-sectional variance and uncertainty of expectations are still the same in this model, and agents should not be responding more strongly to signals about higher-order than first-order beliefs unless the former was much more precise. Similarly, if we allow agents to be "overconfident" about the precision of private signals, as in e.g. Daniel, Hirshleifer, and Subrahmanyam (1998), we can account for the lower forecast uncertainty than cross-sectional dispersion in beliefs found in the data (since the overconfidence of agents reduces their uncertainty relative to what it should be). But by itself, this form of overconfidence still cannot explain the stronger reaction of beliefs in empirical treatments to signals about higher-order beliefs than first-order beliefs. Allowing for measurement error in survey responses or differences in level-*k* thinking, which determines how higher-order beliefs are formed, again can help along one or two dimensions but are generally unable to fully bring the model in line with the empirical evidence. Furthermore, we provide evidence that these mechanisms are either too small to bring data and theory into line or that they make additional predictions which are at odds with the data.

A more promising solution is allowing for the possibility that agents hold different beliefs about long-run levels of inflation toward which they skew their forecasts, as in Patton and Timmermann (2010). In this environment, differences in forecasts across agents reflect not just idiosyncratic signals but also heterogeneity in beliefs about long-run outcomes ("long-run" prirors). Since agents are aware of their own long-run priors, cross-sectional disagreement in forecasts (which is magnified by differences in priors) exceeds forecast uncertainty (which is independent of long-run prior), consistent with the data. If agents are unsure about the exact distribution of others' long-run priors, then the model also implies an imperfect (albeit positive) correlation between first-order and higher-order beliefs, again in line with the data. A key characteristic of this model is that, because agents know about their own long-run prior, they try to undo this effect when forming beliefs about others' beliefs: higher-order beliefs are therefore less dispersed and more precise than in the basic noisy-information model. Long-run priors also provide variation that can account for why information about higher-order beliefs can result in lower estimated weight on priors. However, the additional flexibility afforded by the model implies that its parameters are now under-identified by the moments of our data (but can be potentially identified with additional data) and the mapping between data and theory documented for the basic noisy-information model is more complex. Nonetheless, we interpret this approach as providing a viable avenue to reconciling the data with the broader class of noisy-information models.

Our paper speaks directly to several literatures. The first focuses on providing empirical evidence on the nature of the expectations formation process of different economic agents. The second consists of theoretical work emphasizing the importance of higher-order beliefs in accounting for macroeconomic and financial dynamics. We contribute to both literatures by providing new empirical evidence on both first-order and higher-order beliefs of firms and show how this evidence can be used to test and quantify theoretical models of higher-order beliefs.

There is, by now, an extensive literature on how households, firms, financial market participants and even central bankers form their expectations, especially about aggregate conditions. One strand of this literature has focused primarily on how expectations data can speak to models of the expectations formation process. Coibion and Gorodnichenko (2012, 2015), for example, document a systematic under-reaction of the expectations of these agents to macroeconomic shocks, consistent with models of imperfect information acquisition and processing. More recent work such as Bordalo et al. (2018, 2019, 2020), Broer and Kohlhas (2018), Angeletos, Huo and Shastry (2020), and Afrouzi et al. (2020) provide evidence that there is concurrently an element of over-reaction in their expectations, albeit along different dimensions. A closely related approach relies on randomized information treatments to characterize how agents respond to new information (Armantier et al. 2015, Cavallo et al. 2017). Another strand of this literature has focused on characterizing whether/how agents' macroeconomic expectations affect their decisions. CGK and Coibion, Gorodnichenko and Ropele (2020), for example, show how the inflation expectations of firms feed into their pricing, investment and employment decisions. Crump et el. (2018), Coibion, Gorodnichenko and Weber (2019), and D'Acunto et al. (2020) focus on the link between inflation expectations and consumption decisions of households. Roth and Wolfhart (2020) study the link between households' macroeconomic optimism and their desired spending. Our main contribution relative to this literature is that we are the first to provide direct empirical evidence on the higher-order beliefs of firms and to characterize how these relate to their first-order beliefs as well as the characteristics of both the firm and the manager. We are also the first to include information about higher-order beliefs as a source of information in randomized treatments as well as to study how higher-order beliefs respond to new information.

There has also been, in parallel, a growing body of theoretical work emphasizing the potential importance of higher-order thinking and dynamics in macroeconomics and finance. Angeletos and La'O (2009), for example, highlight the importance of considering higher-order beliefs separately from an agent's own beliefs. Bacchetta and Wincoop (2008) show that the difference between higher-order and own expectations is important for determining the pricing volatility of assets as well as the link between asset pricing and expectations of future asset payoffs. Nimark (2008) emphasizes the role of higher-order expectations in pricing decisions for generating inflation inertia. Angeletos and Huo (2020) show that higher-order expectations can generate dynamics equivalent to myopia and anchoring. Huo and Takayama (2015) study the role of higher-order beliefs in generating persistent effects of confidence shocks. We contribute to this literature by providing a novel set of moments on the higher-order beliefs of firms and show how these can be used to test and quantify models of higher-order beliefs.

The remainder of the paper is organized as follows. We first describe in Section 2 how the survey was implemented as well as the key empirical findings from the survey and information treatments. Section 3 considers how well a basic noisy-information model of higher-order expectations under strategic complementarities in prices and infinite regress in expectations can account for the empirical patterns as well as how the data can be used to calibrate parameters of the model. Section 4 considers extensions and alternatives to the basic noisy-information model that can potentially reconcile theory and data. Section 5 concludes.

II. Survey

This paper utilizes two additional waves of the survey of firm managers in New Zealand described in CGK. The first wave was implemented between 2017Q4 and 2018Q1. The follow-up ran from 2018Q1 to 2018Q2, such that each firm manager from the first wave was invited to participate in the second wave three months after his or her initial interview. The first wave included 1,025 firms, with 515 of these participating in the second wave.

2.1 Sampling Frame and Protocol

We obtained information on the population of firms in New Zealand from two sources: Kompass New Zealand (KNZ) and Equifax (EQ). Following the Australia and New Zealand Standard Industrial Classification 2006 (ANZSIC06), firms are classified into one of four broad industries: manufacturing, trade, construction and transportation, and professional and financial services. Following CGK, we focus on firms with six or more employees. We targeted for two thirds of the sample to come from professional and financial services and manufacturing as these industries account for relatively large shares of New Zealand's GDP (New Zealand Treasury, 2016).[3] The remainder of the sample comes from firms in other industries, i.e. trade, construction, communication and transportation. We excluded industries related to the government, community service, agriculture, fishing and mining, and energy, gas and water from the sample. These sectors are often dominated by a few extensively regulated firms or by very small firms. Within each industry, firms are classified as small (6-19 workers), medium (20-49 workers) and large (50 or more workers). To make the survey population more representative, we oversampled firms with 50-99 workers and 100+ workers in each industry. To this end, we contacted all firms that fall into these two employment size groups. We then computed the relative shares of firms in the remaining employment size groups and include enough firms to match the relative share of their size and industry.

To achieve the target of 1,000 firms in the sample, we invited 10,100 firms to participate in the survey. Each firm's general manager received an email containing an information sheet and survey questionnaire about ten days before receiving a phone call to collect responses. Note that the initial questionnaire sent to managers did not include the treatment information and the subsequent related

[3] New Zealand Treasury (2016), New Zealand: Economic and Financial Overview 2016, Wellington. See https://treasury.govt.nz/sites/default/files/2010-04/nzefo-16.pdf.

questions. We called each firm three times to elicit responses. After the third round of calls, we examined the response rates for sectors, subsectors and employment size groups. We then targeted groups in which responses rates were low. We continued contacting firms until we hit the target sample size. Appendix F reports response rates by industry and size.

Responses were collected over the phone. A research assistant (RA) called the general manager and recorded answers by hand while also recording the phone call. An independent RA then listened to the recording and confirmed the accuracy of the handwritten responses. For the confidentiality of the participants, the recordings were deleted following data collection. The handwritten questionnaires were then entered into a spreadsheet, with two independent RAs verifying that the handwritten and spreadsheet responses matched. As discussed in CGK, responses of managers are consistent with information available from other sources and the quality of the survey is reasonably high.[4]

We provide some summary statistics of the characteristics of respondents in Table 1. The average firm size is relatively small, with about 40 employees. This is representative of the distribution of firm size in New Zealand. Some firms however are much larger, with the biggest having 800 employees. The average firm in our sample is 26 years old and sells most of its products in New Zealand. However, our sample also includes many firms that export extensively. In addition, we collected some characteristics of the individual respondents. Most have been at the firm over 10 years and are well-educated, with average years of schooling exceeding 16 years. Twenty percent of respondents are women. Overall, there is relatively little cross-sectional variation in managers' socioeconomic characteristics.

The second wave (follow-up) of the survey was implemented three months after the initial wave. For the follow-up, we contacted all firms that participated in the main wave of the survey. The response rate was approximately 50 percent. We achieved a high response rate because we provided respondents with a monetary incentive of $50 gift voucher and dinner and entertainment ticket worth $50. Further, respondents

[4] We verified our survey data against the publicly available online information in four ways. First, we verified managers' responses about the age of the firm using the information from the Companies Office or their website. We find that the reported age in the survey match exactly with the information available in the Companies Office or their website for 1012 firms. Information about age of 20 firms is not available in any other source. Second, we verified whether the firm exports or not. Firms that indicated in the survey that they export overseas, this information is available in their websites. Third, we asked in the survey about the number of Directors, number of shareholders and the number of shares issued in the business. There are 862 firms classified as Companies in this survey, i.e. public or private companies. We find that more than 98 percent of these firms' responses match with the information available in the Companies Office. Last, we verified survey responses on firms' products and prices. To do this, we randomly selected around 20 percent of the firms (206 firms) and asked them about their main product and price of the main product. For 203 firms, details about their main products are available in their websites. 43 out of 203 firms list their prices online in their websites. The reported prices of main products do not match the online information for only 4 firms; this is equivalent to 1 percent. For firms whose prices are not listed online, we made phone enquiries about the price of their main products. These were general customer enquiries about their prices. To this end, we made 163 phone enquiries and 94 percent of the firms' reported price matched with the quotes provided.

enter into a pool draw to win a cash prize of $5,000. The main reason for non-participation was that the general manager was too busy to respond in a reasonable time frame. We find (Appendix Table 1) that the observable characteristics of managers/firms do not help predict whether firms participated in the second round.

2.2 Survey questions

After collecting basic demographic information about firms, the survey asks respondents to report their beliefs about future aggregate variables (inflation, unemployment rate, and wages) and about future firm-specific outcomes (employment, fixed assets, prices, and wages). The horizon for aggregate variables is one-year ahead. The horizon for firm-specific variables is three-month ahead (which was determined by the timing of the follow-up) and six-month ahead. Firms were also asked to report their perceptions and nowcasts (e.g., their perception of inflation over the previous twelve months). The survey asks a few hypothetical questions to provide us with estimates of parameters that would be difficult to identify otherwise.

Inflation expectations were elicited in two ways. First, firms were asked to assign probabilities for possible outcomes (see Appendix Table 2 for specific formulation of questions). These distributional questions are similar to the questions asked in the Survey of Consumer Expectations (SCE) run by the Federal Reserve Bank of New York. Second, firms were asked to provide point predictions for future inflation and other variables. We do not restrict responses for this type of questions in any way (e.g., we do not censor responses or prompt respondents to reconsider if responses are outside some range). In contrast to previous surveys, we collect information not only about managers' own expectations about future inflation but also about what managers think about *other managers*' inflation expectations.[5]

The survey has two additional novel parts. First, after the core part of the survey is complete, respondents are invited to participate in a strategic game to infer their level of thinking. This game is similar to Nagel (1995) and we provide more details in section 4.5. Second, after the game, firms are randomly assigned into control and treatment groups. Firms in treatment groups are provided with different pieces of information, while firms in the control group are told nothing. The treatments are described in section 2.4, but some include information about the higher-order expectations of other firms. We use these treatments to study how firms form their expectations and how they use these expectations to set prices, wages, employment, and fixed assets.

2.3 Unconditional Moments of First and Higher-Order Expectations

To gauge firms' expectations of inflation as well as their expectations of what other managers expect about inflation, we rely primarily on probability distribution questions. Firms were first asked to assign probabilities

[5] In the survey, we first asked about managers' first-order expectations than enquired about their higher-order expectations. We verified in a subsequent survey of managers that if we randomly change the ordering of these questions, the resulting distributions of answers are insensitive to the ordering.

to a wide range of different possible outcomes for overall price changes over the next 12 months, following CGK. From the probabilities that they assign, we construct implied forecasts of each manager using mid-point values of each bin (the two end bins are assigned values of +30 and -30). We also measure the uncertainty in their forecast (standard deviation of probabilities across bins). To measure their higher-order expectations, firms were asked an equivalent distributional question (Appendix Table 2) with respect to what they believe "other managers (drawn from all sectors of the New Zealand economy in a representative way) think will happen to overall prices in the economy." Using this question, we construct the implied forecast and uncertainty of each manager for their higher-order expectation. To the best of our knowledge, this is the first time anyone has surveyed firm managers about their higher-order expectations of macroeconomic variables.

Summary results are presented in Table 2. In terms of first-order inflation expectations, the results closely follow CGK. The average forecast of inflation across managers is 3.4%, significantly above actual inflation at the time. Managers are quite uncertain about their forecasts, with an average standard deviation in their forecasts of 1.1 percentage point. They also display significant disagreement: the cross-sectional standard deviation in forecasts is 3%. These results are also similar to the moments of households' inflation expectations in the SCE (Kumar et al. 2015). Respondents were asked to provide forecasts of the unemployment rate and changes in wages over the next twelve months, both using distributional questions. Their responses are uncorrelated with their forecasts of inflation but are similarly dispersed. They anticipate on average relatively low wage growth (1.1%) and relatively high unemployment (a 12-month forecast of unemployment of 4.9% at a time when the actual unemployment rate was 4.4%). Respondents were also asked about planned actions on the part of their firms over the next 3 months. As reported in Table 2, managers expected their firms to raise their prices by less than 1% and raise their wages by very little, whereas they expected their employment to grow by 3% and their investment by close to 2%. In each case, the cross-sectional distribution is very dispersed around these numbers (although much less so for wage growth), with firms reporting a wide range of expected outcomes.

The most novel dimension of the survey is the fact that respondents were asked about their higher-order inflation expectations, i.e. what they thought other managers were predicting for aggregate inflation over the next twelve months. First, we find that the mean higher-order inflation expectation is almost identical to the mean first-order inflation expectation (3.5% vs. 3.4%), and we cannot reject the null of equality for the two. Thus, we do not observe a stronger or weaker bias in higher-order expectations than we do in lower-order inflation expectations. Note that this equality in the average first- and higher-order forecasts is not because managers report identical values for the two. In fact, only 2% report the same first-order and higher-order forecasts.[6] Figure 1 plots the underlying distributions of both first-order forecasts

[6] We also find that 44% of respondents report positive probability for a fewer number of inflation bins in their higher-order inflation expectations than first-order expectations. 28% report the same number of bins. 28% report positive

(Panel A) and higher-order forecasts (Panel B). The two are quite different. For example, first-order forecasts have a large mass of forecasts between 0 and 2% but a significant tail of much higher values, while higher-order forecasts have a smaller mass in the 0-2% range but a larger one in the 3-4% range. The near-equality of the average forecasts is therefore not an artifact of respondents providing the same responses to first-order and higher-order forecasts. Figure 1 also shows that conditional on assigning a positive probability to an inflation bin, assigned probabilities vary considerably and so the average probability assigned to a bin (red circles in the figure) masks dramatic heterogeneity across managers.

Figure 2 plots the distribution of the *within-manager* difference in expectations (first-order expectations minus higher-order expectation) across managers. The figure demonstrates that, even within a given inflation bin, a typical manager often assigns different probabilities to first-order and higher-order expectations. For example, while the mean probability assigned to the [0,2) inflation bin is similar for first-order expectations (32.7) and higher-order expectations (27.7), the standard deviation of the within-manager probability difference for this bin is 32.8. The average difference in assigned probabilities across bins (red circles in the figure) has an inverted-W shape: the difference is negative for bins [2,4) and [4,6) and positive for bins [-4,-2), [-2,0) and [6,8), [8,10). This pattern suggests that, when we examine the within-manager distribution of beliefs, the distribution of higher-order beliefs for a given manager is on average more concentrated than the distribution of first-order beliefs of that manager.

There is, nonetheless, a strong positive correlation between a manager's inflation forecast and their higher-order expectation (Figure 3). Managers who expect higher inflation also tend to believe that other managers expect higher inflation as well. This relationship is imperfect, with a correlation of 0.68, but the slope coefficient between the two is strongly positive at 0.66 and statistically significant at standard levels. First-order expectations are therefore informative about managers' higher-order beliefs, but there is clearly independent variation in the latter that is not explained by the former.

Table 2 reveals two other striking facts about higher-order inflation beliefs of managers in New Zealand. One is that there is significantly lower cross-sectional disagreement about higher-order beliefs than for first-order beliefs: the cross-sectional standard deviation of higher-order beliefs is 2.4%, well below the 3.1% found for first-order beliefs. In other words, while managers disagree a lot about what will happen to inflation over the next year, there is much more agreement about what they think other managers are predicting inflation to be. As we discuss in more detail in sections 3 and 4, this pattern points toward a role for public signals since public signals will lead to coordination in higher-order beliefs.

Another novel characteristic of the data documented in Table 2 is that managers are generally more certain about their higher-order inflation forecasts than they are about their first-order forecasts: the average

probability for a greater number of inflation bins in their higher-order inflation expectations than first-order expectations.

uncertainty in higher-order forecasts is 0.9% whereas it is 1.1% for first-order forecasts. We can again strongly reject the null of equality for these two levels of uncertainty. Our results in Figure 2 suggest that this pattern does not arise from aggregation but rather stems from within-manager differences. One might expect uncertainty to accrue as agents extrapolate from their belief to what others might know, but the opposite is true in the data: confidence is higher about the beliefs of others than about their own beliefs. As discussed in subsequent sections, this striking feature of the data is also consistent with simple models of noisy information in which agents receive public signals along with their private signals.

A final fact apparent from Table 2 is that the average level of uncertainty in managers' forecasts (both first-order and higher-order) is lower than the average cross-sectional dispersion in those same forecasts. For both first-order and higher-order beliefs, we can strongly reject the null of equality in average disagreement and cross-sectional dispersion. Uncertainty and disagreement are sometimes treated as interchangeable in the literature, but we find a clear difference in their levels here, both for first-order and higher-order beliefs. As we will see in sections 3 and 4, this feature of the data will be one of the most difficult to explain in the context of theoretical models, which generally impose a very tight link between uncertainty and disagreement.[7]

These facts hold when we examine sample splits by firm or manager characteristics (Appendix Figure 6 and Appendix Table 3). Consistent with CGK, manager characteristics appear to play little role in generating cross-sectional heterogeneity (see also Appendix Table 7), likely reflecting the fact that managers are much less heterogeneous than the general population. There is more variation in the moments across firm characteristics such as size, age, and the number or competitors but the qualitative patterns are similar across subsamples.

2.4 Effect of Information Treatments on Expectations

In addition to these unconditional moments of respondents' expectations, we conducted an experiment to assess how agents revise both their first- and higher-order beliefs in response to new information. After asking firms about their inflation expectations and higher-order expectations in the initial wave of the survey, we provided randomly selected subsets of firms with different types of information. We divided managers into five groups. Group A is a control group and did not receive any information. The surveys of 300 respondents in Group A were completed first and the resulting moments of this group were used to implement the subsequent information treatments (which require information about firms' beliefs). Group B received information about the average beliefs of survey participants about inflation. Group C received information about the average higher-order inflation expectations of survey participants. Group D's signal consisted of both information about average expectations and average higher-order expectations. Note that average first-

[7] Appendix Figure 5 shows histograms for implied-mean predictions and uncertainty for first- and higher-order expectations.

order and higher-order expectations estimated from the control group, 3.3% and 3.5% respectively, are very similar to the average expectations for the full sample presented in Table 2. We utilize Group E to compare the impact of information about other managers' beliefs to information about lagged inflation, as in CGK.

Immediately after providing firms with information,[8] we asked them to report their point predictions for inflation (one-year ahead) and for their beliefs about what other managers in the economy predict for inflation (one-year ahead). Measuring revisions in expectations immediately after the treatment allows us to obtain the instantaneous effect of the treatment on firms' beliefs. Note that priors are measured as mean expected inflation implied by the reported distribution of future inflation while the posteriors are measured as point predictions. Different formulations of the inflation questions are deliberately used pre- and post-treatment to avoid antagonizing respondents by repeatedly asking them to answer identical distributional questions multiple times.[9] Any difference in responses induced by the formulation of the questions will be captured in the control group's responses and so will not affect the results of the information treatment. In the follow-up wave (three months after the initial wave), we asked firms to report distributions of their beliefs about future inflation. Using responses from the follow-up survey, we construct another measure of posteriors as the mean expected inflation implied by the reported distribution. This set of posteriors provides a sense of the persistence of the treatment effects of information on expectations.

To assess the influence of various information treatments on managers' beliefs, we use the following econometric specification:

$$Posterior_i = constant + b \times Prior_i + error_i \quad (1)$$

where slope b captures the strength of manager i's prior relative to the treatment, and the value associated with the treatment is absorbed into the constant term (since it is common across firms within that group). More informative priors should be associated with high values of b. If the estimated slope $\hat{b}$ is equal to zero, the treatment is interpreted as a completely informative signal which causes managers to discard their priors in favor of the signal. If $0 < \hat{b} < 1$, the treatment is interpreted as a partially informative signal and managers will update their posterior somewhat but will still rely partially on the prior. If $\hat{b}$ is approximately one, managers see the information provided as uninformative and do not update their prior beliefs at all. Because we use point predictions for posteriors and implied means for priors, the estimated slope may be biased up or down depending on how managers respond to probability distribution questions vs. point forecasts (see e.g. Kleinjans and van Soest 2010, Fischhoff and Bruine de Bruin 1999, Bruine de Bruin et

[8] For the control group we simply continue with the questions.

[9] One of the only repeated questions is a first-order point prediction of inflation expectations in the first wave of the survey. The format of this question follows the Michigan Survey of Consumers, which is closer to the post-treatment point prediction for future inflation. We use this information to quantify measurement error in reported inflation expectations. Because we did not elicit point predictions in the follow-up wave of the survey, we use the SCE format for pre-treatment beliefs.

al. 2000), but this will be observable in the estimated b for the control group. Because we are interested in how managers respond not only to new information but also to different kinds of information, we estimate specification (1) for each treatment separately.

Table 3 reports the estimated coefficients on the prior expectation for both own inflation expectations and higher-order inflation expectations in specification (1). We find that when no information is provided, the point estimate of the slope is approximately 0.7 (row 1). This estimate does not mean that firms revise their beliefs in the absence of information treatment by large amounts. Instead, this estimate likely highlights differences between expectations elicited as point predictions and expectations elicited as probability distributions. Indeed, when we use point predictions for future inflation that were elicited before the informational treatment was provided, we find that the slope is close to one for the control group (Appendix Table 5).

With this benchmark in mind, we turn to Treatment B (provide firms with $\overline{E}[\pi]$, row 2 in Table 3). When we elicit expectations immediately after the treatment, firms assign 0.50 weight on the prior when they update their first-order inflation expectations (column 1) and 0.43 weight when they update their higher-order inflation expectations (column 2).[10] These weights are statistically different from the weight assigned by the control group. If we normalize these weights by the weights in the control group, the adjusted weights are approximately 0.7 and 0.6 for first-order and higher-order beliefs respectively. Thus, Treatment B has useful information content that leads firms for revise their beliefs. This finding that firms place some weight on the forecasts of other firms in revising their first-order beliefs is consistent with the experimental evidence in CGK, but the finding that they revise their higher-order beliefs in a comparable manner is completely novel to the literature, to the best of our knowledge.

Another novel dimension of the experiment is that firms in treatment group C received information about the average higher-order belief of other firms (row 3 in Table 3). This previously unexplored information treatment leads to the striking result that the estimated weights on priors are considerably smaller than the information treatment using first-order beliefs: 0.09 and 0.12 for first- and higher-order beliefs. These estimates suggest that *managers perceive the information about firms' higher-order expectations as a very informative signal* that leads them to place little weight on their prior beliefs, both when they update their first-order and higher-order beliefs. The weights on the prior are similar in Treatment D (provide firms with both first- and higher-order information, row 4 in Table 3). We interpret this result as indicating that information in the average first-order inflation forecast has relatively little incremental content relative to information in the average higher-order inflation forecast.

[10] Appendix Figures 1 through 4 shows scatter plots of posteriors against priors.

For firms receiving information about the past realization of inflation (Treatment E, row 5 in Table 3), the weight on the prior is 0.059 for first-order expectations and 0.062 for higher-order expectations. The former confirms an earlier result in CGK and is consistent with other evidence for firms in Italy (Coibion, Gorodnichenko and Ropele, 2020) as well as households in the U.S. (Armantier et al. 2015, Coibion, Gorodnichenko and Weber 2019) and the Netherlands (Coibion, Geogarakos, Gorodnichenko and van Rooij 2019). While the high weight assigned to information about recent inflation in revising first-order beliefs is therefore well-documented, the fact that higher-order beliefs respond in an equivalent manner is completely new to the literature. Strikingly, firms seem to view information about other firms' higher-order beliefs as being almost as informative as information about recent inflation.

Conducting the same analysis using the posterior belief reported in the follow-up wave (three months after the treatment) produces similar results (see columns (4) and (5) of Table 3). We see mean reversion in the reported responses of the control group. Treatments C, D and E result in low weights on priors while Treatment B yields weights approximately half-way between the control group and the other treatment groups. These results indicate that the effect of information is persistent after three months and that the size of treatment effects continue to depend on the type of signal that the firm received.[11,12]

2.5 Summary

Using a novel survey of firm managers in New Zealand, we document six new empirical facts (summarized in Table 4) relating first-order and higher-order inflation expectations of business managers and CEOs. To the best of our knowledge, these represent the first empirical characterization of higher-order beliefs of firms from a survey. These facts can potentially speak not just to the way in which agents form their beliefs about the aggregate economy, but also to the role played by higher-order beliefs in this process. In the next section, we consider to what extent these facts are consistent with simple noisy-information models in which higher-order beliefs can play an important role as well as how these stylized facts can potentially be used to shed new light on underlying parameters of these models.

III. Interpreting Survey Results through a Noisy-Information Model

Our results demonstrate that not only can one measure the higher-order expectations of economic agents

[11] CGK and Cavallo et al. (2017) find that the difference in beliefs for treatment and control groups largely disappears six months after the treatment. We reconcile these results by using the findings in Coibion, Gorodnichenko and Ropele (2020) who study a long panel of firms to document that informational treatments have significant effects on expectations after three months but vanish after six months. In a similar spirit, Coibion, Gorodnichenko and Weber (2019) find that information treatment effects are detectable 3 months after the treatment but not discernable after 6 months.

[12] While the response of actions to information treatments is not the main focus of this paper (we are interested in how beliefs are formed), we document in Appendix G that managers act on the beliefs revised in response to the information treatments. Thus, changes in beliefs are translated into changes in actions.

but also that these expectations can play an important role in shaping beliefs. This is illustrated for example by the large revisions in firms' first-order inflation expectations when presented with information about other firms' higher-order expectations. How should we think about these results on the higher-order beliefs of firms? Are they consistent with what theory would predict? In general, strategic complementarities in pricing behavior require that firms think not only of their own expectations of a fundamental, but also of other firms' expectations and actions. Firm A must think about the fundamental and what Firm B thinks of the fundamental. Firm B then anticipates the fundamental, what firm A thinks of the fundamental, and what Firm A thinks that Firm B thinks. Firm A's expectations must respond accordingly, etc. As firms anticipate each other's actions, they must form higher-order beliefs that involve iterating a problem to progressively higher levels of reasoning. In this section, we use the static model of Morris and Shin (2002)—which is a workhorse model in this literature—to demonstrate how the expectations and higher-order expectations of the firms in our survey compare to the predictions of a model of strategic complementarities where firms perform infinite regress in their expectations.

3.1 A Simple Model of Expectations Formation and Price-Setting

Firm $i \in [0,1]$ chooses to set its optimal price, p_i, as a linear combination of its expectation of a fundamental, m, and its expectation of the aggregate price level in the economy, $\overline{p}$:

$$p_i = (1-\alpha)E_i[m] + \alpha E_i[\overline{p}], \tag{2}$$

where parameter $\alpha \in (0,1)$ describes the degree of complementarity in pricing. Because $\overline{p} \equiv \int_0^1 p_j dj$, manager i can iterate the optimal price equation forward by substituting the average optimal price equation for the aggregate price level to obtain:

$$p_i = (1-\alpha)E_i[m] + \alpha E_i\left[\int p_j dj\right] \tag{2'}$$

Define the average expectation in the economy for variable m as $\overline{E}[m] \equiv \left[\int_0^1 E_j(m)dj\right]$ and let $E_i\left[\overline{E}[m]\right]$ be the expectation of manager i about the average expectation in the economy. In a similar spirit, $E_i[\overline{p}]$ is the first-order ("own") expectation about the price level, $E_i\left[\overline{E}[\overline{p}]\right]$ is a higher-order expectation about the price level in the sense that this is an expectation of manager i about what other managers think about the price level. We can iterate these expectations to the k^{th} higher order recursively: $\overline{E}^k[X] \equiv \left[\int_0^1 E_j\left(\overline{E}^{k-1}[X]\right)dj\right]$.

Using the definition of the price level and repeated substitutions in equation (2'), we find that the aggregate price level becomes an average of progressively higher-order expectations of the fundamental, weighted by the complementarities present at each step:

$$\overline{p} = (1-\alpha)\overline{E}[m] + \alpha(1-\alpha)\overline{E}^2[m] + \alpha^2(1-\alpha)\overline{E}^3[m] + \cdots . \tag{3}$$

It follows that the optimal choice of p_i depends on the manager's expectation of each event in equation (3).

$$p_i = (1-\alpha)E_i[m] + \alpha(1-\alpha)E_i\left[\overline{E}[m]\right] + \alpha^2(1-\alpha)E_i\left[\overline{E}^2[m]\right] + \cdots. \quad (4)$$

This captures the notion that optimal decisions of a firm depend not just on their expectations of the fundamental but also what they think other firms think about the fundamental, etc.

To characterize how firms form their expectations about the fundamental as well as how they form their higher-order expectations, we assume that they operate under imperfect information. Rather than observing m perfectly, they each receive one noisy public signal and one private signal. Each signal reflects the true value of m combined with some noise. Specifically, the public signal about the fundamental takes the following form: $y = m + \varepsilon$ where $\varepsilon \sim N(0, \kappa_y^{-1})$ and is common across all firms. In addition, each firm i also receives its own private signal about m: $x_i = m + v_i$ with $v_i \sim N(0, \kappa_x^{-1})$ where ε and v_i are uncorrelated and v_i is i.i.d. across managers. κ_x and κ_y capture the precision of each type of signal. Firms weigh their signals according to the relative noise in each to obtain an individual expectation of m:

$$E_i[m] = \frac{\kappa_y}{\kappa}y + \frac{\kappa_x}{\kappa}x_i = (1-\delta)y + \delta x_i, \quad (5)$$

where $\kappa = \kappa_x + \kappa_y$ and, for ease of notation, we denote $\frac{\kappa_x}{\kappa}$ and $\frac{\kappa_y}{\kappa}$ as δ and $1-\delta$, respectively, for the remainder of the paper. As the private signal becomes more precise relative to the public signal, the firm places relatively more weight on it in forming beliefs about the fundamental. Aggregating equation (5) across managers gives the average expectation about the fundamental in the economy:

$$\overline{E}\,[m] = (1-\delta)y + \delta m. \quad (6)$$

Manager i's expectation about the average expectation of other managers in the economy is

$$E_i\left[\overline{E}\,[m]\right] = (1-\delta)y + \delta E_i[m] = (1-\delta^2)y + \delta^2 x_i. \quad (7)$$

One can obtain progressively higher-order expectations of m by continuing to substitute $E_i[m]$ for m to find:

$$E_i\left[\overline{E}^k[m]\right] = (1-\delta^{k-1})y + \delta^{k-1}E_i\left[\overline{E}^{k-1}\,[m]\right] = (1-\delta^k)y + \delta^k x_i. \quad (8)$$

Equation (8) shows that higher-orders of reasoning will depend progressively more on the public signal as that signal is common across firms.

Using the firm's optimal price-setting in equation (4), we can substitute for manager i's expectations of m at various orders to obtain the optimal price as a function of received signals:

$$p_i = (1-\alpha)\sum_{k=0}^{\infty}\alpha^k\left[[1-\delta^{k+1}]y + \delta^{k+1}x_i\right]. \quad (9)$$

It follows that every agent sets the optimal price at:

$$p_i = \phi_y y + \phi_x x_i, \quad (10)$$

where $\phi_y = \frac{1-\delta}{(1-\alpha)\delta + (1-\delta)}$ and $\phi_x = \frac{(1-\alpha)\delta}{(1-\alpha)\delta+(1-\delta)}$. The realization of the aggregate price is the integral of equation (10) across the support of all managers:

$$\overline{p} \equiv \int_0^1 p_j dj = \phi_y y + \phi_x m. \tag{11}$$

Using these results, we can derive the first-order expectation of manager i about the price level:

$$E_i[\overline{p}] = \phi_y y + \phi_x\big((1-\delta)y + \delta x_i\big) = (1-\phi_x\delta)y + \phi_x\delta x_i. \tag{12}$$

Aggregating across agents gives the average first-order expectation about the price level:

$$\overline{E}[\overline{p}] = \phi_y y + \phi_x\big((1-\delta)y + \delta m\big) = (1-\phi_x\delta)y + \phi_x\delta m. \tag{13}$$

The individual expectation of the left-hand side in equation (13) characterizes an individual manager's higher-order expectation:

$$E_i\left[\overline{E}[\overline{p}]\right] = \phi_y y + \phi_x[(1-\delta^2)y + \delta^2 x_i] = (1-\phi_x\delta^2)y + \phi_x\delta^2 x_i. \tag{14}$$

Aggregating equation (15) gives the mean of the higher-order expectation:

$$\overline{E}^2[\overline{p}] = \phi_y y + \phi_x[(1-\delta^2)y + \delta^2 m] = (1-\phi_x\delta^2)y + \phi_x\delta^2 m. \tag{15}$$

These derivations demonstrate that firms in the noisy-information model have two sources of uncertainty: noise in the public signal and noise in the private signal. When firms make inferences about the fundamental m or when we consider unconditional distributions of $\bar{p}$, both sources of uncertainty appear. However, for a given period, firms observe y and thus know this portion of other firms' information sets. Indeed, equation (11) demonstrates that the price level is a function of public signal y (observed) and fundamental m (unobserved). Because for firm i the expected value of m is a linear combination of y and private signal x_i, the only source of uncertainty about $\bar{p}$ is the realized private signals of other firms. As a result, uncertainty about $\bar{p}$ is described by distributions conditional on y. Specifically, one can show that the distributions of these conditional expectations are:

$$E_i[\overline{p}]|y \sim N\big(\big[\phi_y + \phi_x(1-\delta)\big]y + \phi_x\delta m, (\phi_x\delta)^2\kappa_x^{-1}\big), \tag{16a}$$

$$\overline{E}[\overline{p}]|y \sim N\big(\big[\phi_y + \phi_x(1-\delta)\big]y + \phi_x\delta m, 0\big), \tag{16b}$$

$$E_i\left[\overline{E}[\overline{p}]\right]|y \sim N\big(\big[\phi_y + \phi_x(1-\delta^2)\big]y + \phi_x\delta^2 m, (\phi_x\delta^2)^2\kappa_x^{-1}\big), \tag{16c}$$

$$\overline{E}^2[\bar{p}]|y \sim N\big(\big[\phi_y + \phi_x(1-\delta^2)\big]y + \phi_x\delta^2 m, 0\big). \tag{16d}$$

Jointly, these equations allow us to compare this basic noisy-information model to the first five empirical moments identified in section 2. Specifically, equations (12) and (14) characterize the first and higher-order expectations of firm *i* respectively, while the average first-order and higher-order expectations of prices are in equations (13) and (15) respectively. Uncertainty in first-order and higher-order expectations come from (16a) and (16c), as do the levels of cross-sectional dispersion in each.

3.2 Comparing Predictions of the Model to Moments in the Data

We now compare the empirical facts about higher-order expectations from the survey to predictions of the model.

Fact 1: The mean higher-order expectation is almost identical to the mean first-order expectation. The model predicts that the mean of the distribution of firms' own expectations of the aggregate price level ($E_i[\overline{p}]$) can be similar to that of the firms' higher-order expectation of the aggregate price level $\left(E_i\left[\overline{E}[\overline{p}]\right]\right)$, that is, their expectation of other managers' expectation. Specifically, the difference between $E_i[\overline{p}]$ and $E_i\left[\overline{E}[\overline{p}]\right]$ depends on how far y is from m:

$$\left(\overline{E}[\overline{p}] - \overline{E}^2[\overline{p}]\right)|y = \phi_x(1-\delta)\delta[m-y]. \tag{17}$$

Similar average first-order and higher-order expectations will therefore arise whenever the public signal is close to the fundamental. Hence, the basic noisy-information model can readily accommodate this first fact. In addition, in the model, any difference between the mean first-order and higher-order expectations speak directly to the sign of $[m-y]$. As shown in Table 2, among firms in New Zealand at the time of survey, $\overline{E}[\pi]|y$ is 3.41 and $\overline{E}^2[\pi]|y$ is 3.50. The small negative difference between the two is consistent with y being greater than m (that is, the public signal is more "inflationary" than the fundamental). Whether the difference is large or small depends on the magnitudes of ϕ_x and δ, but since $\phi_x, \delta \in (0,1)$ we expect that $m-y<-0.09$. As we show later, $\delta \approx 0.8$ and $\phi_x \approx 0.55$ so that $m-y \approx -1$. Incidentally, the survey responses were collected at a time when oil prices were rising, possibly sending an "inflationary" public signal to firms in New Zealand.[13]

Fact 2: First-order and higher-order expectations are positively but imperfectly correlated across firms. Because the private signal x_i is the only source of cross-sectional variation in expectations, our model predicts a *perfect* correlation between higher- and lower-order expectations. To see this, note that equations (12) and (14) can be combined to express higher-order beliefs of firm *i* solely in terms of that firm's first-order belief and the public signal:

$$E_i\left[\overline{E}[\overline{p}]\right] = (1-\delta)y + \delta E_i[\overline{p}]$$

As a result, the basic model cannot explain the imperfect correlation between first-order and higher-order beliefs across firms. In principle, measurement error in survey responses could be one natural reason for an imperfect correlation, and we explore this explanation in detail in section 4.1 as well as other potential sources for the imperfect correlation.

[13] Coibion and Gorodnichenko (2015) and Coibion et al. (2020) document that households' inflation expectations are sensitive to the price of oil, gasoline and similar goods. Kumar et al. (2015) and CGK present suggestive evidence of high sensitivity of managers' inflation expectations to changes in oil prices and other energy products frequently purchased by consumers.

Despite this inconsistency, the positive average relationship between firms' first-order and higher-order beliefs can still be related to the model. For example, Figure 3 documented that the slope of the relationship between the two in the survey is approximately 0.6. That would imply a value of $\delta = 0.6$, so that the precision of the private signal should be 50% greater than that of the public signal. However, to the extent that there might be measurement error in the survey responses, this would imply that the empirical regression in Figure 3 understates the true slope of the relationship between first-order and higher-order beliefs and therefore 0.6 is a lower bound for the value of δ in the theory. We return to this point in more detail in section 4.1.

The joint distribution of first- and higher-order beliefs can also inform us about the level of the public signal y and the fundamental m. Note that when $E_i[\overline{p}] = E_i\left[\overline{E}[\overline{p}]\right]$, equations (12) and (14) imply that $E_i[\overline{p}] = E_i\left[\overline{E}[\overline{p}]\right] = x_i = y$. Our regression estimates suggest that this point (that is, when the fitted regression line crosses the 45-degree line in Figure 3) occurs at $E_i[\overline{p}] = E_i\left[\overline{E}[\overline{p}]\right] = 3.6$ and therefore $y = 3.6\%$. Since, as shown earlier, $m - y \approx -1$, we can infer $m \approx 2.6$. In other words, the underlying fundamental inflation in New Zealand should have been approximately 2.6 percent during this period but firms systematically believe inflation is higher because of an inflationary public signal. Strikingly, the implied fundamental is close to actual inflation: CPI and PPI inflation rates in 2018Q1 were 2.2 and 2.7 percent respectively.

Fact 3: The cross-sectional dispersion in first-order expectations is greater than the dispersion in higher-order expectations.

Private signals are the reason why agents disagree about macroeconomic variables in the noisy-information model. Equations (16a) and (16c) predict that the cross-sectional variance of higher-order expectations is given by $Var\left(E_i\left[\overline{E}[\overline{p}]\right]|y\right) = (\phi_x\delta^2)^2\kappa_x^{-1}$ while the cross-sectional variance in first-order expectations is given by $Var(E_i[\overline{p}]|y) = (\phi_x\delta)^2\kappa_x^{-1}$. The ratio of cross-sectional variances for $E_i\left[\overline{E}[\overline{p}]\right]|y$ and $E_i[\overline{p}]|y$ is therefore given by

$$\frac{Var\left(E_i\left[\overline{E}[\overline{p}]\right]|y\right)}{Var(E_i[\overline{p}]|y)} = \delta^2 < 1. \qquad (18)$$

This implies that the basic noisy-information model correctly predicts that the cross-sectional variance of higher-order beliefs is smaller than that of first-order beliefs. This is because higher-order beliefs place more weight on the public signal than first-order beliefs and are therefore more tightly concentrated.

Furthermore, the basic noisy-information model relates the relative level of these variances to δ, as was the case with fact 2. In this case, the ratio of the two dispersion measures in the survey implies $\delta \approx 0.80$ (bootstrap s.e. 0.02), that is, the precision of the private signal is about four times larger than the precision of the public signal. Note that both facts 2 and 3 pin down the same parameter δ with two different moments, so we can think of this as an over-identification test for the model. To the extent that we need two different values of δ to match both facts, one could interpret this as a rejection of the model. But as explained previously, we view the implied δ needed to match fact 2 as a lower bound, so there need not necessarily be an inconsistency in the required parameter values.

Because $Var(E_i[\overline{p}]|y) = (\phi_x \delta)^2 \kappa_x^{-1}$, we can go even further and use the amount of disagreement in the data to precisely identify the levels of precision in each signal (κ_x and κ_y) if we know both δ and ϕ_x. δ is pinned down by the relative cross-sectional variances. The other parameter is given by $\phi_x = \frac{(1-\alpha)\delta}{(1-\alpha)\delta+(1-\delta)}$, so we can assign a value to ϕ_x if we have an estimate of strategic complementarity α. While we cannot obtain α directly from moments of inflation expectations, we can recover this parameter from a series of hypothetical questions that were also included in the survey:

> **For the next three questions, suppose that neither you nor your competitors face any costs in changing your prices. Also suppose that you get news that the general level of prices went up by 10% in the economy:**
> **a.** By what percentage do you think your competitors would raise their prices on average?
> **b.** By what percentage would your firm raise its price on average?
> **c.** By what percentage would your firm raise its price if your competitors did not change their price at all in response to this news?

As explained in Afrouzi (2018), these hypothetical questions capture the components of the individual firm's price setting equation: $p_i = (1-\alpha)E_i[m] + \alpha E_i[\overline{p}]$. Part a. of the question captures the response of firm i's price to a change in $E_i[\overline{p}]$, or the expected price increase of competitors. Part b. captures the whole right-hand side of the equation, and part c. measures $(1-\alpha)E_i[m]$, or the amount that firms would adjust prices if unconstrained by competitors' response. Given these component pieces, it follows that α is the slope in the regression of {the answer in "b" minus the answer in "c"} on {the answer in "a"}. When we implement this regression in our sample, we find $\hat{\alpha} \approx 0.7$ (s.e. 0.02). It follows that $\phi_x \approx 0.55$ (bootstrap s.e. 0.06) given $\delta = 0.8$, that is, firms put 55% weight on their private signals and 45% on the public signal when setting prices.

With this value of ϕ_x, we can then further identify the precision of both private and public signals. Because in the data disagreement is $Var(E_i[\overline{p}]|y) = (\phi_x \delta)^2 \kappa_x^{-1} = 3.06^2$, it follows that $\kappa_x \approx 0.02$ (bootstrap s.e. 0.003). Using $\delta \equiv \frac{\kappa_x}{\kappa_x+\kappa_y}$, we find that $\kappa_y = 0.005$ (bootstrap s.e. 0.001). These estimates suggest that both signals could be rather imprecise. However, this imprecision agrees with the notion that firms should pay little attention to inflation if inflation is stable and low (e.g., Sims 2003, Mackowiak and Wiederholt 2009), which is the case in New Zealand, an early adopter of inflation targeting. Hence, not only can the model correctly replicate the empirical fact that disagreement across firms is lower for higher-order than first-order expectations, these moments can help identify all of the relevant structural parameters in the model, when combined with an independent estimate of strategic complementarity.

Fact 4: The average uncertainty in first-order expectations is greater than the uncertainty in higher-order expectations.

The average uncertainty in firms' first-order and higher-order expectations are given in equations (16a) and (16c), such that $\Omega_{\{E_i[\bar{E}[\bar{p}]]|y\}} = (\phi_x \delta^2)^2 \kappa_x^{-1}$ (where $\Omega_{\{E_i[\bar{E}[\bar{p}]]|y\}}$ is the conditional variance of firm i's forecast about other firms' expectations) and $\Omega_{\{E_i[\overline{p}]|y\}} = (\phi_x \delta)^2 \kappa_x^{-1}$ (where $\Omega_{\{E_i[\overline{p}]|y\}}$ is the conditional variance of firm i's first-order expectation). As with disagreement, the ratio of uncertainty in higher-order expectations to uncertainty in first-order expectations also pins down δ since

$$\Omega_{\{E_i[\bar{E}[\bar{p}]]|y\}} / \Omega_{\{E_i[\overline{p}]|y\}} = \delta^2 < 1.$$

Thus, the basic noisy-information model correctly predicts that average uncertainty about higher-order expectations should be lower than the corresponding uncertainty in first-order expectations. This is because higher-order beliefs place relatively more weight on public signals, which are known with certainty by firms, than private signals, which are unobserved by other firms, relative to first-order expectations.

In addition, data on the relative amount of uncertainty in first-order and higher-expectations again identifies δ. Strikingly, column (4) of Table 2 implies that the ratio of standard deviations implied by the reported distributions for own expectations of inflation (=1.11) and for expectations about other managers (=0.89) is 0.81 (bootstrap s.e. 0.03), the same value as found using data on disagreement. Hence, using either moments of uncertainty or moments of disagreement leads to the same value of δ. This can therefore be interpreted as an over-identification restriction implied by the theory which is consistent with the data.

Fact 5: The cross-sectional variance in first-order (higher-order) inflation expectations is greater than the average uncertainty in first-order (higher-order) expectations.

Equations (16a) and (16c) imply that the level of disagreement in either first-order or higher-order expectations should be identical to the average uncertainty in either first-order or higher-order expectations. In other words, $Var\left(E_i\left[\overline{E}[\overline{p}]\right]|y\right) = \Omega_{\left\{E_i\left[\overline{E}[\overline{p}]\right]|y\right\}} = \Omega_{\{E_i[\overline{p}]|y\}} = Var(E_i[\overline{p}]|y)$ because both uncertainty and disagreement are determined by the variance of the noise in the private signals. As documented in Table 2 however, uncertainty is about a third of disagreement in the survey of firms in New Zealand, both for first-order expectations as well as higher-order expectations. Hence this restriction implied by the basic noisy-information model is clearly at odds with the data.

Fact 6: The response of both first-order and higher-order inflation expectations is greater to an information treatment about the higher-order beliefs of other firms than it is to a treatment about the first-order beliefs of other firms.

In the basic noisy-information model, the average first-order and higher-order price expectations are both a linear combination of the public signal y and the fundamental m. Telling a firm in the model what either average expectation is would therefore allow them to fully infer the fundamental (since they already know the public signal), and all firms provided with this information would therefore form the exact same beliefs. In the survey, providing firms with information treatments about either average first-order or higher-order inflation expectations does not lead to a full convergence of beliefs after the provision of these information treatments. To avoid this extreme prediction from the model, we assume that firms interpret the treatment information as noisy signals (e.g., because we give moments from the survey that have sampling errors).

Specifically, when thinking about the information treatment involving the average inflation expectation of other firms, we interpret this as providing a signal for $\overline{E}[\overline{p}]$ given by

$$s_B = \overline{E}[\overline{p}] + \xi_B, \tag{19}$$

where $\xi_B \sim N(0, \kappa_B^{-1})$ and ξ_B is uncorrelated with noise ε and ν_i.[14] Note that because $\overline{E}[\overline{p}] = (1-\phi_x\delta)y + \phi_x\delta m$ and firms observe y directly, signal s_B has the same content as signal $\tilde{s}_B = \phi_x\delta m + \xi_B = H_B m + \xi_B$ with $H_B \equiv \phi_x\delta$. Using Bayes rule, we can derive beliefs about m after observing $\tilde{s}_B$

$$E_i(m|\tilde{s}_B, x_i, y) = E_i(m|x_i, y) + P_B\left(\tilde{s}_B - \phi_x\delta E_i(m|x_i, y)\right), \tag{20}$$

where $P_B = \delta\kappa^{-1}\phi_x\delta(\kappa_B^{-1} + (\phi_x\delta)^2\delta\kappa^{-1})^{-1}$ is the gain of the Kalman filter and κ is the precision of the prior $E_i(m|x_i, y)$. We can re-write this equation as:

$$E_i^{post}(m) = (1 - P_B H_B)E_i^{pre}(m) + P_B\tilde{s}_B = \frac{\kappa_B^{-1}}{\kappa_B^{-1} + (\phi_x\delta)^2\delta\kappa^{-1}} E_i^{pre}(m) + P_B\tilde{s}_B, \tag{21}$$

where $E_i^{post}(m)$ denotes expectations after receiving the additional information while $E_i^{pre}(m)$ denotes expectations before receiving additional information. Importantly, the coefficient on the prior belief $E_i^{pre}(m)$ can identify κ_B, the perceived precision of the signal $\tilde{s}_B$.

We can generalize the form of this signal across treatment types to:

$$E_i^{Post}[m] = (1 - PH)E_i^{Pre}[m] + P\tilde{s} \tag{22}$$

which one can estimate by regressing post-treatment expectations on pre-treatment expectations and a constant. A low coefficient on pre-treatment (prior) expectations indicates that managers strongly respond to a signal (i.e., a high weight on $\tilde{s}$ and a low weight on the prior). The response of firm i's expectations of $\overline{p}$ and $\overline{E}[\overline{p}]$ to information is given by

$$E_i^{Post}\begin{bmatrix}\overline{p}\\ \overline{E}[\overline{p}]\end{bmatrix} = PH\begin{bmatrix}\phi_y\\ \phi_y + \phi_x(1-\delta)\end{bmatrix}y + (1 - PH)E_i^{Pre}\begin{bmatrix}\overline{p}\\ \overline{E}[\overline{p}]\end{bmatrix} + \begin{bmatrix}\phi_x\\ \delta\phi_x\end{bmatrix}P\tilde{s}. \tag{23}$$

Note that weight on the prior $(1 - PH)$ is the same for first- and higher-order expectations about the price

[14] Although s_B does not have index i (we drop it to simplify notation), we interpret s_B as a private signal because we do not tell a firm receiving this signal that other firms receive this signal too.

level. Equation (23) implies that, for a given signal, agents should place the same weight on their prior beliefs when updating both their first-order and higher-order beliefs about inflation. This is remarkably consistent with what we observe in Table 2, where we cannot reject the null of equality for responses of first-order and higher-order beliefs to each type of information treatment. So the model is consistent with the fact that first-order and higher-order beliefs respond similarly to information treatments.

Turning to the treatment about the higher-order expectations of other firms, we interpret this as a signal of the same form as equation (19) and given by:

$$\tilde{s}_C = \phi_x \delta^2 m + \xi_C = H_C m + \xi_C \quad (24)$$

with $\xi_C \sim N(0, \kappa_C^{-1})$ and $H_C \equiv \phi_x \delta^2$ so that

$$E_i^{post}(m) = (1 - P_C H_C) E_i^{pre}(m) + P_C \tilde{s}_C = \frac{\kappa_C^{-1}}{\kappa_C^{-1} + (\phi_x \delta^2)^2 \delta \kappa^{-1}} E_i^{pre}(m) + P_C \tilde{s}_C \quad (25)$$

where $P_C = \delta \kappa^{-1} \phi_x \delta^2 (\kappa_C^{-1} + (\phi_x \delta^2)^2 \delta \kappa^{-1})^{-1}$ is the gain from the Kalman filter. If signals about first-order and higher-order expectations were perceived as equally precise ($\kappa_C^{-1} = \kappa_B^{-1}$) by firms, we would have $P_B > P_C$, that is, signal s_B (treatment with the average first-order expectation) would receive a *higher* weight than signal s_C (treatment with the average higher-order expectation) when firms update their beliefs. Intuitively, because higher-order beliefs are more concentrated on the public signal, which is consistent with fact 3, they are interpreted by firms as containing less information about the unobserved m and would therefore get less attention from managers. This result implies that, under equally precise signals, we would expect to observe a stronger response of both first-order and higher-order beliefs to information about the average first-order belief than about the information about the average higher-order belief. As shown in Table 3, we observe exactly the opposite in the survey of firms: their expectations respond more strongly to treatments about higher-order expectations of other firms than they do to treatments of first-order expectations of other firms. Note that facts 3 and 6 therefore deliver a contradiction that is hardwired into the baseline model. They cannot hold simultaneously as the first requires higher-order beliefs to be less dispersed while the second requires higher-order beliefs to be more dispersed. In the context of the basic noisy-information model, the only way to match fact 6 simultaneously with fact 3 would be for managers to interpret signals about higher-order beliefs as being far more precise than signals about first-order beliefs (we would need $\kappa_C \approx 10\kappa_B$ to match the responses). Because there is a priori no reason to expect this to be true, we interpret this empirical fact as being at odds with the basic noisy-information model.

3.3 Summary

We use a basic model of noisy information with infinite regress of expectations to interpret the data on first- and higher-order expectations from the survey. Despite its simplicity, this basic model is consistent with a number of the empirical facts from the survey, as summarized in Table 4. For example, it can explain why both disagreement and average uncertainty are lower for higher-order than first-order inflation expectations. The simple model is consistent with, and provides an interpretation for, the fact that average first-order and higher-order inflation expectations are so close in the data. And it can explain why, in the cross-section, a firm with larger first-order expectations tends to have larger higher-order expectations. We also show that the moments from the survey can be used to recover the realized shocks and the underlying parameters of the model, a feature which can be useful to discipline these models in the future. Furthermore, the model makes over-identifying restrictions on parameters in the sense that different moments can be used to identify the same parameters. Strikingly, we find that these different moments are generally consistent with one another in terms of the parameter values they imply. The model can also rationalize the fact that agents update both their first-order and higher-order expectations by similar amounts in response to an information treatment.

However, we also document several limitations of the canonical noisy-information model with infinite regress in terms of its ability to rationalize the data. First, while first- and higher-order expectations are highly correlated across firms in the data, there is significant heterogeneity that the canonical model cannot easily explain. Second, the model predicts that the level of uncertainty and cross-sectional dispersion of expectations should be the same, a feature we can strongly reject in the data. Third, if we view the information treatments as being noisy signals, it is difficult to rationalize the difference in response across types of treatments. This interpretation of the data would require firms to believe that the signal about the higher-order expectations of other firms be much more precise than the signal about the first-order expectations of other firms. As a result of these contradictions between data and theory, we consider whether variants on the basic noisy-information model can better come to grips with the empirical facts on the higher-order beliefs of firms.

IV. Extensions

There are a number of ways that one can deviate from the basic noisy-information model to potentially address the differences between our empirical results and the basic model with infinite regress. The first approach we consider simply incorporates measurement error in survey responses. A second is to augment the signal space with a semi-public signal. Both can help generate additional idiosyncratic variation in expectations and weaken the predicted link between first-order and higher-order expectations. Another approach is to allow for heterogeneity in “long-run” priors, which would generate additional variation in the beliefs of agents even before they receive their idiosyncratic signals. A fourth approach is to relax the assumption that agents utilize the signals in the optimal way, allowing instead for overreaction to some signals. The final strategy we consider introduces behavioral/cognitive constraints that prevent agents from engaging in the infinite regress used in our stylized model. In this section, we characterize the extent to which each of these alternatives helps reconcile theory and empirics.

4.1 Measurement Error in Survey Responses

Perhaps the simplest way to potentially explain some of the departures of the basic model from the data is to allow for reporting/measurement error. Specifically, we assume that instead of reporting their true expectations, firms report this expectation plus an independent noise term of the form $v_{FO} \sim N(0, \kappa_{FO}^{-1})$ for first-order expectations and $v_{HO} \sim N(0, \kappa_{HO}^{-1})$ for higher-order expectations. Since these errors are mean zero, they do not change the ability of the model to explain the first stylized fact, namely the near equality of average first- and higher-order expectations. The introduction of uncorrelated error terms to both first- and higher-order expectations can immediately help reconcile our model with the second stylized fact, namely the finding that the cross-sectional correlation between first- and higher-order expectations is less than one in the data. As long as the variance of the noise in first- and higher-order reported forecasts is sufficiently similar ($\kappa_{HO} \approx \kappa_{FO}$), then fact 3 will still hold as the cross-sectional variance of higher-order forecasts will still be given by a fraction of the cross-sectional variance of first-order forecasts. Fact 4 also continues to hold, assuming that measurement error affects only reported mean forecasts and not the distributions around the forecasts, so that reported uncertainty in both first- and higher-order forecasts is unchanged. Under this condition, the noisy-information model with measurement error can also explain fact 5, since the cross-sectional variance of forecasts will be greater than the average uncertainty in both first- and higher-order forecasts:

$$Var(E_i[\overline{p}]|y) = (\phi_x\delta)^2\kappa_x^{-1} + \kappa_{FO}^{-1} > (\phi_x\delta)^2\kappa_x^{-1} = \Omega_{\{E_i[\overline{p}]|y\}}$$

$$Var\left(E_i\left[\overline{E}[\overline{p}]\right]|y\right) = (\phi_x\delta^2)^2\kappa_x^{-1} + \kappa_{HO}^{-1} > (\phi_x\delta^2)^2\kappa_x^{-1} = \Omega_{\{E_i[\overline{E}[\overline{p}]]|y\}}.$$

Introducing measurement error can therefore help the model match the first five empirical facts qualitatively. However, measurement error cannot help match the effect of information treatments, unless one were prepared to assume that measurement error was perceived as significantly lower for higher-order beliefs than first-order beliefs. In addition, it is difficult for measurement error to *quantitatively* explain the differences between the basic noisy-information model and empirical facts 2 and 5.

To see this, note that we have repeated, within-survey measurements of first-order beliefs (point predictions) in the first wave of the survey and we can use these repeated measurements to assess the quantitative importance of measurement errors (that is, a respondent reports his/her belief plus a reporting error). We find that the correlation between pre-treatment point predictions and post-treatment point predictions (within the control group) is 0.98, so the implied amount of measurement error is quite small ($\kappa_{FO}^{-1} \approx 0.02 \times (\phi_x\delta)^2\kappa_x^{-1}$). In contrast, the amount of measurement error needed to explain the data is quite large. For example, to explain the difference between average uncertainty in first-order expectations and the cross-dispersion in those expectations, one would need $\kappa_{FO}^{-1} \approx 6.5 \times (\phi_x\delta)^2\kappa_x^{-1}$ so that

$\frac{Var(E_i[\bar{p}]|y)}{\Omega_{\{E_i[\bar{p}]|y\}}} = 1 + \frac{\kappa_{FO}^{-1}}{(\phi_x \delta)^2 \kappa_x^{-1}} \approx 7.5$ as in the data.[15] In short, while allowing for measurement error in the survey data is qualitatively helpful along some (but not all) dimensions as summarized in Table 4, it is quantitatively insufficient to account for departures between the model and the data.

4.2 Semi-Public Signals

Another way to break the perfect correlation between firms' first- and higher-order expectations is to allow for multiple sources of idiosyncratic variation in managers' expectations rather than a single idiosyncratic noise term in the firm's private signal. For example, we consider the case where firms receive a semi-public signal (rather than the fully public one in section 3) in addition to their private signal. The semi-public signal case looks very similar to the basic model presented in Section 3 with the twist that the public signal is no longer perfectly observed, but includes a manager-specific error term:[16]

$$x_i = m + \nu_{i,1} \tag{26}$$

$$y_i = y + \nu_{i,2} = m + \varepsilon + \nu_{i,2} \tag{27}$$

where $\nu_{i,1} \sim N(0, \kappa_x^{-1})$, $\varepsilon \sim N(0, \kappa_y^{-1})$, and $\nu_{i,2} \sim N(0, \kappa_z^{-1})$.

In this setting, because first- and higher-order expectations are formed as different mixtures of signals x_i and y_i, the implied correlation between $E_i[\bar{p}]$ and $E_i[\bar{E}[\bar{p}]]$ becomes less than one, as we see in the data (fact 2). While the inclusion of a semi-public signal therefore helps explain the observed imperfect correlation between the two expectations in the data, it does less well in reconciling the remaining discrepancies between the data and the basic model (see Appendix C for derivations). For example, introducing a semi-public signal does not explain the difference between cross-sectional disagreement and uncertainty in the model: in this modification, uncertainty should be *higher* than disagreement because now managers are uncertain not only about the fundamental m but also the common component y in the semi-public signal y_i. As a result, the model continues to be strongly at odds with fact 5. Furthermore, the presence of a semi-public signal rather than a public signal also cannot explain why information treatments involving higher-order beliefs of other firms lead to larger revisions in forecasts than treatments involving first-order beliefs. With a semi-public signal, agents continue to place relatively more weight on the latter signal than on private signals when

[15] We could also consider the amount of measurement error needed to reconcile the model with an estimated slope of 0.6 in Figure 3, assuming the true $\delta = 0.8$, as needed to match fact 4. In this case, measurement error in first-order expectations would need to be $\kappa_{FO}^{-1} \approx 0.25 \times (\phi_x \delta)^2 \kappa_x^{-1}$, again much larger than what we estimate from revisions in first-order point forecasts.

[16] With minor modifications, the model with a semi-public signal may be interpreted as allowing for different public signals across industries. Specifically, rather than assigning each manager a public signal with idiosyncratic noise, we partition the semi-public signal into industry-specific public signals. In this case, each firm in industry j receives $y_j \sim N(y, \kappa_z^{-1})$. This structure allows managers to consider only the public information of their direct competitors and to form higher-order expectations in a manner consistent with this motivation.

forming higher-order beliefs, and because they know all other agents do the same and do so based on a correlated signal, uncertainty about the higher-order beliefs of other firms is lower than uncertainty about the first-order beliefs of other firms. Receiving a signal about first-order beliefs of other firms should therefore (assuming equally precise treatment signals) lead to a stronger response in beliefs than receiving an equivalent signal about the higher-order beliefs of other firms. Therefore, this extension of the basic model cannot explain why managers respond more to the higher-order signal than the first-order signal. In short, as summarized in Table 4, the semi-public signal fares even worse than measurement error in reconciling theory and data.

4.3 Heterogeneity in Long-Run Priors

An alternative extension to the basic noisy-information model of Section 3 is to introduce heterogeneity in prior beliefs. Specifically, we follow Patton and Timmermann (2010), who propose a modification to the basic noisy-signal model to rationalize large disagreement in short-horizon forecasts. They posit that forecasters shrink their optimal inflation forecast (that is, the forecast based on information in objective signals) toward their prior beliefs about long-run inflation. In other words, forecasters "anchor" their reported predictions to their long-run forecasts, or "long-run" priors. As we show below, this modification helps us address several issues in the basic noisy-information model. To keep the exposition concise, we focus only on key results here and present detailed derivations in Appendix D.

The signal structure about m is unchanged and managers still set prices as in equation (2) but, as in Patton and Timmermann (2010), their reported expectation of the aggregate price level (which they use to set prices) is now skewed by the manager's "long-run" prior μ_i:

$$E_i^*[\bar{p}] = \omega\mu_i + (1-\omega)E_i\left[\int p_j dj\right] = \omega\mu_i + (1-\omega)\left\{(1-\alpha)E_i\left[\bar{E}[m]\right] + \alpha E_i\left[\bar{E}[\bar{p}]\right]\right\} \quad (28)$$

where $E_i^*[\bar{p}]$ denotes the skewed first-order expectation of $\bar{p}$ ($E_i[\bar{p}]$ continues to denote the correct mathematical expectation of $\bar{p}$), $\mu_i \sim N\left(\bar{\mu}, \kappa_\mu^{-1}\right)$ and $\omega = \frac{Var(E_i[\int p_j dj])}{\gamma^2 + Var(E_i[\int p_j dj])}$. $\gamma^2 \geq 0$ is a parameter measuring the extent to which the managers prefer their own priors. κ_μ^{-1} and $\bar{\mu}$ measure the dispersion and average level of the "long-run" priors. We assume that only first-order expectations of managers are skewed directly. We further allow the average prior $\bar{\mu}$ to be unobserved so that the dispersion of μ_i extends into higher-order expectations. An individual firm now sets its price as a function of its "long-run" prior, a sum of progressively higher-order expectations of the aggregate prior, $\bar{\mu}$, and a sum of progressively higher-order expectations of m. We impose further structure on $E_i\left[\bar{E}^k[\bar{\mu}]\right]$, the k^{th}-order expectation of the average prior by assuming that managers do not know $\bar{\mu}$, but each observes a private signal of the mean $\varsigma_i \sim N(\bar{\mu}, \kappa_\varsigma^{-1})$. We further allow the manager's own "long-run" prior to skew their view of the aggregate "long-run" prior: $E_i[\bar{\mu}] = \omega' \mu_i + (1-\omega')\varsigma_i$ where $\omega' = \frac{\kappa_\varsigma^{-1}}{\gamma^2 + \kappa_\varsigma^{-1}}$.[17] The uncertainty about the aggregate prior, κ_ς^{-1}, is necessary to bring priors and the dispersion they provide into higher-order expectations. If $\bar{\mu}$ were known with certainty, $\omega' = 0$ and $\varsigma_i = \bar{\mu} \; \forall \; i$. This would mean that disagreement and uncertainty about higher-order expectations would be equal, which is rejected in the data (fact 5). Apart from extending the effect of "long-run" priors into higher-order expectations, this modelling approach features $\bar{E}[\bar{\mu}] = \bar{\mu}$ so that $E_i\left[\bar{E}^k[\bar{\mu}]\right]$ is the same $\forall \; k$.

With heterogeneous priors, there are now two sources of variation between first- and higher-order beliefs: information in the private signal as well as the difference between the agent's prior belief and the prior belief they assign to others. Since the two sources are uncorrelated by assumption and the weights on signals are different for first- and higher-order expectations, heterogeneity in "long-run" priors generates an imperfect correlation in the cross-section between the first-order and second-order beliefs of agents and the slope in the regression of higher-order expectations on first-order expectations is less than 1, as observed in the data (fact 2). However, the difference between the average first-order expectations and average higher-order expectations remains determined as in the basic model, so fact 1 can still be satisfied.

In addition, we can show that if $\kappa_\mu^{-1} > 0$ and $\kappa_\varsigma^{-1} > 0$ (i.e., "long-run" priors are heterogeneous with an uncertain mean level), then average uncertainty in both first- and higher-order expectations must be lower than the cross-sectional disagreement in first- and higher-order expectations respectively. For example, uncertainty ($\Omega_{\{E_i[\bar{p}]|y\}}$) and disagreement ($Var[E_i[\bar{p}]]$) in first-order beliefs are related as follows:

$$Var[E_i[\bar{p}]] = \Omega_{\{E_i[\bar{p}]|y\}} + (\omega + (1-\omega)\theta\omega')^2 \kappa_\mu^{-1}.$$

Hence, fact 5 is satisfied (unlike in the basic noisy-information model) because the dispersion in "long-run" priors provides another source of cross-sectional variation in expectations that is not reflected in uncertainty around forecasts since managers take their "long-run" priors as given.

In contrast to the basic noisy-information model which unambiguously predicts that $\Omega_{\{E_i[\bar{p}]|y\}} > \Omega_{\{E_i[\bar{E}[\bar{p}]]|y\}}$ and $Var[E_i[\bar{p}]] > Var\left[E_i[\bar{E}[\bar{p}]]\right]$, the ranking of first- and higher-order uncertainty and disagreement in the noisy-information model with "long-run" priors depends on parameter values. As we show in Appendix D, if κ_x^{-1} and κ_μ^{-1} are large relative to κ_ς^{-1} and, the model with "long-run" priors can reproduce $\Omega_{\{E_i[\bar{p}]|y\}} > \Omega_{\{E_i[\bar{E}[\bar{p}]]|y\}}$ and $Var[E_i[\bar{p}]] > Var\left[E_i[\bar{E}[\bar{p}]]\right]$ and thus match facts 3 and 4. When κ_x^{-1} is high, there is a lot of dispersion in private signals. Moving from first-order to higher-order beliefs, managers place less weight on their own priors and on their private signals, as in the basic noisy information model, which

[17] Setting $\omega' = 1$ nests the case in which managers assume that all other managers share their own prior.

tends to reduce dispersion and uncertainty in higher-order beliefs relative to first-order beliefs. A high κ_{μ}^{-1} indicates a lot of heterogeneity in long-run priors about average prices. Since managers understand that they have a biased first-order belief and that other managers report biased first-order beliefs too, when thinking about what other managers believe, each manager tries to remove their own bias (i.e., their "long-run" prior) from the reported value. This tends to reduce dispersion in higher-order beliefs relative to first-order beliefs. When κ_{ς}^{-1} is high on the other hand, there is a lot of dispersion in beliefs about other managers' priors. This therefore tends to increase both uncertainty and dispersion in higher-order forecasts relative to first-order forecasts. Reproducing facts 3 and 4 therefore requires that this last force be weaker than the prior two.

To understand how the model with long-run priors can potentially match fact 6 as well, one should recall why the basic noisy-information model could not simultaneously match the strong estimated response of expectations (i.e., a low value of the slope coefficient in specification (1)) to treatment C (relative to treatment B) as well as the levels of disagreement and uncertainty observed in the data. Intuitively, matching a low slope in response to treatment C requires a higher value of κ^{-1} (the combined precision of private and public signals) so that agents are less confident in their priors about higher-order beliefs and therefore respond strongly to higher-order signals. Matching the levels of disagreement and uncertainty instead requires a lower value of κ^{-1}, so that agents are relatively more confident in their higher-order beliefs thus reaching a contradiction.[18] In contrast, the model with long-run priors decouples these moments by allowing another source of variation (long-run priors).

Because there are now two unobserved states m and $\bar{\mu}$ and we have a common public signal only for fundamental m, the weights placed on these fundamentals in forming expectations change as we increase the order of expectations: the weight on m is shrinking as managers put increasingly more weight on the public signal y, while the weight on $\bar{\mu}$ does not. In other words, as we increase the order of expectations in the signals, the content of the signals is increasingly skewed (in relative terms) toward $\bar{\mu}$. Also note that our signals s_B and s_C effectively have a higher order of expectations than $E_i[\bar{p}]$ and $E_i\left[\bar{E}[\bar{p}]\right]$ respectively. This leads to a discrepancy in the weights assigned to the fundamentals in the expectations that are getting updated relative to the weights embedded in the signals. When we extrapolate the posterior beliefs for a given order of expectations (e.g., $E_i^{Posterior}[\bar{p}]$) from the prior beliefs for that order (e.g., $E_i^{Posterior}[\bar{p}]$) in response to a signal that measures a higher order of expectations (e.g., signal B that provides $\bar{E}[\bar{p}] + noise$), this extrapolation overstates the contribution due to $\bar{\mu}$ and understates the contribution due to m.

[18] As we discuss above, the basic noisy-information model can match the estimated regression coefficients in specification (1) for treatments C and B if signal C is more precise than signal B. However, this would require signal C to be an order of magnitude more precise than signal B (i.e., $\kappa_C \approx 10\kappa_B$). Relaxing the equality of precision in signals B and C also helps the model with "long-run" priors to match quantitatively the estimated differences in slopes for signals B and C but the required difference in precision is much more modest: it is enough to have $\kappa_C \approx 2\kappa_B$.

This feature of the experiment implies that from the point of the model, equation (1) is generally misspecified: one cannot express the observed posterior first (higher) order expectations for $\bar{p}$ as a function of only the signal received in the treatment and the observed prior first (higher) order expectations for $\bar{p}$. [19] The misspecification introduces omitted terms that depend on prior beliefs about m and $\bar{\mu}$. Furthermore, given the structure of the model (specifically the fact that higher-order signals put smaller weights on m), the omitted term for $E_i^{Prior}(m)$ creates a positive bias in the estimated slope on observed prior first (higher) order expectations for $\bar{p}$, while the omitted term for $E_i^{Prior}(\bar{\mu})$ creates a negative bias. As a result, on the one hand, the *true* response to signal C in this modified model will tend to be weaker than the *true* response to signal B because we need to match the fact that uncertainty in higher-order expectations is lower than uncertainty in first-order expectations, just as was the case in the basic noisy-information model. On the other hand, provided that we have large variation in μ_i, the misspecification can lead to large negative biases in the *estimated* slopes in response to treatment C relative to treatment B thus matching fact 6 (see Appendix D for more details). In summary, at least qualitatively, this model provides one potential way to reconcile theory and data.

While heterogeneity in priors can thus potentially account for all the deviations between the basic noisy-information model and the data, this more sophisticated model does not permit easy identification of structural parameters from the empirical moments. For example, with heterogeneous "long-run" priors, the ratio of higher-order disagreement and first-order disagreement does not uniquely pin down δ as in equation (18). The mapping from moments to parameters is more complex and requires solving a system of nonlinear equations. Furthermore, given the data available in our survey, the system is under-identified and, therefore, one needs additional assumptions to calibrate model parameters (Appendix D provides an example). Alternatively, one could bring in additional data. Particularly useful would be first- and higher-order *long-run* inflation expectations to measure μ_i and $E_i[\bar{\mu}]$ to recover γ, γ', and κ_ς (see Appendix D for more details), but unfortunately these data were not part of our survey. The inability to pin down more parameters of the model implies that, without additional input, it is difficult to determine whether this model is quantitatively able to match all features of the data. Nonetheless, the structure of the model and the patterns observed in the data suggest that the model with long-run priors should assign a prominent role to variation in long-run priors $(\kappa_\mu^{-1}, \kappa_\varsigma^{-1})$ because we need this variation to generate a negative bias in the estimated slope coefficients in response to treatment C.

4.4 Overconfidence

The data show considerable heterogeneity in managers' inflation expectations as well as cross-sectional

[19] There is one exception when we study the response of $E_i\left[\bar{E}[\bar{p}]\right]$ to signal B. In this case, the order of expectations is the same for measured expectations $E_i\left[\bar{E}[\bar{p}]\right]$ and for the signal.

disagreement exceeding subjective uncertainty. This suggests that managers disagree with each other about the level of inflation but do not realize how much they disagree. In the previous section, we explained this disagreement with heterogeneity in long-run priors. As an alternative to heterogeneity in priors, we can explain the large degree of heterogeneity across managers and the disparity between disagreement and subjective uncertainty by modeling managers as overconfident about the precision of their signals. Over-confidence means that managers have excessive faith that their signals reflect the truth (Moore and Healy 2008). In this extension, we allow managers to hold beliefs about signal precision that differ from the truth. In order to generate the same patterns seen in the data, managers must overestimate the precision of the *private* signal, κ_x, as in Daniel, Hirshleifer and Subrahmanyam (1998). Specifically, managers continue to receive public and private signals as in section 3 where $\nu_i \sim N(0, \kappa_x^{-1})$ is the noise in the private signal and $\varepsilon \sim N(0, \kappa_y^{-1})$ is the noise in the public signal. However, we allow them to overestimate the precision of the private signal such that $E_i[\kappa_x] > \kappa_x$ and they therefore overestimate its relative precision as well: $\tilde{\delta} = \frac{E[\kappa_x]}{\kappa_y + E[\kappa_x]} > \frac{\kappa_x}{\kappa_y + \kappa_x} = \delta$. This means that firms' perceived level of the fundamental will be overly sensitive to the private signal but insufficiently sensitive to the public signal. We provide derivations in Appendix E.

The model with overconfidence can correctly reproduce facts 1, 3 and 4 following the same logic as the basic noisy-information model. With respect to fact 2, there is still only one idiosyncratic source of information, so while the model can reproduce the positive average relationship between first-order and higher-order expectations of firms, it generates the same counterfactual prediction of a perfect correlation between the two as in the basic noisy-information model. Allowing for overconfidence, however, helps explain fact 5: the greater cross-sectional disagreement in first- and higher-order expectations relative to the uncertainty in those expectations. This is due to the fact that the low uncertainty that firms anticipate reflects the perceived precision of the private signal, which is greater than its true precision. The dispersion in forecasts however is driven by the actual dispersion in private signals, and therefore exceeds the uncertainty perceived by firms.[20]

Quantitatively, the degree of overconfidence needed to match this fact is relatively large. The disagreement and uncertainty terms for each first-order and higher-order expectations give us four independent moments to pin down three parameters: κ_x, κ_y, and $E_i[\kappa_x]$. As before, we use a value of α based on a separate set of survey questions following Afrouzi (2018). The ratio of cross-sectional variances of

[20] An alternative to the overconfidence approach is the diagnostic model of Bordalo et al. (2018, 2019, 2020) in which agents over-respond to *all* signals. In this context, we can formalize this as perceiving both public and private signals as being more precise than they actually are. If the over-precision is proportional across signals, agents will allocate the same relative weights to public and private signals as in the basic noisy-information model of Section 3. The cross-sectional variance of beliefs will be unchanged but the average uncertainty about both first-order and higher-order beliefs will be lower, since these rely on perceived precision of the signals. As a result, this alternative formulation of overconfidence or diagnostic expectations can also explain why uncertainty is lower than disagreement in the data.

higher-order and first-order expectations implies that $\tilde{\delta} \approx 0.8$.[21] Given this estimate and our estimated value of α, we obtain $\tilde{\phi}_x \approx 0.55$. This in turn implies that the perceived precision of the private signal is $E_i[\kappa_x] \approx 0.15$. This significantly exceeds the value of $\kappa_x \approx 0.02$ implied by the cross-sectional disagreement. In other words, managers should perceive private signals to be roughly an order of magnitude more precise than they actually are. From $\tilde{\delta}$ and $E_i[\kappa_x]$, we find that $\kappa_y \approx 0.04$. The relative weights assigned to private and public signals in forming beliefs about the price level are quantitatively very different from the optimal ones: the optimal weight on private signals in forming first-order beliefs is $\delta\phi_x \approx 0.04$ while the actual weight used by overconfident agents is $\tilde{\delta}\tilde{\phi}_x \approx 0.44$. Because the actual private signals are in fact quite imprecise, the large weight the managers assign to them results in a large degree of heterogeneity and disagreement.

With respect to information treatments, the structure of the basic noisy-information model is preserved in this setting, but with perceived signal precisions (and associated response parameters) in lieu of actual signal precisions. Thus, it's still the case that managers should respond more strongly to Signal B (first-order treatment) than Signal C (higher-order treatment) which is counterfactual. By itself, overconfidence therefore only helps the model match fact 5. However, there is, at least, one potential way to reconcile the model of overconfidence with our results on the effects of information treatments. Specifically, if one were willing to consider models of overconfidence in which agents are potentially overconfident about the quality of the signals introduced in the information experiment, then this could reconcile the model with the data. Specifically, this would require managers to have differential overconfidence in signals B and C. In that sense, such a model would resemble the basic noisy-information model in requiring differential precisions for signals B and C, that is, both models require an extra degree of freedom to rationalize the observed reactions to signals. But a priori it's not clear why one might expect overprecision of this type.

4.5 Level-k Thinking

The basic noisy-information models assumes that agents undertake infinite degrees of reasoning about the pricing decisions of others. Reasoning of this sort is, however, difficult and computationally intensive. Managers are therefore likely, due to either cognitive constraints or recognizing the costs of such reasoning, to limit their degrees of thinking to levels well below infinity. In this section, we introduce cognitive constraints via level-k thinking, i.e. restricting how far individuals go in terms of higher-order thinking.

To make our model of expectations consistent with level-k thinking, we revise the optimal pricing equation in equation (9) such that firm i will weigh the public and private signals according to

[21] The model is overidentified as the ratio of higher-order uncertainty to first-order uncertainty also pins down an estimate of $\tilde{\delta}$. This additional restriction again implies that $\tilde{\delta} \approx 0.8$.

$$p_i(k) = \frac{\sum_{r=0}^{k} \alpha^r \left[\left[1-\delta^{r+1}\right]y + \delta^{r+1} x_i\right]}{\sum_{r=0}^{k} \alpha^r},$$

where k is the firm's type. We allow firms to fall into one of three different thinking types such that $k = 0,1,2$. A level-0 firm will have pricing strategies in equation (10) with $\phi_{x,0} = \delta$ and $\phi_{y,0} = 1 - \delta$. These strategies ignore the strategic complementarity in prices and rely only on the relative precision of the public and private signals. One can show that the strategies for level-1 and level-2 firms will shift weight towards the public signal; that is, $\phi_{x,0} > \phi_{x,1} > \phi_{x,2}$ and $\phi_{y,0} < \phi_{y,1} < \phi_{y,2}$ as $\phi_{x,k} = \delta \frac{1+\alpha\delta+\cdots+(\alpha\delta)^k}{1+\alpha+\cdots+\alpha^k}$ and $\phi_{y,k} = 1 - \phi_{x,k}$. The aggregate price-level will then be a weighted average of the pricing behavior of each type of firm $\overline{p} = \sum_{k=0}^{2} \omega_k \overline{p(k)}$ where ω_k is the proportion of firms thinking at level-k and $\overline{p(k)} = \phi_{x,k} m + \phi_{y,k} y$. Heterogeneity in strategies means that firms must consider the distribution of types in forming their expectations. For simplicity, we model that all firms behave as if all other firms are of their own type.

Due to their cognitive constraints, level-0 and level-1 firms are unable to iterate expectations past their first-order expectation. As a result, their higher-order expectations are the same as their first-order expectations. For level-2 firms, higher-order expectations reflect an extra level of iteration relative to their first-order expectations but do not go through the full iteration of firms in the basic noisy-information model. Nonetheless, it is still the case that $E_{i,2}\left[\overline{E}[\overline{p}]\right] = (1-\delta)y + \delta E_{i,2}[\overline{p}]$ as in the basic model where $E_{i,2}$ denotes the expectation of firm i of level-2.

The model with level-k thinking can readily accommodate the same facts as the noisy-information model. For example, one can show (Appendix B) that for level-0 and level-1 firms, higher-order and first-order expectations are the same, so fact 1 holds exactly for them and can hold for level-2 firms just like for firms in the basic noisy-information model. Similarly, the dispersion of higher-order expectations will be lower for level-2 firms than the dispersion in their first-order expectations, while it will be identical for level-0 and level-1 firms, so fact 3 will hold as long as there are some level-2 firms. The same logic holds for fact 3 in terms of average uncertainty. Unlike the basic model, fact 2 will also now be satisfied. This is because level-0 and level-1 firms have a slope relationship of one between their first-order and higher-order beliefs while level-2 firms have a slope relationship of $\delta < 1$. Because we mix groups of firms with different slopes, there will be an imperfect correlation between first- and higher-order beliefs across all firms, with a positive average slope that is less than 1.

The level-k model does not have a clear prediction for the difference between disagreement and uncertainty. However, using the proportions of types observed in our data, any feasible calibration of δ yields disagreement and uncertainty estimates that are approximately equal (see Appendix B for derivations). Finally, the model with level-k thinking will not be able to match fact 6, the stronger response of beliefs to higher-order than first-order treatments. This is because level-2 firms will respond like firms in the basic noisy-information model, i.e. more strongly to lower-order signals. Level-0 and level-1 firms will respond equally strongly to the two signals since these firms do not distinguish between first-order and higher-order moments.

While level-k thinking helps match some empirical facts that the basic model cannot, it also makes testable predictions regarding firms' beliefs and their level of thinking. For example, key to the model's ability to match fact 2 is that higher-order and first-order expectations have a slope relationship of 1 when k=0 or 1 but a slope less than one for k=2. In the survey, we asked questions that allow us to characterize the degree of level-k thinking of firms and thereby test these additional predictions of the model. Specifically, following Nagel (1995), Nagel and Duffy (1997) and many others, we characterize managers' degree of reasoning by asking the following question:

> *"Please choose a number from zero to 100. We will take your number as well as the numbers chosen by other managers to calculate the average pick. The winning number will be the number that is closest to two-thirds (2/3) of the average. The individual(s) with the winning number will receive (or share with other winners in case of tie) $500."*

A k^{th}-level thinker provides the following guess: $g(k) = \left(\frac{2}{3}\right)^k \times 50$. The distribution of managers' guesses appears in Figure 4. Guesses appear throughout the entire interval (0-100). However, when we restrict the sample to those managers who spent at least 20 seconds on the question, the guesses pile on integers associated with reasoning types as defined by the equation above between $k = 1$ and $k = 5$, with the number of managers of each type declining with k. Accordingly, we classify these managers by their guess and assign $k = 0$ to those who answer the question in less than 20 seconds.[22,23] The average guess in our sample is 33 when we use all responses and 21 when we use guesses with response time of 20 or more seconds. In our survey, 36.8 percent of managers are $k = 0$, as opposed to 20 to 27.3 percent in the experimental studies. Our sample is also more heavily weighted towards higher thinking types ($k \geq 3$) than other papers, with roughly a quarter of the sample performing such high degrees of reasoning.[24] Individual levels of thinking are generally uncorrelated with the observable characteristics of firms and managers as well as industry fixed effects (Appendix Table 6).

Table 5 documents how various moments of survey expectations from New Zealand vary with the level of k of each agent. Mean expected inflation and disagreement about future inflation decrease in k, while uncertainty is approximately constant across k. These patterns hold for first- and higher-order

[22] The guesses associated with $k = 0$ therefore fall throughout the [0,100] interval, rather than at 50 as in Nagel (1995).
[23] As a robustness check, we consider an alternative treatment of guesses with short response times: we code responses as level-zero thinking if response times are less than 20 seconds and responses are between 47 and 53; we set level of thinking to missing for other responses with response time less than 20 seconds. We denote this alternative coding with k'.
[24] Camerer (1997) reports that average responses for CEOs at Cal Tech's Board of Trustees, for portfolio managers, and for Wharton's MBA students are 38, 24, and 38 respectively. However, Camerer (1997) reports generally lower average scores for subjects participating in experimental settings.

inflation expectations and are broadly consistent with theoretical predictions: low-k firms should disagree more (since they place more weight on private signals) and should be more likely to have expectations with larger departures from fundamentals. The precision of signals (κ_x, κ_y), the relative precision of the private signal (δ), and the weight on the private signal (ϕ_x) exhibit an inverted-U shape in k whereas the theory predicts that $\phi_{x,k}$ should decrease in k monotonically. Finally, while in our theoretical setting the cross-sectional correlation between first- and higher-order inflation expectations is perfect (recall that private signals x_i is the only source of variation in the cross-section), one might more generally expect the correlation to be stronger for low-level thinkers because these thinkers do not distinguish between low- and high-level expectations. In fact, we find that the correlation between first- and higher-order inflation expectations is weakly increasing in k. In short, we do not observe clear links between the level of thinking done by managers and their reported first-order and higher-order inflation expectations.

This evidence should be interpreted with caution for several reasons. First, sample sizes are relatively small so the sampling uncertainty in the estimates is relatively large. Second, the amount of predicted variation in sensitivity of beliefs and possible actions to new information across different k can be quantitatively small depending on underlying parameter values. The fact that we cannot uncover meaningful differences depending on the level of thinking may therefore reflect underlying parameter values rather than a failure of the model. Third, while beauty contest questions are commonly used to assess the level of thinking in the experimental literature, these measures may not necessarily be appropriate to measure the levels of thinking used by managers when they revise their inflation expectations or make decisions about employment, investment, etc. In this case, the fact that we do not find variation in expectations or behavior for different levels of k could simply reflect a poor identification of relevant k. Future work could consider alternative approaches to measure cognitive abilities of firm managers to assess whether these affect their beliefs and decisions.

V. Concluding remarks

This paper presents novel survey evidence on the higher-order inflation expectations of firms. Higher-order expectations are often perceived to be crucial to many decisions and play a key role in many macroeconomic models. Yet empirical evidence on them has been completely lacking. Our survey therefore provides unprecedented empirical evidence on how firms form higher-order beliefs about inflation and how these relate to their first-order beliefs. The resulting empirical facts can be used to both test and quantify models with higher-order beliefs. For example, we illustrate how our empirical moments can be used to pin down the parameters of a basic noisy-information model. These results can therefore be of immediate practical use for future work using this class of models.

Our results can also help identify along which dimensions noisy-information models could be extended. Indeed, while simple models of noisy-information can go a long way in rationalizing observed expectations of firms, the mapping from expectations to actions is more complex than is commonly postulated by these models. One increasingly popular departure from the noisy-information model with infinite regress is to assume cognitive constraints on agents in the form of level-*k* thinking. But our results suggest that this approach is unlikely to be fruitful in accounting for apparent deviations between the data and theory. While we are able to identify the level of thinking associated with each manager, we find little evidence that any important dimension of the data is related to these differing levels of thinking. Other extensions to the basic noisy-information model seem more promising. One such avenue is incorporating heterogeneity in priors. This extension can help explain many of the otherwise puzzling empirical results, but the additional flexibility introduced by this model also implies that our data moments are insufficient to pin down all the parameters of the model. Another promising avenue is allowing for overconfidence or diagnostic expectations on the part of agents, which also allows the model to conform more closely to our survey results.

While these results present a theoretical challenge to basic noisy-information models, they nonetheless should be of immediate interest to policymakers. For example, communication-based policy tools (e.g., forward guidance) often rely on moving not only first-order but also higher-order expectations. These results provide a rationale for utilizing survey measures of inflation expectations in policymaking as well as a foundation for policies operating via information interventions (e.g., forward-guidance). Our findings therefore contribute to a broader research agenda explaining the expectations formation of agents and utilizing these expectations in policymaking.

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Table 1. Descriptive statistics for the initial wave of the survey.

	Mean (1)	St.dev. (2)
Firm characteristics		
Employment	37.70	67.98
Age	25.97	19.23
Share of domestic sales in total sales	97.19	7.69
Number of competitors	8.78	6.26
Manager characteristics		
Tenure at the firm	11.48	7.32
Gender (female=1)	0.19	0.39
Years of schooling	16.71	1.92
Sectoral shares		
Manufacturing	0.31	
Construction	0.08	
Transport and communication	0.07	
Trade	0.17	
Other services	0.36	

Notes: The table provides summary characteristics of respondents in the first wave of survey. All statistics are unweighted. The number of observations is 1,032.

Table 2. Expectations of future inflation and other managers' inflation expectations.

	# obs.	Mean	St.dev. (disagreement)	Uncertainty	Correlation with expected inflation
	(1)	(2)	(3)	(4)	(5)
Initial wave (pre-experiment)					
Expected inflation, 12-month ahead	1,032	3.41	3.06	1.11	1.00
Expected inflation expectation of other managers, 12-month ahead	1,032	3.50	2.43	0.89	0.68
p-value for equality of moment		0.18	0.00	0.00	
Initial wave (post experiment)					
Expected inflation, 12-month ahead	1,032	3.25	1.76	-	1.00
Expected inflation expectation of other managers, 12-month ahead	1,032	3.23	1.42	-	0.62
p-value for equality of moment		0.79	0.00	-	
Follow-up wave					
Expected inflation, 12-month ahead	515	3.03	2.11	0.89	1.00
Expected inflation expectation of other managers, 12-month ahead	515	3.49	1.74	1.14	0.70
p-value for equality of moment		0.00	0.00	0.00	
Memorandum					
Expected inflation, 12-month ahead, point prediction, initial wave	1,032	3.76	2.52	-	0.63
Perceived inflation, previous 12 months, point prediction, initial wave	1,032	4.11	2.55	-	0.93
Expected unemployment rate, 12-month ahead	1,032	4.90	0.55	0.40	-0.01
Expected aggregate wage growth rate, 12-month ahead	1,032	1.14	1.12	1.27	0.03
Expected price change for firm products, 3-month ahead	1,032	0.86	2.04	-	0.07
Expected wage change for firm employees, 3-month ahead	1,032	0.30	0.77	-	0.13
Expected change for firm fixed assets, 3-month ahead	1,032	1.75	3.47	-	0.15
Expected change for firm employment, 3-month ahead	1,032	3.05	5.07	-	-0.08

Notes: The table reports basic moments of first-order and higher-order expectations of inflation. Column (3) reports the cross-sectional standard deviation of mean inflation forecasts. Column (4) reports the average (across managers) standard deviation of the reported distribution for future inflation. Column (5) for memorandum items reports correlation with expected inflation (12-month ahead) in the first row of the table.

Table 3. Effect of Information Treatment on Expectations. .

Row	Treatment	Initial wave			Follow-up wave		
		Own Expectations	Higher-order Expectations	p-value equality	Own Expectations	Higher-order Expectations	p-value equality
		(1)	(2)	(3)	(4)	(5)	(6)
(1)	Group A, Control	0.727***	0.699***	0.35	0.744***	0.708***	0.45
		(0.020)	(0.021)		(0.038)	(0.038)	
(2)	Group B, $\overline{E}[\pi_t]$	0.502***	0.430***	0.21	0.461***	0.513***	0.45
		(0.041)	(0.039)		(0.065)	(0.049)	
(3)	Group C, $\overline{E}^2[\pi_t]$	0.090***	0.118***	0.36	0.116***	0.146***	0.61
		(0.018)	(0.024)		(0.043)	(0.047)	
(4)	Group D, $\overline{E}[\pi_t]$ and $\overline{E}^2[\pi_t]$	0.096***	0.071***	0.37	0.155***	0.097**	0.18
		(0.022)	(0.019)		(0.038)	(0.042)	
(5)	Group E, π_{t-1}	0.059***	0.062***	0.90	0.088**	-0.006	0.14
		(0.015)	(0.021)		(0.043)	(0.040)	
	Observations	1,032	1,032		515	515	
	R^2	0.757	0.759		0.653	0.602	

Notes: The table reports the coefficient on managers' pre-treatment inflation expectations in specification (1). The dependent variable in each column is the post-treatment inflation expectation. All inflation expectations are measured at the one-year-ahead horizon. Group B was provided information about the average first-order inflation expectation of other firms ($\overline{E}[\pi_t]$), group C was provided information about the average higher-order inflation expectation ($\overline{E}^2[\pi_t]$), group D received both pieces of information, while group E was told the most recent inflation rate (π_{t-1}). Group A is the control group and received no information. Columns (1) and (2) present results for post-treatment inflation expectations measured immediately after treatment. Columns (4) and (5) present results for post-treatment inflation expectations measured three months after treatment. Columns (1) and (4) are for firms' own inflation expectations Columns (2) and (5) present the same results for the expectation of other firms' inflation expectations. Column (3) reports p-values of the null hypothesis that columns (1) and (2) are equal. Column (6) reports p-values of the null hypothesis that columns (4) and (5) are equal. Robust standard errors are reported in parentheses. ***, **, and * indicate significance at the 0.01, 0.05 and 0.10 percent levels, respectively.

Table 4. Empirical Stylized Facts and Theory Predictions.

Empirical Stylized Facts	Basic noisy-information model with with private and public signals	Noisy-information model with:				
		reporting error in survey	heterogeneous long-run priors	overconfidence in private signals	semi-public signal	level-k thinking
Fact 1: The average first-order inflation expectations is approximately equal to average higher-order inflation expectations	Yes	Yes	Yes	Yes	Yes	Yes
Fact 2: First-order and higher-order inflation expectations are positively but not perfectly correlated in the cross-section with a slope coefficient less than one.	No: Perfect correlation with slope less than one	Yes	Yes	No: Perfect correlation with slope less than one	Yes	Yes
Fact 3: The cross-sectional dispersion in first-order inflation expectations is greater than the dispersion in higher-order inflation expectations.	Yes	Yes	Yes	Yes	Yes	Yes
Fact 4: The average uncertainty in first-order inflation expectations is greater than the uncertainty in higher-order inflation expectations.	Yes	Yes	Yes	Yes	Yes	Yes
Fact 5: The average level of uncertainty in first-order (higher-order) inflation expectations is smaller than the cross-sectional dispersion in first-order (higher-order) inflation expectations.	No	Yes	Yes	Yes	No	No
Fact 6: The response of both first-order and higher-order inflation expectations is greater to an information treatment about the higher-order inflation expectations of others than to an information treatment about the first-order inflation expectations of others.	No	No	Yes	No	No	No

Notes: The table summarizes the empirical stylized facts from the survey (column 1) and the extent to which the models of sections 3 and 4 are consistent with these moments. In terms of matching fact 1, all of the "Yes" answers mean that the models can be consistent with fact 1 for specific realizations. The "Yes" response of the model with heterogeneous long-run priors to fact 3 is conditional on parameter values (see section 4.3). For fact 6, the "No" responses mean that models cannot match the fact without assuming that signals about the higher-order beliefs of other firms would have significantly less noise than signals about first-order beliefs of firms (see section 3.2). The "No" response for level-k thinking to Fact 5 is conditional on the distribution of firms in the data (see section 4.5).

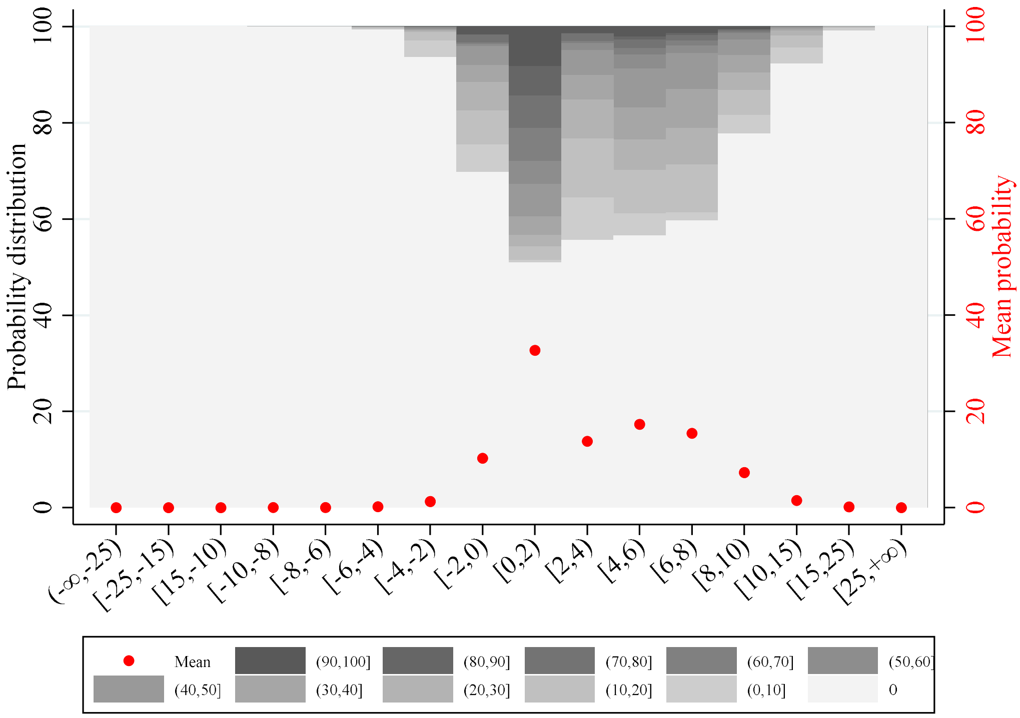
Table 5. Moments of inflation expectations and implied parameter values by level of thinking.

	Level of thinking						
	$k = 0$	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	memorandum $k' = 0$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Moment of own (first-order) inflation expectations							
Mean	5.16	2.60	2.24	2.40	2.46	1.54	3.53
Disagreement	2.83	2.91	2.50	2.71	2.60	2.49	2.87
Uncertainty	1.29	1.06	0.95	0.92	1.02	1.03	1.10
Moment of higher-order inflation expectations							
Mean	4.87	2.83	2.74	2.69	2.62	2.13	3.87
Disagreement	2.09	2.40	2.17	2.32	2.20	1.58	2.54
Uncertainty	0.88	0.95	0.80	0.92	0.93	0.86	0.90
Correlation between first- and higher-order inflation expectations	0.48	0.66	0.68	0.70	0.71	0.79	0.65
Slope in the regression of higher-order beliefs on first-order beliefs	0.36	0.54	0.59	0.60	0.60	0.50	0.58
	(0.04)	(0.06)	(0.05)	(0.08)	(0.06)	(0.05)	(0.06)
Strategic complementarity in pricing, α	0.68	0.69	0.75	0.56	0.84	0.82	0.82
	(0.04)	(0.05)	(0.06)	(0.06)	(0.05)	(0.10)	(0.07)
Implied parameters							
δ	0.74	0.82	0.87	0.86	0.85	0.63	0.89
ϕ_x	0.47	0.59	0.62	0.72	0.47	0.24	0.58
κ_x	0.015	0.028	0.047	0.052	0.023	0.004	0.032
κ_y	0.004	0.007	0.012	0.013	0.006	0.001	0.008
Observations	378	216	160	134	110	34	72

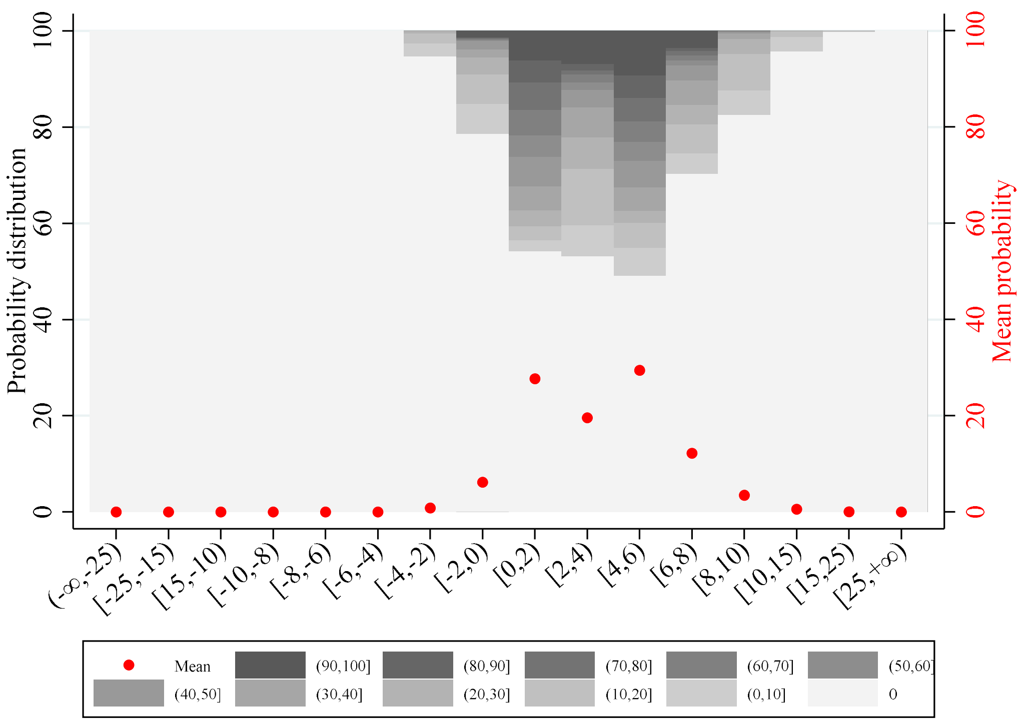
Notes: The table reports moments of inflation expectations by level of thinking k. Classification of managers into various level of k is described in section 5.1. Coding k' for level of thinking sets $k' = 0$ for guesses in the beauty contest with response time of 20 seconds or more and responses close to 50 and response time less than 20 seconds. The coding of k and k' is identical for $k > 0$. Disagreement is the cross-sectional standard deviation of mean inflation forecasts. Uncertainty is the average (across managers) standard deviation of the reported distribution for future inflation. Parameters $\alpha, \delta, \phi_x, \kappa_x, \kappa_y$ implied by these moments are calculated as in section 4. Precision of signals κ_x, κ_y is calculated using disagreement in first- and higher-order inflation expectations. Figures in parentheses report heteroskedasticity-robust standard errors for estimated regression coefficients.

Figure 1. Probability distributions of first-order and higher-order inflation expectations.

Panel A: First-order expectations

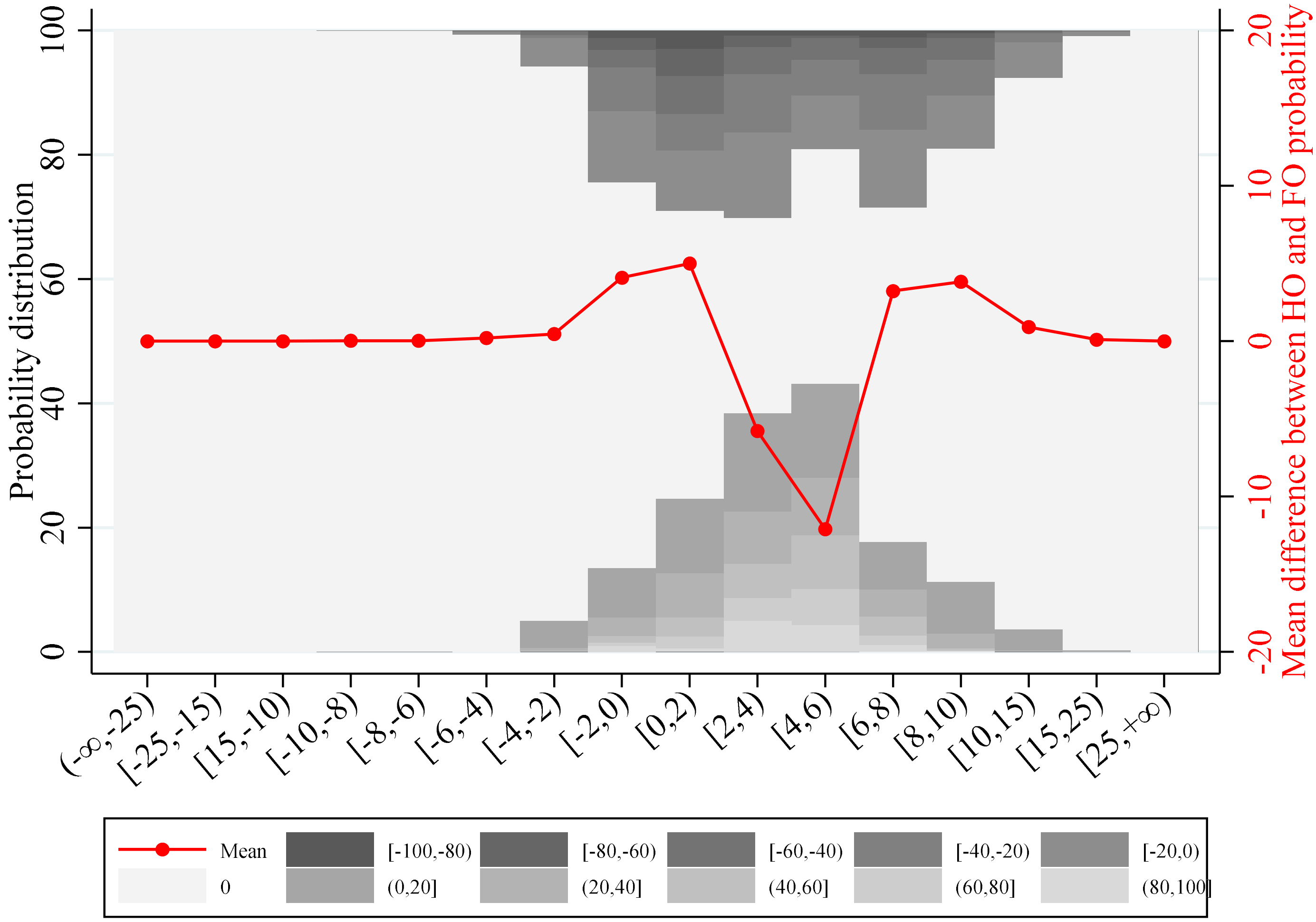


Panel B: Higher-order expectations



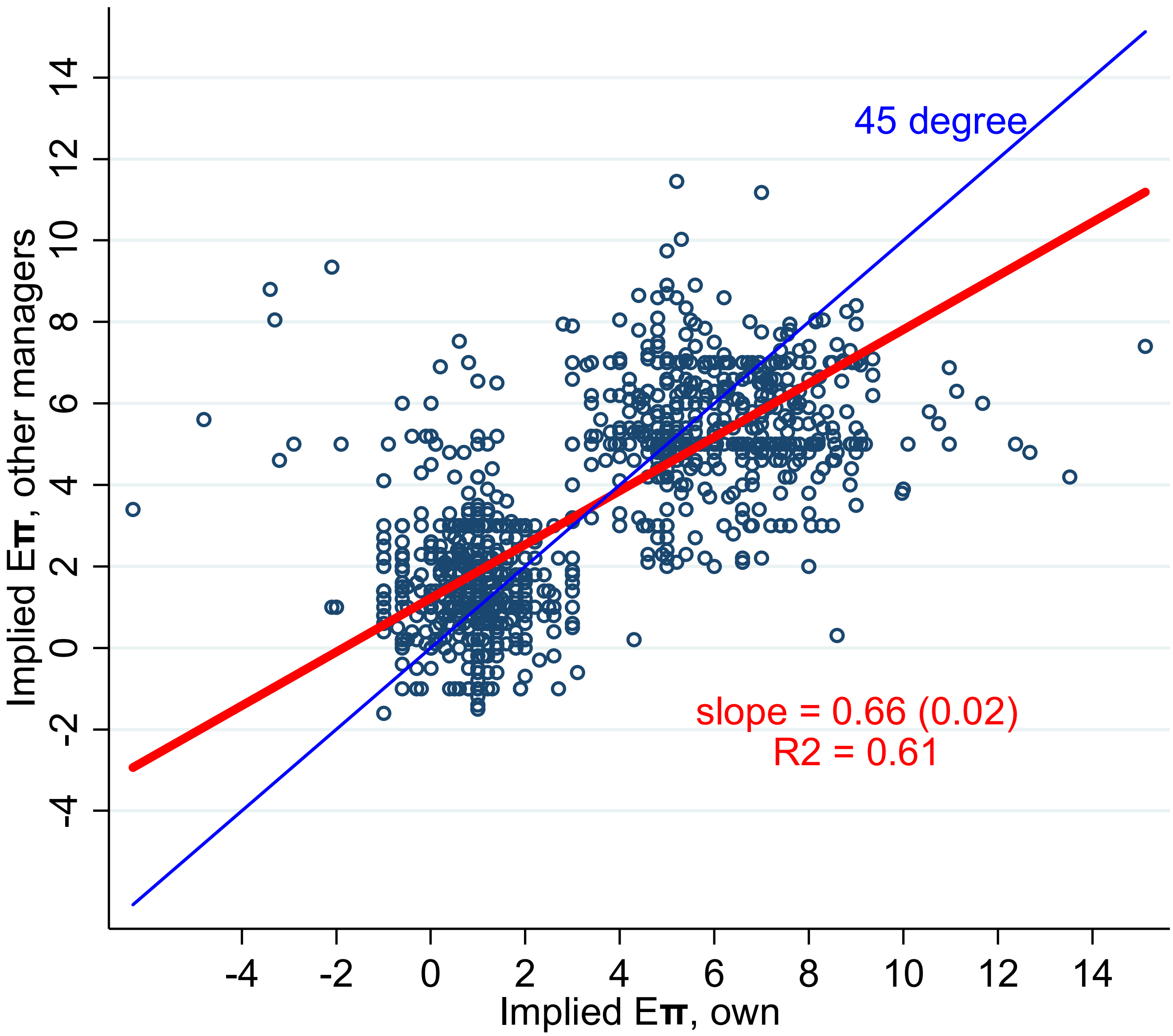
Notes: The red circles show the average probability assigned to a given bin across all respondents. The sum of red circles is 100%. The shaded areas show the share of respondents reporting a given probability range in a given range. For example, take the [0,2] bin. Light-shade area shows the share of respondents that assigned zero probability that inflation next year will be in the [0,2] bin. A slightly darker area shows the share of respondents that assigned (0%,10%] probability that inflation next year will be in the [0,2] bin. An even darker area shows the share of responses that assigned (10%,20%] probability that inflation next year will be in the [0,2] bin. And so on. The darkest area shows the share of respondents that assigned (90%,100%] probability that inflation next year will be in the [0,2] bin. By construction, the shared areas sum up to 100%. Average probabilities (red circles in the figure) and the corresponding standard derivations across bins are reported in Appendix Table 4.

Figure 2. Distribution of within-respondent differences in probabilities assigned to first- and -higher-order beliefs.



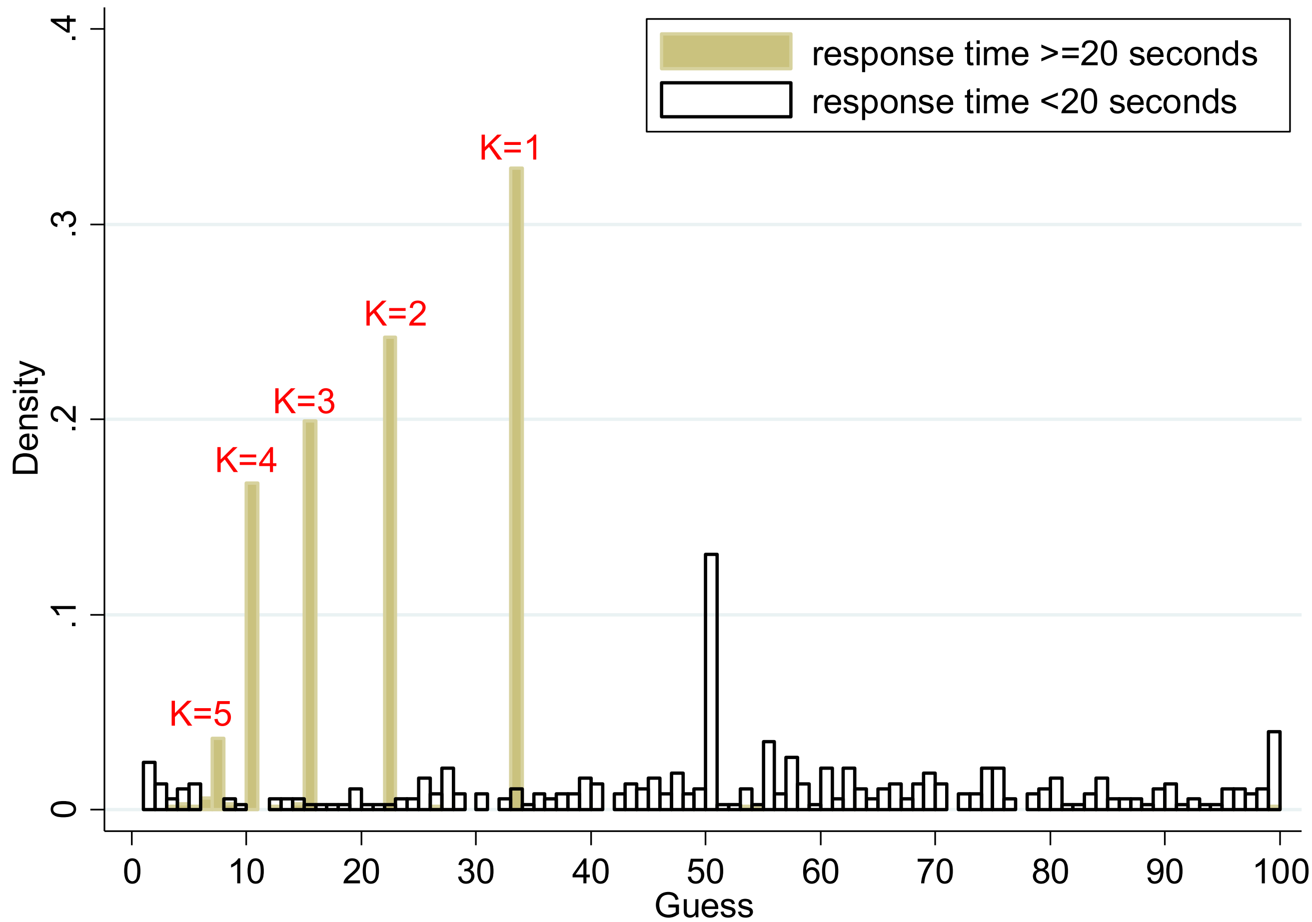
Notes: The red circles show the average *difference* in probability assigned to a given bin across all respondents. The sum of red circles is 0%. The W-shape of the difference in probabilities means that people assign lower probabilities for tail outcomes when they think about others' expectations. In other words, the distribution of higher-order beliefs is more concentrated than the distribution of first-order expectations. The shaded areas show the share of respondents reporting a given range in probability difference. For example, take the [0,2] bin. Light-shade area at the bottom of the bar chart shows the share of respondents that have a difference (between higher-order and first-order probabilities assigned to the [0,2] bin) between 80% and 100%. For instance, if somebody assigns 100% probability for first-order (own) expectations for this bin and 0% probability for higher-order expectation, this person will be in this group. A slightly darker area shows the share of respondents that assigned (60%,80%] difference in probability that inflation next year will be in the [0,2] bin. And so on. By construction, the shared areas sum up to 100%. Average difference in probabilities (red circles in the figure) and the corresponding standard derivations of the differences across bins are reported in Appendix Table 4.

Figure 3. Own Expectations and Higher-order Expectations.



Notes: The figure reports the relationship between a manager's own expectation of inflation and their higher-order expectation of inflation. Expectations are measured as mean expectations implied by the reported probability distributions for future inflation (see Appendix Table 2 for the wording of the questions). Expectations are for the one-year-ahead horizon.

Figure 4. Responses to Beauty Contest Question.



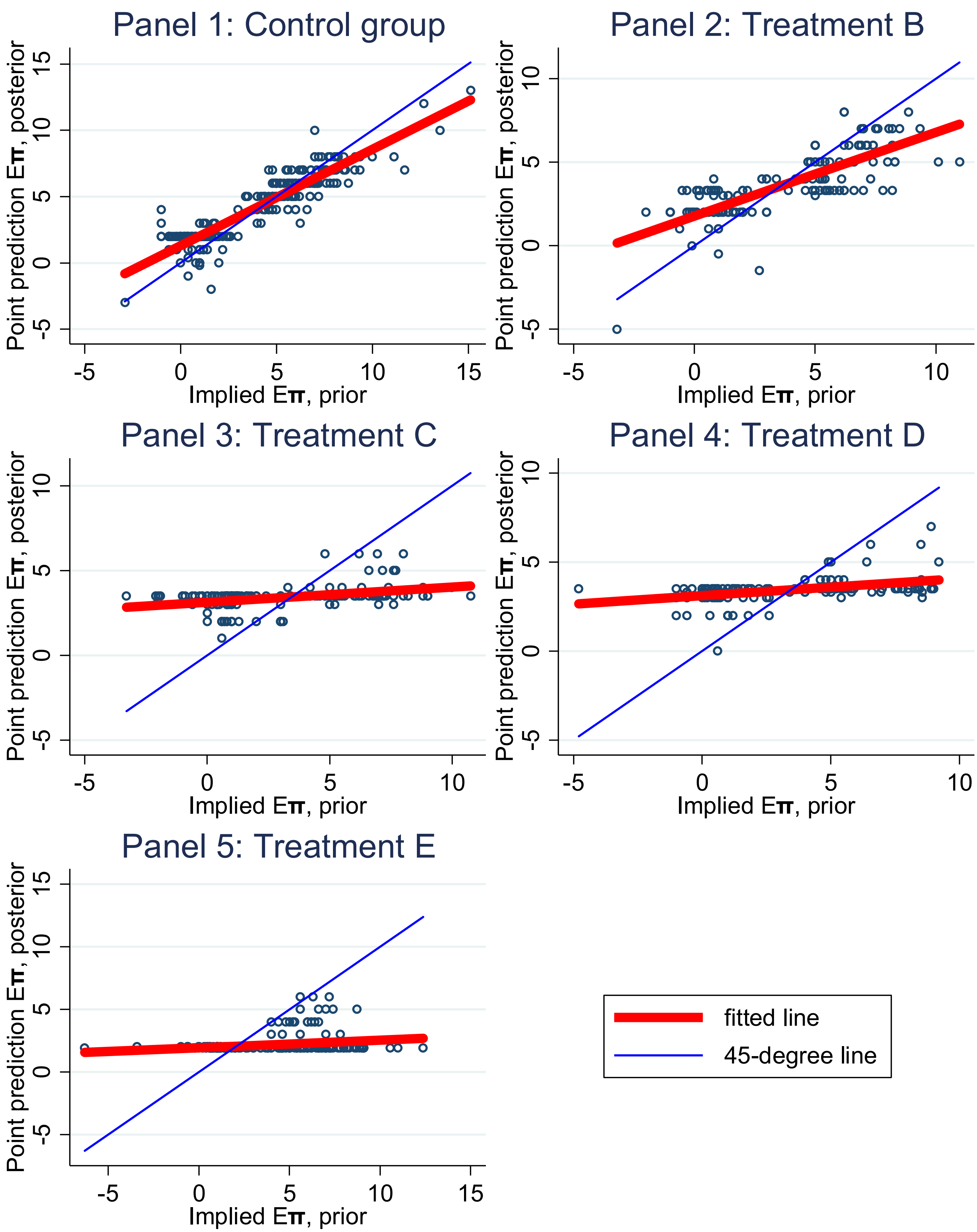
Notes: This figure shows the distribution of guesses from the beauty contest game. We asked managers to provide a guess between zero and 100 with the guess closest to 2/3 of the average guess receiving a prize. For managers who spent at least 20 seconds in considering their guess, we see clumping of guesses at those points which correspond neatly with level-*k* types as defined in Nagel (1995). Those managers who answered the question in less than 20 seconds made guesses dispersed across the full interval.

Online Appendix

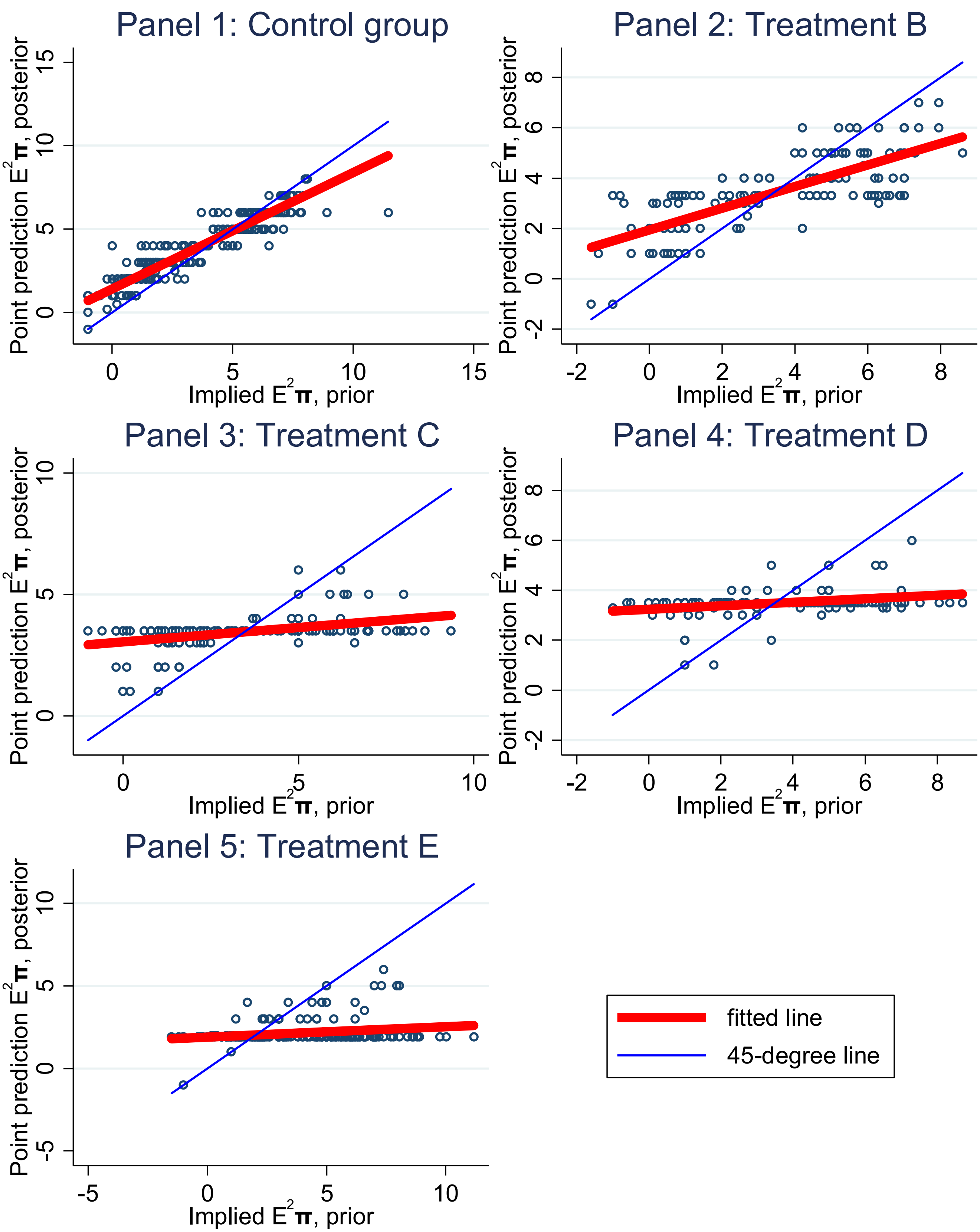
Appendix A:

Additional Tables and Figures

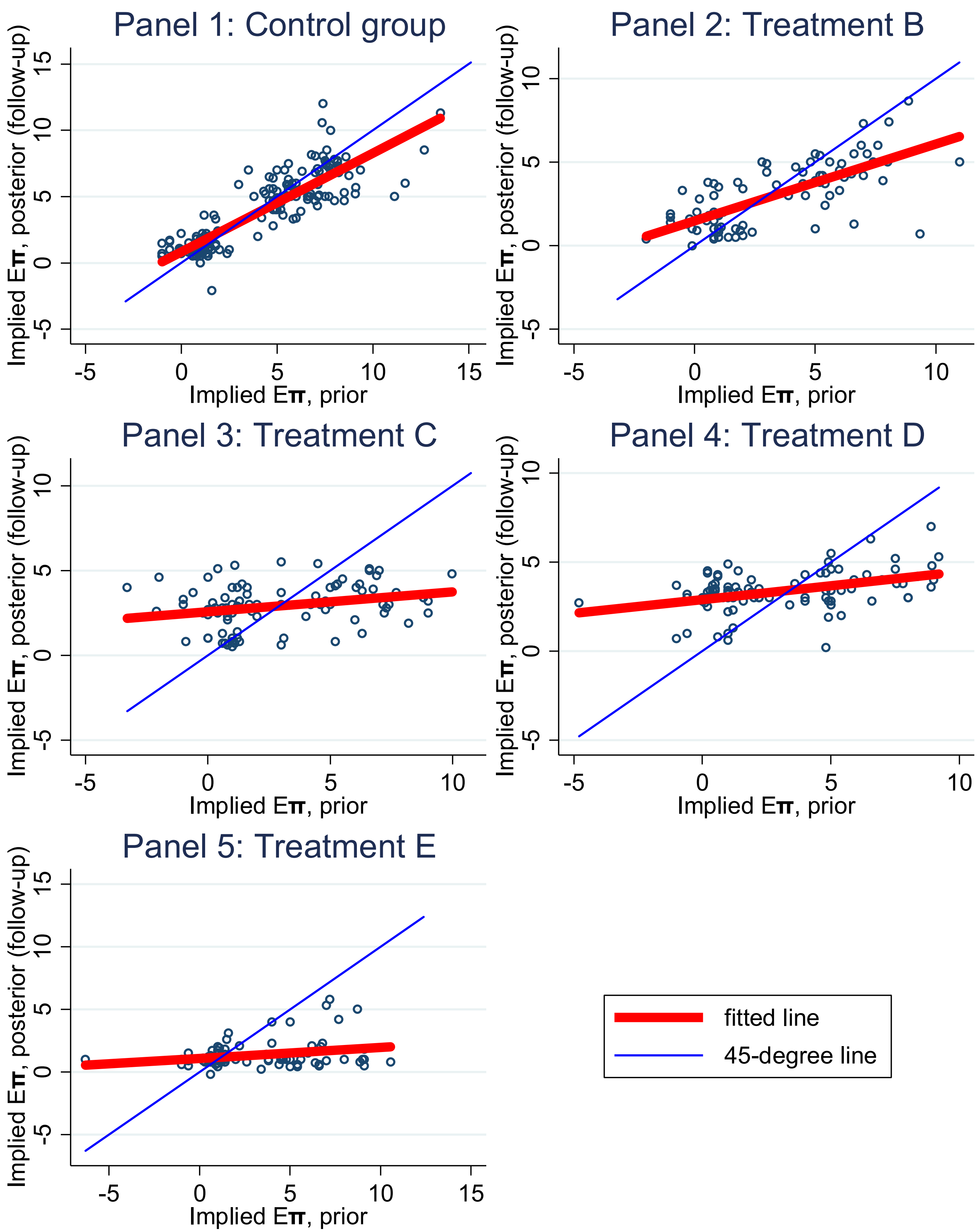
Appendix Figure 1. Revision of beliefs immediately after treatment: first-order beliefs.



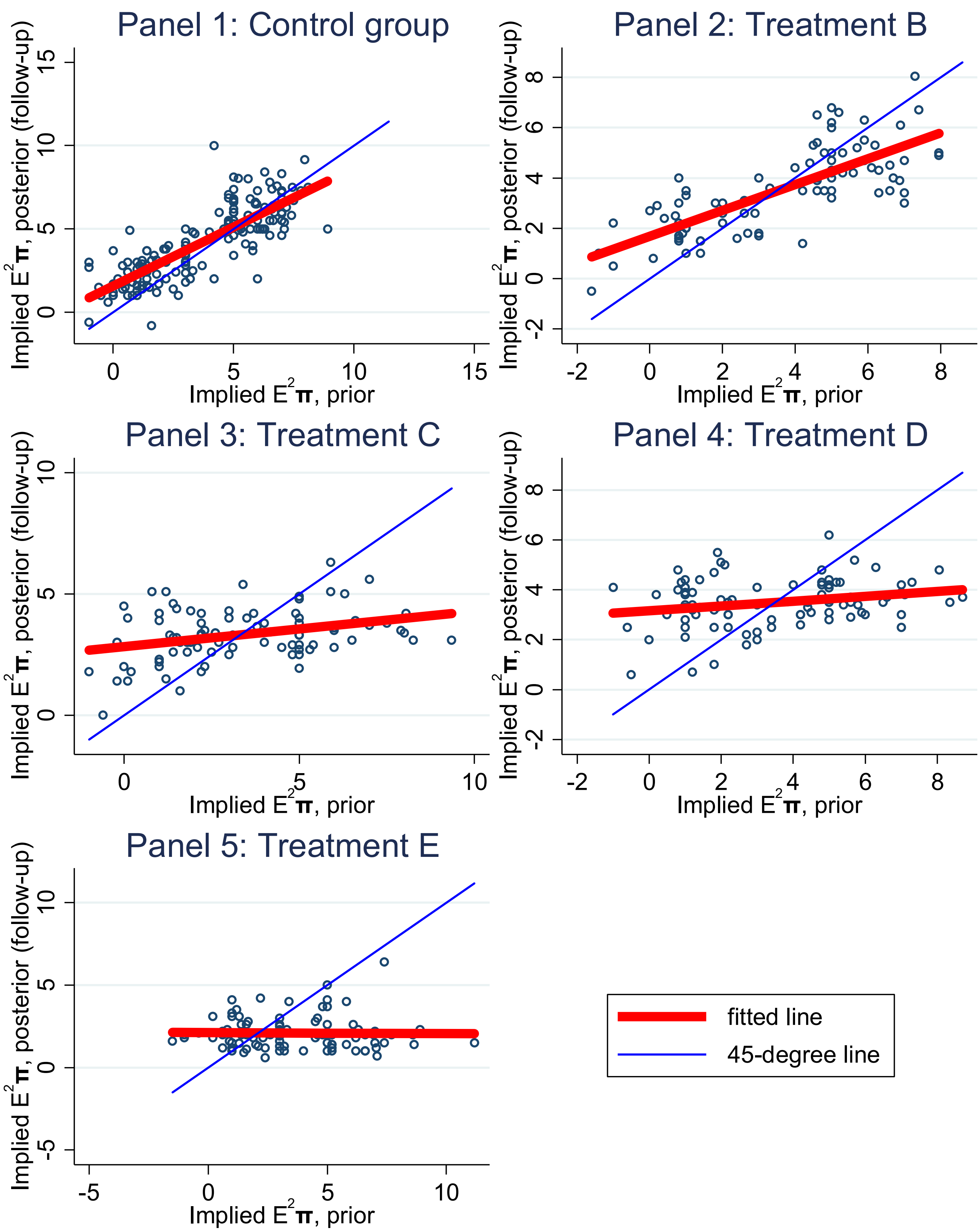
Appendix Figure 2. Revision of beliefs immediately after treatment: higher-order beliefs.



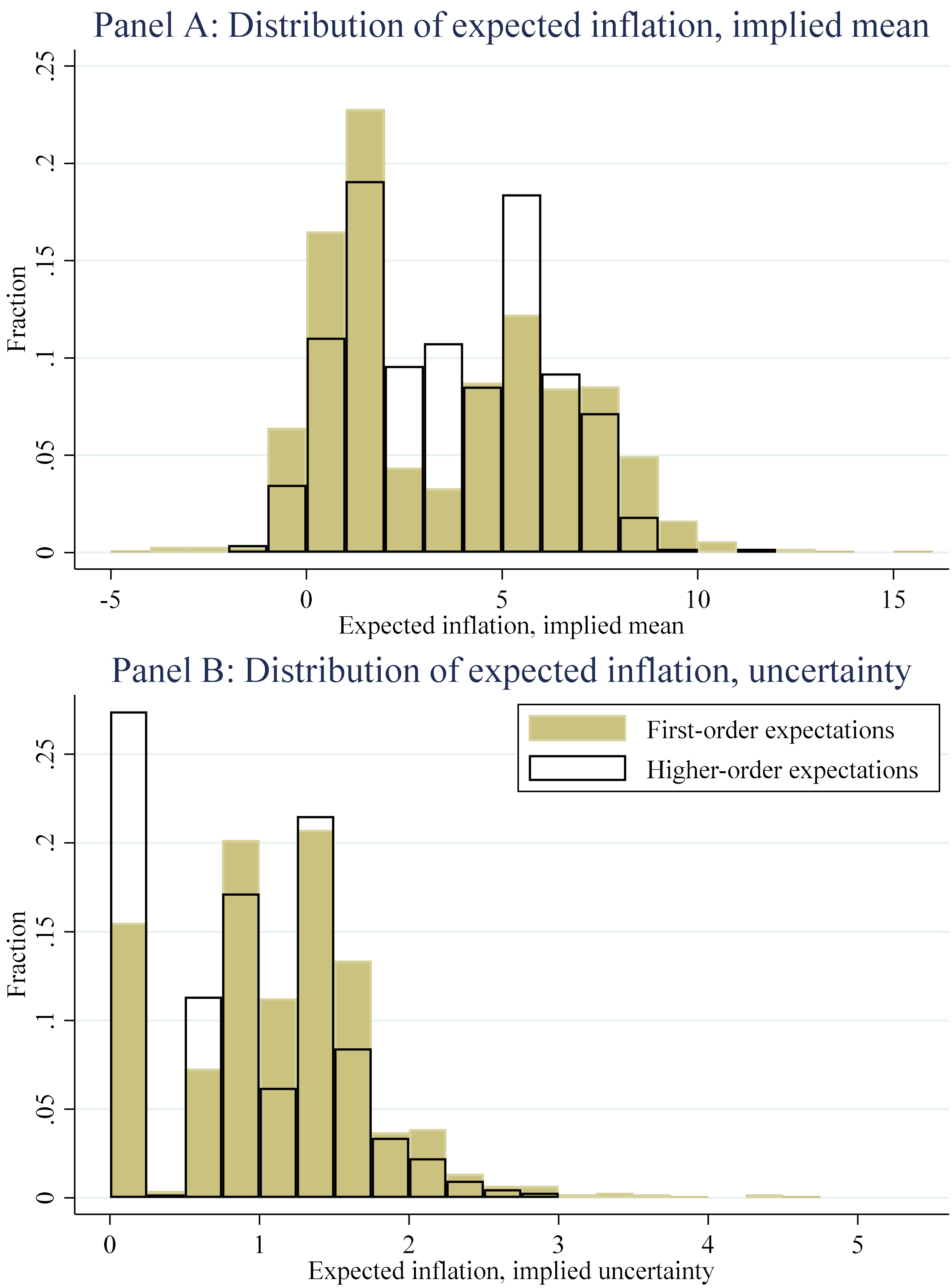
Appendix Figure 3. Revision of beliefs in the follow-up survey: first-order beliefs.



Appendix Figure 4. Revision of beliefs in the follow-up survey: higher-order beliefs.

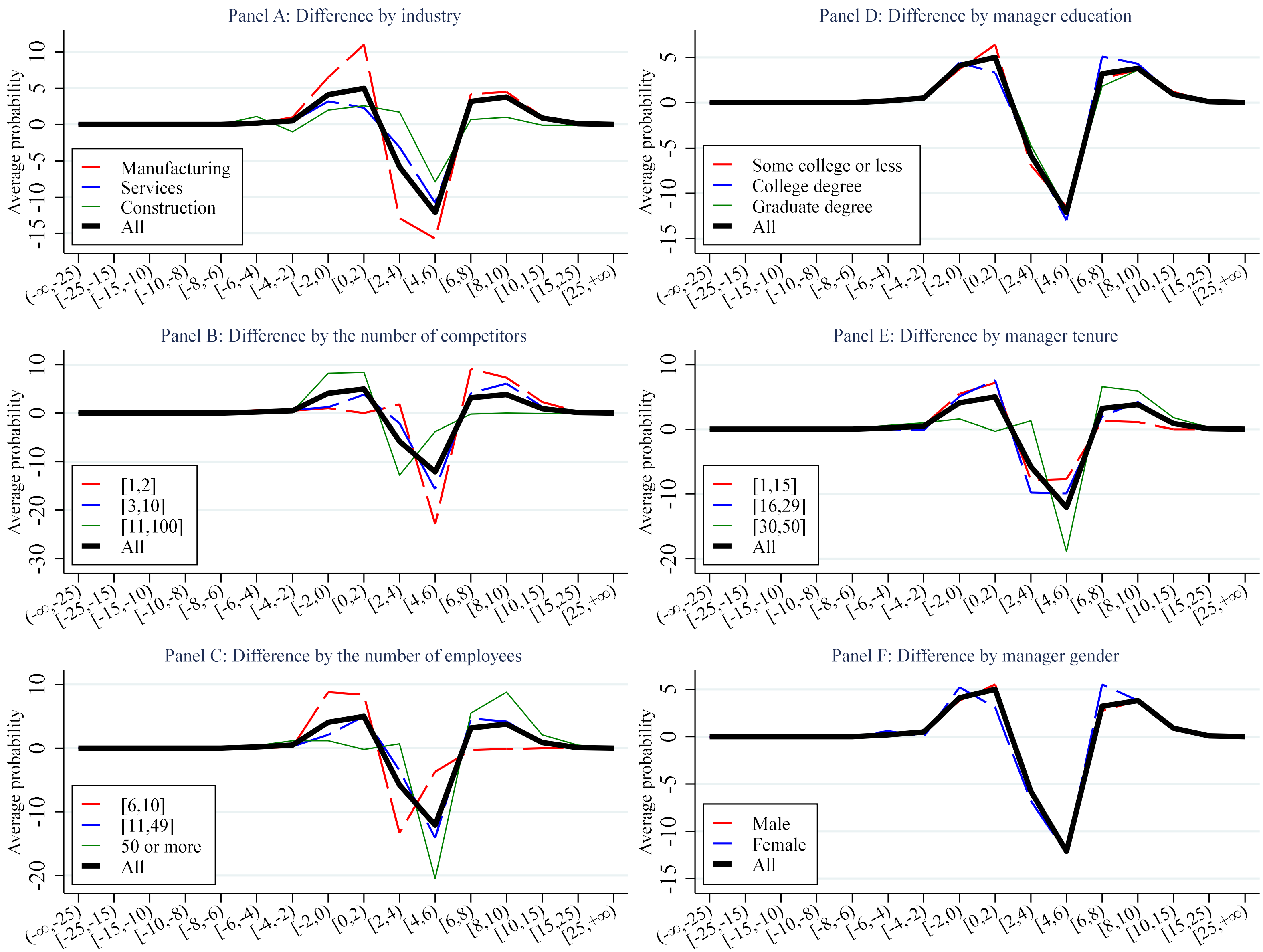


Appendix Figure 5. Distribution of moments for first- and higher-order inflation expectations.



Notes: the figure shows the distribution of implied means (Panel A) and implied uncertainty (standard deviation; Panel B) across managers. Both moments are computed using probability distributions for expectation inflation. The moments are computed for first-order ("own") and higher-order ("others'") expectations.

Appendix Figure 6. Distribution of the difference between first-order and higher-order expectations across inflation bins by subsample.



Notes: each panel reports the distribution of {average probability assigned for a given inflation bin by first-order inflation expectations} minus {average probability assigned for a given inflation bin by higher-order inflation expectations}.

Appendix Table 1. Predictors of selection into the follow-up wave of the survey.

	Dependent variable: Participation in the follow-up wave of the survey			
	(1)	(2)	(3)	(4)
Ln(Employment)	-0.026	-0.022	-0.020	-0.023
	(0.023)	(0.023)	(0.024)	(0.024)
Ln(Age)	-0.028	-0.029	-0.029	-0.027
	(0.018)	(0.018)	(0.018)	(0.018)
Share of domestic sales	-0.002	-0.002	-0.002	-0.003
	(0.002)	(0.002)	(0.002)	(0.002)
Number of competitors	-0.004	-0.005	-0.005	-0.005
	(0.003)	(0.004)	(0.004)	(0.004)
Manager's tenure at the firm		-0.004	-0.004	-0.004
		(0.002)	(0.002)	(0.002)
Manager's gender (male = 1)		-0.015	-0.015	-0.023
		(0.039)	(0.039)	(0.040)
Manager's years of schooling		0.001	0.001	0.001
		(0.008)	(0.008)	(0.008)
Level of thinking, *k*			0.004	0.004
			(0.011)	(0.011)
Constant	0.892***	0.926***	0.913***	1.000***
	(0.222)	(0.259)	(0.262)	(0.268)
Observations	1,032	1,032	1,032	1,032
R-squared	0.005	0.007	0.007	0.011
Industry FE	No	No	No	Yes

Notes: the table reports estimates of the linear probability model to check selection on observable characteristics of firms and managers. Participation is the dummy variable equal to one if a firm participates in the follow-up and zero otherwise. Industry fixed effects are at the one-digit level. Robust standard errors are reported in parentheses. ***, **, and * indicate significance at the 0.01, 0.05 and 0.10 percent levels, respectively.

Appendix Table 2. Distribution of probability bins

Panel A. Own expectations

Please assign probabilities (from 0-100) to the following ranges of possible overall price changes for the New Zealand economy over the next 12 months: (Note that the probabilities in the column should sum to 100)

Percentage Price Changes PER YEAR over the next 12 months.	**Probabilities**	
More than 25%:	**………………**	**%**
From 15 to 25%:	**………………**	**%**
From 10 to 15%:	**………………**	**%**
From 8 to 10%:	**………………**	**%**
From 6 to 8%:	**………………**	**%**
From 4 to 6%:	**………………**	**%**
From 2 to 4%:	**………………**	**%**
From 0 to 2%:	**………………**	**%**
From -2 to 0%:	**………………**	**%**
From -4 to -2%:	**……………..**	**%**
From -6 to -4%:	**……………..**	**%**
From -6 to -8%:	**………………**	**%**
From -8 to -10%:	**………………**	**%**
From -10 to -15%:	**………………**	**%**
From -15 to -25%:	**………………**	**%**
Less than -25%:	**………………**	**%**
Total (the column should sum to 100%):	**100**	**%**

Panel B. Expectations of Other Managers' Beliefs

We would like to know what your opinion is about what *other* managers (drawn from all sectors of the New Zealand economy in a representative way) think will happen to overall prices in the economy. Please assign probabilities (from 0-100) **to the following ranges of beliefs that other managers might hold about overall price changes in the economy over the next 12 months for New Zealand:** (Note that the probabilities in the column should sum to 100)

Percentage Price Changes PER YEAR over the next 12 months.	**Probabilities**	
More than 25%:	**………………**	**%**
From 15 to 25%:	**………………**	**%**
From 10 to 15%:	**………………**	**%**
From 8 to 10%:	**………………**	**%**
From 6 to 8%:	**………………**	**%**
From 4 to 6%:	**………………**	**%**
From 2 to 4%:	**………………**	**%**
From 0 to 2%:	**………………**	**%**
From -2 to 0%:	**………………**	**%**
From -4 to -2%:	**……………..**	**%**
From -6 to -4%:	**……………..**	**%**
From -6 to -8%:	**………………**	**%**
From -8 to -10%:	**………………**	**%**
From -10 to -15%:	**………………**	**%**
From -15 to -25%:	**………………**	**%**
Less than -25%:	**………………**	**%**
Total (the column should sum to 100%):	**100**	**%**

Appendix Table 3. Moments for first- and higher-order inflation expectations by subsamples.

Sample		Mean		Disagreement		Uncertainty		Correlation of HO and FO expectations
		First-order	Higher-order	First-order	Higher-order	First-order	Higher-order	
		(1)	(2)	(3)	(4)	(5)	(6)	(7)
Full sample		3.41	3.50	3.06	2.43	1.11	0.89	0.68
Industry	Manufacturing	3.37	3.66	3.06	2.12	1.07	0.90	0.64
	Service	3.41	3.38	3.09	2.54	1.14	0.89	0.71
	Construction	3.52	3.78	2.85	2.70	0.99	0.91	0.67
Firm size	[6,10]	0.88	1.50	0.93	1.19	0.87	0.90	-0.00
	[11,49]	3.98	3.96	2.97	2.44	1.17	0.88	0.66
	50 or more	5.96	5.51	2.54	1.47	1.33	0.90	0.01
Manager education	Some college or less	3.46	3.55	3.07	2.44	1.09	0.86	0.72
	College	3.39	3.40	2.98	2.37	1.11	0.90	0.68
	Graduate studies	3.36	3.54	3.14	2.50	1.11	0.93	0.65
Gender	Male	3.33	3.44	3.05	2.44	1.10	0.90	0.68
	Female	3.74	3.75	3.06	2.37	1.15	0.86	0.67
Firm age	10 years or less	2.99	3.19	2.91	2.31	1.08	0.81	0.75
	(10,25] years	3.18	3.43	2.99	2.49	1.04	0.91	0.67
	More than 25 years	3.62	3.61	3.14	2.41	1.14	0.90	0.66
Manager tenure	15 years or less	1.86	2.31	2.36	2.19	0.97	0.92	0.62
	[16,29] years	2.79	2.92	2.85	2.27	1.05	0.85	0.74
	29 years or more	5.61	5.33	2.63	1.70	1.30	0.92	0.23
Number of competitors	[1,2]	5.97	5.48	2.27	1.41	1.33	0.92	0.11
	[3,10]	4.72	4.55	2.97	2.29	1.20	0.90	0.52
	11 or more	1.00	1.61	1.18	1.39	0.91	0.88	0.31

Notes: Columns (1) and (2) report average inflation expectations for first-order (FO) and higher-order (HO) expectations. Inflation expectations are measured as implied means from the reported distributions. Columns (3) and (4) report the standard deviation (disagreement) for FO and HO inflation expectations (implied means) across firms. Columns (5) and (6) report average uncertainty for FO and HO inflation expectations across firms. Uncertainty is measured as the standard deviation implied by the reported probability distribution. Column (7) reports the correlation between FO and HO inflation expectations (implied means) across firms.

Appendix Table 4. Distribution of first-order and higher-order expectations across inflation bins.

Inflation bin	Average probability (standard deviation of probability)		
	First-order expectations	Higher-order expectations	First-order minus higher-order expectations
	(1)	(2)	(3)
[25, ∞)	0.0	0.0	0.0
	(0.0)	(0.0)	(0.0)
[15,25)	0.2	0.1	0.1
	(1.8)	(1.0)	(2.1)
[10,15)	1.5	0.6	0.9
	(6.2)	(3.0)	(6.8)
[8,10)	7.3	3.5	3.8
	(16.7)	(8.8)	(17.6)
[6,8)	15.4	12.2	3.2
	(22.7)	(24.5)	(27.3)
[4,6)	17.3	29.4	-12.1
	(25.0)	(36.6)	(35.1)
[2,4)	13.8	19.6	-5.8
	(21.1)	(29.5)	(36.7)
[0,2)	32.7	27.7	5.0
	(37.6)	(35.3)	(32.8)
[-2,0)	10.3	6.2	4.1
	(20.5)	(16.3)	(23.9)
[-4,-2)	1.3	0.8	0.5
	(6.5)	(3.8)	(7.4)
[-6,-4)	0.2	0.0	0.2
	(3.2)	(0.0)	(3.2)
[-8,-6)	0.0	0.0	0.0
	(0.8)	(0.0)	(0.8)
[-10,-8)	0.0	0.0	0.0
	(0.9)	(0.0)	(0.9)
[-15,-10)	0.0	0.0	0.0
	(0.0)	(0.0)	(0.0)
[-25,-15)	0.0	0.0	0.0
	(0.0)	(0.0)	(0.0)
(-∞,-25)	0.0	0.0	0.0
	(0.0)	(0.0)	(0.0)

Notes: The table reports average probability (standard deviation in parentheses) assigned by managers to future inflation bins indicated in the left column.

Appendix Table 5. Effect of Information Treatment on Expectations.

Row	Treatment	Initial wave	Follow-up wave
		Own Expectations	Own Expectations
		(1)	(4)
(1)	Group A, Control	0.968***	0.973***
		(0.014)	(0.045)
(2)	Group B, $\overline{E}[\pi_t]$	0.625***	0.574***
		(0.051)	(0.076)
(3)	Group C, $\overline{E}^2[\pi_t]$	0.122***	0.157**
		(0.026)	(0.061)
(4)	Group D, $\overline{E}[\pi_t]$ and $\overline{E}^2[\pi_t]$	0.115***	0.175***
		(0.031)	(0.049)
(5)	Group E, π_{t-1}	0.073***	0.096*
		(0.020)	(0.054)
	Observations	1,032	515
	R^2	0.840	0.672

Notes: the table replicates analysis in Table 3 with the regressor being the point prediction for inflation rather than implied mean. See note to Table 3 for more details.

Appendix Table 6. Predictors of level of thinking.

Dependent variable: k, level of thinking	Sample					
	All responses		Responses with $k > 0$		Responses with non-missing k'	
	(1)	(2)	(3)	(4)	(5)	(6)
Firm characteristics						
Ln(Employment)	-0.314***	-0.315***	-0.023	-0.021	-0.131	-0.140
	(0.066)	(0.068)	(0.088)	(0.090)	(0.093)	(0.095)
Ln(Age)	-0.038	-0.035	0.022	0.023	-0.023	-0.017
	(0.048)	(0.048)	(0.055)	(0.055)	(0.057)	(0.057)
Share of domestic sales	-0.013**	-0.011	-0.012*	-0.013*	-0.017**	-0.019**
	(0.006)	(0.007)	(0.007)	(0.008)	(0.007)	(0.008)
Number of competitors	0.044***	0.043***	0.002	0.002	0.008	0.007
	(0.011)	(0.011)	(0.012)	(0.012)	(0.012)	(0.012)
Manager characteristics						
Manager's tenure at the firm	-0.011*	-0.012*	-0.007	-0.006	-0.002	-0.003
	(0.007)	(0.006)	(0.009)	(0.009)	(0.009)	(0.009)
Manager's gender (female = 1)	0.006	-0.005	0.093	0.084	0.007	-0.031
	(0.110)	(0.111)	(0.126)	(0.129)	(0.133)	(0.137)
Manager's years of schooling	0.008	0.010	0.024	0.024	0.021	0.021
	(0.023)	(0.023)	(0.026)	(0.026)	(0.027)	(0.027)
Observations	1,032	1,032	654	654	726	726
R^2	0.144	0.148	0.009	0.011	0.019	0.022
Industry FE	No	Yes	No	Yes	No	Yes

Notes: The table report results of regressing level of thinking k on firm and manager characteristics. Industry fixed effects are at the one-digit level. Coding k' for level of thinking sets $k' = 0$ for responses with response time of 20 seconds or more and responses close to 50 and response time less than 20 seconds. The coding of k and k' are identical for $k > 0$. Robust standard errors are reported in parentheses. ***, **, and * indicate significance at the 0.01, 0.05 and 0.10 percent levels, respectively.

Appendix Table 7. Predictors of inflation expectations.

	Dependent variable:			
	Expected inflation, implied mean		Expected inflation, implied uncertainty	
	First-order	Higher-order	First-order	Higher-order
	(1)	(2)	(3)	(4)
Firm characteristics				
Sector (omitted category: Manufacturing)				
Services	0.082	-0.278**	0.080*	0.001
	(0.143)	(0.111)	(0.047)	(0.047)
Construction	-0.148	-0.100	-0.108	0.028
	(0.258)	(0.213)	(0.077)	(0.092)
Employment (omitted category: [6,10] employees)				
11 to 49 employees	1.419***	1.142***	0.166***	-0.051
	(0.141)	(0.135)	(0.050)	(0.055)
50 or more employees	2.333***	1.826***	0.248***	-0.053
	(0.245)	(0.181)	(0.072)	(0.077)
Number of competitors (omitted category: 1 or 2 competitors)				
3 to 10 competitors	-0.729***	-0.488***	-0.078	-0.022
	(0.213)	(0.144)	(0.065)	(0.065)
11 competitors or more	-3.072***	-2.262***	-0.227***	-0.065
	(0.236)	(0.183)	(0.075)	(0.076)
Firm age (omitted category: 10 years old or less)				
11 to 25 years old	0.126	0.185	-0.047	0.095
	(0.171)	(0.148)	(0.055)	(0.058)
26 or more years old	0.437**	0.173	0.083	0.087
	(0.175)	(0.145)	(0.057)	(0.057)
Manager characteristics				
Gender				
Male	-0.050	-0.069	-0.007	0.042
	(0.174)	(0.133)	(0.051)	(0.056)
Education (omitted category: some college or less)				
College diploma	-0.065	-0.135	0.013	0.046
	(0.148)	(0.122)	(0.047)	(0.053)
Graduate studies	-0.131	-0.029	0.014	0.071
	(0.163)	(0.128)	(0.053)	(0.054)
Tenure in the industry (omitted category: 15 years or less)				
16 to 29 years	0.424***	0.227*	0.035	-0.082
	(0.144)	(0.134)	(0.047)	(0.052)
30 years or more	1.162***	1.082***	0.101*	-0.013
	(0.217)	(0.175)	(0.060)	(0.072)
Observations	1,032	1,032	1,032	1,032
R-squared	0.547	0.542	0.094	0.009

Notes: the table reports estimated coefficients for the regression of a given moment for inflation expectations on a set of indicator variables capturing firm and manager characteristics. The dependent variables are indicated in column titles. Robust standard errors are reported in parentheses. ***,**,* indicate statistical significance at 1, 5, and 10 percent levels.

Appendix B:
Response to Information by Level-*k*

Our discussion in Section 3 assumes that firms perform infinite iterations of the optimal pricing function. That is, firms are capable of infinite degrees of reasoning, an assumption which models of level-k thinking challenge. To make our model of expectations consistent with level-k thinking, we revise the optimal pricing equation in equation (10) such that firm i will weigh the public and private signals according to

$$p_i(k) = \frac{\sum_{r=0}^{k} \alpha^r \left[\left[1-\delta^{r+1}\right] y + \delta^{r+1} x_i \right]}{\sum_{r=0}^{k} \alpha^r}, \tag{B1}$$

where k is the firm's type. We allow firms to fall into one of three different thinking types such that $k = 0,1,2$. A level-0 firm will have pricing strategies in equation (11) with $\phi_{x,0} = \delta$ and $\phi_{y,0} = 1 - \delta$. These strategies ignore the strategic complementarity in prices and rely only on the relative precision of the public and private signals. One can show that the strategies for level-1 and level-2 firms will shift weight towards the public signal; that is, $\phi_{x,0} > \phi_{x,1} > \phi_{x,2}$ and $\phi_{y,0} < \phi_{y,1} < \phi_{y,2}$ as $\phi_{x,k} = \delta \frac{1+\alpha\delta+\cdots+(\alpha\delta)^k}{1+\alpha+\cdots+\alpha^k}$ and $\phi_{y,k} = 1 - \phi_{x,k}$.

The aggregate price-level will then be a weighted average of the pricing behavior of each type of firm

$$\bar{p} = \sum_{k=0}^{2} \omega_k \overline{p(k)} \tag{B2}$$

where ω_k is the proportion of firms thinking at level-k and $\overline{p(k)} = \phi_{x,k} m + \phi_{y,k} y$.

Heterogeneity in strategies means that firms must consider the distribution of types in forming their expectations. Our data on the expectations of firms about the distribution of other types suggests that firms assign the greatest weight to firms of their own type. For simplicity, we model that all firms behave as if all firms are of their own type.

Level-0 firms will form expectations of the aggregate price level

$$E_{i,0}[\bar{p}] = \phi_{x,0} x_i + \left(1 - \phi_{x,0}\right) y = \delta x_i + (1-\delta) y. \tag{B3a}$$

Just as these firms do not iterate expectations in the price-setting equation, they do not iterate on expectations of the price level. Namely, they fail to substitute their expectation of m into equation (B2).

Level-1 and level-2 firms are capable of iterating their expectation of the price level. Accordingly, their first-order expectations are:

$$E_{i,1}[\bar{p}] = \phi_{x,1} \delta x_i + \left(1 - \phi_{x,1}\delta\right) y \tag{B3b}$$

$$E_{i,2}[\bar{p}] = \phi_{x,2} \delta x_i + \left(1 - \phi_{x,2}\delta\right) y \tag{B3c}$$

The aggregate expectation for each type is therefore:

$$\bar{E}_0[\bar{p}] = \phi_{x,0} m + \left(1 - \phi_{x,0}\right) y \tag{B4a}$$

$$\bar{E}_1[\bar{p}] = \phi_{x,1} \delta m + \left(1 - \phi_{x,1}\delta\right) y \tag{B4b}$$

$$\bar{E}_2[\bar{p}] = \phi_{x,2} \delta m + \left(1 - \phi_{x,2}\delta\right) y \tag{B4c}$$

Aggregating across types and firms gives:

$$\bar{E}[\bar{p}] = \sum_{k=0}^{2} \omega_k \bar{E}_k[\bar{p}] = (\omega_0(1-\delta)\phi_{x,0} + \bar{\phi}_x \delta) m + \left(1 - (\omega_0(1-\delta)\phi_{x,0} + \bar{\phi}_x \delta)\right) y, \tag{B5}$$

where $\bar{\phi}_x = \sum_{k=0}^{2} \omega_k \phi_{x,k}$. Because by definition level-0 and level-1 firms are unable to iterate expectations past their first-order expectation, their higher-order expectation is the same as their first-order expectation:

$$E_{i,0}\left[\bar{E}[\bar{p}]\right] = \phi_{x,0} x_i + \left(1 - \phi_{x,0}\right) y = \delta x_i + (1-\delta) y, \tag{B6a}$$

$$E_{i,1}\left[\bar{E}[\bar{p}]\right] = \phi_{x,1}E_i[m] + (1-\phi_{x,1})y = \phi_{x,1}\delta x_i + \left(1-\phi_{x,1}\delta\right)y. \quad \text{(B6b)}$$

Unlike level-0 and level-1 firms, level-2 firms will be able to iterate their expectations for a second time, substituting $E_i[m] = \delta x_i + (1-\delta)y$:

$$E_{i,2}\left[\bar{E}[\bar{p}]\right] = \phi_{x,2}\delta E_i[m] + \left(1-\phi_{x,2}\delta\right)y = \phi_{x,2}\delta^2 x_i + \left(1-\phi_{x,2}\delta^2\right)y. \quad \text{(B6c)}$$

The aggregate higher-order expectations for each type are:

$$\bar{E}_0^2[\bar{p}] = \phi_{x,0}m + \left(1-\phi_{x,0}\right)y \quad \text{(B7a)}$$

$$\bar{E}_1^2[\bar{p}] = \phi_{x,1}\delta m + \left(1-\phi_{x,1}\delta\right)y \quad \text{(B7b)}$$

$$\bar{E}_2^2[\bar{p}] = \phi_{x,2}\delta^2 m + \left(1-\phi_{x,2}\delta^2\right)y \quad \text{(B7c)}$$

The average higher-order expectation is then:

$$\bar{E}^2[\bar{p}] = \sum_{k=0}^{2}\omega_k \bar{E}_k^2[\bar{p}].$$

The cross-sectional disagreement for first-order and higher-order beliefs is given by:

$$Var[E_i[\bar{p}]|y] = \left(\omega_0(1-\delta)\phi_{x,0} + \bar{\phi}_x\delta\right)^2\kappa_x^{-1}, \quad \text{(B8)}$$

$$Var\left[E_i\left[\bar{E}[\bar{p}]\right]|y\right] = \left(\sum_{k=0}^{2}\omega_k\phi_{x,k}\delta^k\right)^2\kappa_x^{-1}, \quad \text{(B9)}$$

while uncertainty is given as a weighted sum of the uncertainty of each type of agent:

$$\Omega_{\{E_i[\bar{p}]|y\}} = \sum_{k=0}^{2}\omega_k\Omega_{\{E_k[\bar{p}]|y\}} = \left(\omega_0\phi_{x,0}^2 + \omega_1\left(\phi_{x,1}\delta\right)^2 + \omega_2\left(\phi_{x,2}\delta\right)^2\right)\kappa_x^{-1}, \quad \text{(B10)}$$

$$\Omega_{\left\{E_i\left[\bar{E}[\bar{p}]\right]|y\right\}} = \sum_{k=0}^{2}\omega_k\Omega_{\left\{E_k\left[\bar{E}[\bar{p}]\right]|y\right\}} = \sum_{k=0}^{2}\omega_k\left(\phi_{x,k}\delta^k\right)^2\kappa_x^{-1}. \quad \text{(B11)}$$

The relationship between disagreement and uncertainty is ambiguous. Expanding on equations (B8) and (B10) gives

$$\begin{aligned} Var[E_i[\bar{p}]|y] - \Omega_{\{E_i[\bar{p}]|y\}} &= \{[\omega_0(\omega_0-1)\phi_{x,0} + \omega_1(\omega_1-1)\phi_{x,1}\delta + \omega_2(\omega_2-1)\phi_{x,2}\delta] \\ &\quad + 2[\omega_0\omega_1\phi_{x,0}\phi_{x,1}\delta + \omega_0\omega_2\phi_{x,0}\phi_{x,2}\delta + \omega_1\omega_2\phi_{x,1}\phi_{x,2}\delta^2]\}\kappa_x^{-1} \end{aligned}$$

The first set of square brackets is unambiguously negative. The second set of square brackets is unambiguously positive, but the sign of the sum is uncertain and depends on several parameters. A similar result obtains for higher-order expectations:

$$Var\left[E_i\left[\bar{E}[\bar{p}]\right]|y\right] - \Omega_{\{E_i[\bar{E}[\bar{p}]]|y\}} = \left\{\left[\sum_{k=0}^{2}\omega_k(\omega_k-1)\phi_{x,k}\delta^k\right] + 2\left[\omega_0\omega_1\phi_{x,0}\phi_{x,1}\delta + \omega_0\omega_2\phi_{x,0}\phi_{x,2}\delta^2 + \omega_1\omega_2\phi_{x,1}\phi_{x,2}\delta^3\right]\right\}\kappa_x^{-1}$$

Using our data on the distribution of k, we assign $\omega_0 = 0.362$, $\omega_1 = 0.213$, and $\omega_2 = 0.424$. Given these values and our estimated value of $\alpha = 0.7$, the disagreement and uncertainty terms will change only with the value of δ, which is used to calculate $\phi_{x,0}$, $\phi_{x,1}$, and $\phi_{x,2}$ and is bounded between 0 and 1. For any value in this range, disagreement and uncertainty are similar for both first-and higher-order expectations.

As Section 3.2 outlines, managers can transform signals about the average first-order and higher-order inflation expectation (signals B and C) into signals about m. Signal B, in Equation (19), will be differently perceived by managers at different k levels.

$$\tilde{s}_{B,0} = H_{B,0}m + \xi_B = \phi_{x,0}m + \xi_B \quad \text{(B12a)}$$

$$\tilde{s}_{B,1} = H_{B,1}m + \xi_B = \phi_{x,1}\delta m + \xi_B \tag{B12b}$$

$$\tilde{s}_{B,2} = H_{B,2}m + \xi_B = \phi_{x,2}\delta m + \xi_B \tag{B12c}$$

Note that the interpretation of signals may be incorrect because agents' perception of the data generating process (DGP) may deviate from the actual DGP. For example, for level-0 firms perception of DGP is given by equation (B3a) while actual DGP is given by equation (B2). Indeed, only agents with the highest k have the correct perception. As a result, although agents believe they should interpret signals as in equations (B8), the effective signals are different. For level-0 firms:

$$\begin{aligned}\tilde{s}_{B,0} &= (\omega_0(1-\delta)\phi_{x,0} + \bar{\phi}_x\delta)m + \left(1 - (\omega_0(1-\delta)\phi_{x,0} + \phi_x\delta)\right)y + \xi_B - (1-\phi_{x,0})y \\ &= \phi_{x,0}m + \left((\omega_0 - 1)\phi_{x,0} + \delta(\omega_1\phi_{x,1} + \omega_2\phi_{x,2})\right)(m-y) + \xi_B \\ &= \phi_{x,0}m + \tilde{\xi}_{B,0}\end{aligned} \tag{B12a'}$$

where $\tilde{\xi}_{B,0} \equiv \left((\omega_0 - 1)\phi_{x,0} + \delta(\omega_1\phi_{x,1} + \omega_2\phi_{x,2})\right)(m-y) + \xi_B$. Thus, level-0 firms interpret $\tilde{\xi}_{B,0}$ as uncorrelated noise, but in fact the "noise" is correlated with fundamental m and public signal y. This interpretation of the signal means that, in the long run, level-0 firms may be overconfident in their expectations because $Var(\tilde{\xi}_{B,0}) > Var(\xi_B)$ and, relatedly, these firms may have more disagreement because they may overreact to the perceived signals.

Likewise, for level-1 and level-2 firms:

$$\tilde{s}_{B,1} = H_{B,1}m + \tilde{\xi}_{B,1} = \phi_{x,1}\delta m + \tilde{\xi}_{B,1} \tag{B12b'}$$

$$\tilde{s}_{B,2} = H_{B,1}m + \tilde{\xi}_{B,1} = \phi_{x,2}\delta m + \tilde{\xi}_{B,2} \tag{B12c'}$$

where $\tilde{\xi}_{B,1} \equiv (\omega_0\phi_{x,0} + (\bar{\phi}_x - \phi_{x,1})\delta)(m-y) + \xi_B$ and $\tilde{\xi}_{B,2} \equiv (\omega_0\phi_{x,0} + (\bar{\phi}_x - \phi_{x,2})\delta)(m-y) + \xi_B$.

For each group, we can then show:

$$E_{i,0}^{Post}[m] = \left(\frac{\kappa_B^{-1}}{\kappa_B^{-1} + (\phi_{x,0})^2\delta\kappa^{-1}}\right)E_i^{Pre}[m] + P_{B,0}\tilde{s}_{B,0} \tag{B13a}$$

$$E_{i,1}^{Post}[m] = \left(\frac{\kappa_B^{-1}}{\kappa_B^{-1} + (\phi_{x,1}\delta)^2\delta\kappa^{-1}}\right)E_i^{Pre}[m] + P_{B,1}\tilde{s}_{B,1} \tag{B13b}$$

$$E_{i,2}^{Post}[m] = \left(\frac{\kappa_B^{-1}}{\kappa_B^{-1} + (\phi_{x,2}\delta)^2\delta\kappa^{-1}}\right)E_i^{Pre}[m] + P_{B,2}\tilde{s}_{B,2} \tag{B13c}$$

where the coefficient on the prior corresponds to $(1 - PH)$ in equation (23). Because $\phi_{x,2} < \phi_{x,1} < \phi_{x,0}$, we can predict that the weight on the prior increases in level of thinking k.

Combining equations (B3), (B6) and (B13) gives:

$$E_{i,0}^{Post}\begin{bmatrix}\bar{p} \\ \bar{E}[\bar{p}]\end{bmatrix} = \left(\frac{(\phi_{x,0})^2\delta\kappa^{-1}}{\kappa_B^{-1} + (\phi_{x,0})^2\delta\kappa^{-1}}\right)\begin{bmatrix}1-\phi_{x,0} \\ 1-\phi_{x,0}\end{bmatrix}y + \left(\frac{\kappa_B^{-1}}{\kappa_B^{-1} + (\phi_{x,0})^2\delta\kappa^{-1}}\right)E_{i,0}^{Pre}\begin{bmatrix}\bar{p} \\ \bar{E}[\bar{p}]\end{bmatrix} + \begin{bmatrix}\phi_{x,0} \\ \phi_{x,0}\end{bmatrix}P_{B,0}\tilde{s}_{B,0}$$

$$E_{i,1}^{Post}\begin{bmatrix}\bar{p} \\ \bar{E}[\bar{p}]\end{bmatrix} = \left(\frac{(\phi_{x,1}\delta)^2\delta\kappa^{-1}}{\kappa_B^{-1} + (\phi_{x,1}\delta)^2\delta\kappa^{-1}}\right)\begin{bmatrix}1-\phi_{x,1}\delta \\ 1-\phi_{x,1}\delta\end{bmatrix}y + \left(\frac{\kappa_B^{-1}}{\kappa_B^{-1} + (\phi_{x,1}\delta)^2\delta\kappa^{-1}}\right)E_{i,1}^{Pre}\begin{bmatrix}\bar{p} \\ \bar{E}[\bar{p}]\end{bmatrix} + \begin{bmatrix}\phi_{x,1}\delta \\ \phi_{x,1}\delta\end{bmatrix}P_{B,1}\tilde{s}_{B,1}$$

$$E_{i,2}^{Post}\begin{bmatrix}\bar{p} \\ \bar{E}[\bar{p}]\end{bmatrix} = \left(\frac{(\phi_{x,2}\delta)^2\delta\kappa^{-1}}{\kappa_B^{-1} + (\phi_{x,2}\delta)^2\delta\kappa^{-1}}\right)\begin{bmatrix}1-\phi_{x,1}\delta \\ (1-\delta(\omega_0(1-\delta)\phi_{x,0} + \bar{\phi}_x\delta))\end{bmatrix}y + \left(\frac{\kappa_B^{-1}}{\kappa_B^{-1} + (\phi_{x,2}\delta)^2\delta\kappa^{-1}}\right)E_{i,1}^{Pre}\begin{bmatrix}\bar{p} \\ \bar{E}[\bar{p}]\end{bmatrix} + \begin{bmatrix}\phi_{x,2}\delta \\ \phi_{x,2}\delta^2\end{bmatrix}P_{B,2}\tilde{s}_{B,2}$$

Note that the difference in weight on priors across k is largely governed by variation in $H_{B,k}$ across k. Given our parameter estimates for α and δ as well as the distribution of types, we find that $\phi_{x,0} \approx 0.80$, $\phi_{x,1}\delta \approx 0.59$, $\phi_{x,2}\delta \approx 0.55$. Thus, while the model predicts differentiated responses to signals across k, the differences could be rather small.

We can derive similar expressions for signal C, which gives firms the average higher-order expectations. Because firms incorrectly perceive the DGP, signals must be translated into the effective signals:

$$\tilde{s}_{C,0} = H_{C,0}m + \tilde{\xi}_{C,0} = \phi_{x,0}m + \tilde{\xi}_{C,0} \quad \text{(B14a')}$$

$$\tilde{s}_{C,1} = H_{C,1}m + \tilde{\xi}_{C,1} = \phi_{x,1}\delta m + \tilde{\xi}_{C,1} \quad \text{(B14b')}$$

$$\tilde{s}_{C,2} = H_{C,2}m + \tilde{\xi}_{C,2} = \phi_{x,2}\delta^2 m + \tilde{\xi}_{C,2} \quad \text{(B14c')}$$

where $\tilde{\xi}_{C,0} \equiv \left[(1-\omega_0)\phi_{x,0} + \omega_1\phi_{x,1}\delta + \omega_2\phi_{x,2}\delta^2\right](y-m) + \xi_C$, $\tilde{\xi}_{C,1} \equiv \left[\omega_0\phi_{x,0} + (1-\omega_1)\phi_{x,1}\delta + \omega_2\phi_{x,2}\delta^2\right](y-m) + \xi_C$, and $\tilde{\xi}_{C,2} \equiv \left[\omega_0\phi_{x,0} + \omega_1\phi_{x,1}\delta + (1-\omega_2)\phi_{x,2}\delta^2\right](y-m) + \xi_C$.

Firms then update their expectations of the fundamental according to:

$$E_{i,0}^{Post}[m] = \left(\frac{\kappa_C^{-1}}{\kappa_C^{-1} + (\phi_{x,0})^2\delta\kappa^{-1}}\right) E_i^{Pre}[m] + P_{C,0}\tilde{s}_{C,0} \quad \text{(B15a)}$$

$$E_{i,1}^{Post}[m] = \left(\frac{\kappa_C^{-1}}{\kappa_C^{-1} + (\phi_{x,1}\delta)^2\delta\kappa^{-1}}\right) E_i^{Pr}\ [m] + P_{C,1}\tilde{s}_{C,1} \quad \text{(B15b)}$$

$$E_{i,2}^{Post}[m] = \left(\frac{\kappa_C^{-1}}{\kappa_C^{-1} + (\phi_{x,2}\delta^2)^2\delta\kappa^{-1}}\right) E_i^{Pre}[m] + P_{C,2}\tilde{s}_{C,2} \quad \text{(B15c)}$$

Equations (B15a) and (B15b) imply that, provided $\kappa_B = \kappa_C$, the weight on priors for level-0 and level-1 firms is the same when firms are presented with signals B and C because these firms cannot perform a second iteration on expectations.

Combining equations (B3), (B6) and (B15) gives:

$$E_{i,0}^{Post}\begin{bmatrix}\overline{p}\\ \overline{E}[\overline{p}]\end{bmatrix} = \left(\frac{(\phi_{x,0})^2\delta\kappa^{-1}}{\kappa_C^{-1} + (\phi_{x,0})^2\delta\kappa^{-1}}\right)\begin{bmatrix}1-\phi_{x,0}\\ 1-\phi_{x,0}\end{bmatrix} y + \left(\frac{\kappa_C^{-1}}{\kappa_C^{-1} + (\phi_{x,0})^2\delta\kappa^{-1}}\right) E_{i,0}^{Pre}\begin{bmatrix}\overline{p}\\ \overline{E}[\overline{p}]\end{bmatrix} + \begin{bmatrix}\phi_{x,0}\\ \phi_{x,0}\end{bmatrix} P_{C,0}\tilde{s}_{C,0}$$

$$E_{i,1}^{Post}\begin{bmatrix}\overline{p}\\ \overline{E}[\overline{p}]\end{bmatrix} = \left(\frac{(\phi_{x,1}\delta)^2\delta\kappa^{-1}}{\kappa_C^{-1} + (\phi_{x,1}\delta)^2\delta\kappa^{-1}}\right)\begin{bmatrix}1-\phi_{x,1}\delta\\ 1-\phi_{x,1}\delta\end{bmatrix} y + \left(\frac{\kappa_C^{-1}}{\kappa_C^{-1} + (\phi_{x,1}\delta)^2\delta\kappa^{-1}}\right) E_{i,1}^{Pre}\begin{bmatrix}\overline{p}\\ \overline{E}[\overline{p}]\end{bmatrix} + \begin{bmatrix}\phi_{x,1}\delta\\ \phi_{x,1}\delta\end{bmatrix} P_{C,1}\tilde{s}_{C,1}$$

$$E_{i,2}^{Post}\begin{bmatrix}\overline{p}\\ \overline{E}[\overline{p}]\end{bmatrix} = \left(\frac{(\phi_{x,2}\delta^2)^2\delta\kappa^{-1}}{\kappa_C^{-1} + (\phi_{x,2}\delta^2)^2\delta\kappa^{-1}}\right)\begin{bmatrix}1-\phi_{x,2}\delta\\ 1-\phi_{x,2}\delta^2\end{bmatrix} y + \left(\frac{\kappa_C^{-1}}{\kappa_C^{-1} + (\phi_{x,2}\delta^2)^2\delta\kappa^{-1}}\right) E_{i,1}^{Pre}\begin{bmatrix}\overline{p}\\ \overline{E}[\overline{p}]\end{bmatrix} + \begin{bmatrix}\phi_{x,2}\delta\\ \phi_{x,2}\delta^2\end{bmatrix} P_{C,2}\tilde{s}_{C,2}$$

Given our parameter estimates for α and δ as well as the distribution of types, we find that $\phi_{x,0} \approx 0.80$, $\phi_{x,1}\delta \approx 0.59$, $\phi_{x,2}\delta^2 \approx 0.44$. These estimates determine the difference in the weight given to priors across k.

Firms in Group D receive both signals. If the noise terms in both signals are uncorrelated

$$E_{i,k}^{post}(m) = \left(1 - P_{D,k}H_{D,k}\right)E_i^{pre}(m) + P_{D,k}\tilde{s}_{D,k}, \quad \text{(B16)}$$

where $H_{D,0} = [\phi_{x,0} \quad \phi_{x,0}]'$, $H_{D,1} = [\phi_{x,1}\delta \quad \phi_{x,1}\delta]'$, $H_{D,2} = [\phi_{x,2}\delta \quad \phi_{x,2}\delta^2]'$, $R_D = diag\{\kappa_B^{-1}, \kappa_C^{-1}\}$, and $P_{D,k} = \delta\kappa^{-1}H'_{D,k}\left(R_D + \delta\kappa^{-1}H_{D,k}H'_{D,k}\right)^{-1}$.

All thinking types are able to correctly process Signal E, which contains an estimate of past inflation. Intuitively, signal E provides direct information about the fundamental and updating beliefs does not require thinking about the behavior of other agents in the economy. We therefore do not expect to see any difference between responses to this signal across k.

Appendix C:
Extension of the basic noisy-information model:
Semi-public signal

A. Setup

As in equation (2), we assume that managers set prices as a linear combination of their expectations of the aggregate price level and a fundamental:

$$p_i = (1-\alpha)E_i[m] + \alpha E_i[\bar{p}] \quad \text{(C.1)}$$

In contrast to the basic noisy-information model, we allow managers to observe two signals with idiosyncratic noise, a private signal x_i and a semi-public signal y_i:

$$x_i = m + v_{i,1} \quad \text{(C.2)}$$

$$y_i = y + v_{i,2} = m + \varepsilon + v_{i,2}. \quad \text{(C.3)}$$

with $v_{i,1} \sim N(0, \kappa_x^{-1})$, $\varepsilon \sim N(0, \kappa_y^{-1})$, and $v_{i,2} \sim N(0, \kappa_z^{-1})$. The optimal price level is set as a sum of iteratively higher-order expectations of m as in Section 3:

$$p_i = (1-\alpha)E_i[m] + \alpha(1-\alpha)E_i\left[\bar{E}[m]\right] + \alpha^2(1-\alpha)E_i\left[\bar{E}^2[m]\right] + \cdots. \quad \text{(C.4)}$$

Manger i's expectation of m is given by:

$$E_i[m] = (1-\delta')y_i + \delta' x_i \quad \text{(C.5)}$$

where $\delta' = \frac{\kappa_x}{\kappa'} = \frac{\kappa_x}{\kappa_x+\kappa_y+\kappa_z}$ represents the relative precision of the private signal. The aggregate expectation of m is then

$$\bar{E}[m] = (1-\delta')y + \delta' m \quad \text{(C.6)}$$

In forming higher-order expectations about m, managers substitute in their expectations of both m and y. Because managers receive only one signal about y diluted by Gaussian noise, a manager's best expectation of the public signal is his own semi-public signal. That is, $E_i[y] = y_i$. The mangers expectation about the average expectation of other managers in the economy is therefore:

$$E_i\left[\bar{E}\,[m]\right] = (1-\delta')y_i + \delta' E_i[m] = \left(1-\delta'^2\right)y_i + \delta'^2 x_i. \quad \text{(C.7)}$$

One can obtain progressively higher-order expectations of m by continuing to substitute $E_i[m]$ for m and y_i for y to find:

$$E_i\left[\bar{E}^k[m]\right] = \left(1-\delta'^{k-1}\right)y_i + \delta^{k-1}E_i\left[\bar{E}^{k-1}\,[m]\right] = \left(1-\delta'^k\right)y_i + \delta'^k x_i. \quad \text{(C.8)}$$

We can substitute this into equation (C.4) to find the optimal price as a function of the private and semi-public signals.

$$p_i = (1-\alpha)\sum_{k=0}^{\infty}\alpha^k\left[\left[1-\delta'^{k+1}\right]y + \delta'^{k+1}x_i\right]. \quad \text{(C.9)}$$

It follows that every agent sets the optimal price at:

$$p_i = \phi_y' y + \phi_x' x_i, \quad \text{(C.10)}$$

where $\phi_y' = \frac{1-\delta'}{(1-\alpha)\delta'+(1-\delta')}$ and $\phi_x' = \frac{(1-\alpha)\delta'}{(1-\alpha)\delta'+(1-\delta')}$. Note that the structure of the problem is identical to the structure we have for the basic noisy-information model. The only difference is that for the semi-public signal version of the model we have δ' and in the basic noisy-information model we have δ. Therefore:

$$\bar{p} \equiv \int_0^1 p_j dJ = \phi_y' y + \phi_x' m \quad \text{(C.11)}$$

A manager's optimal expectation of this is:

$$E_i[\bar{p}] = \phi_y' y_i + \phi_x'\left((1-\delta')y_i + \delta' x_i\right) = (1-\phi_x'\delta')y_i + \phi_x'\delta' x_i \quad \text{(C.12)}$$

Aggregating across agents gives the average expectation about the price level:

$$\bar{E}[\bar{p}] = \phi_y' y + \phi_x'\big((1-\delta')y + \delta' m\big) = m + (1-\phi_x'\delta')\varepsilon \tag{C.13}$$

The manager's higher-order expectation is then given by:

$$E_i\left[\bar{E}[\bar{p}]\right] = \phi_y' y_i + \phi_x'\left((1-\delta'^2)y_i + \delta'^2 x_i\right) = (1-\phi_x'\delta'^2)y_i + \phi_x'\delta'^2 x_i \tag{C.14}$$

and the average higher-order expectation is:

$$\bar{E}^2[\bar{p}] = (1-\phi_x'\delta'^2)y + \phi_x'\delta'^2 m = m + (1-\phi_x'\delta'^2)\varepsilon \tag{C.15}$$

Because y is a common component of y_i, ε does not contribute to cross-sectional disagreement:

$$Var[E_i[\bar{p}]|y] = (1-\phi_x'\delta')^2\kappa_z^{-1} + (\phi_x'\delta')^2\kappa_x^{-1} \tag{C.16}$$

$$Var\big[E_i[\bar{E}[\bar{p}]|y]\big] = (1-\phi_x'\delta'^2)^2\kappa_z^{-1} + (\phi_x'\delta'^2)^2\kappa_x^{-1} \tag{C.17}$$

Uncertainty will incorporate uncertainty about both m and y. While y is shared across agents, it is unknown to the individual because the semi-public signal includes idiosyncratic noise:

$$\Omega_i[E_i[\bar{p}]|y_i] = (1-\phi_x'\delta')^2\kappa_z^{-1} + (\phi_x'\delta')^2\kappa_x^{-1} + \big(\phi_x'(1-\delta')\big)^2\kappa_y^{-1}, \tag{C.18}$$

$$\Omega_i\left[E_i\left[\bar{E}[\bar{p}]\right]\middle|y_i\right] = (1-\phi_x'\delta'^2)^2\kappa_z^{-1} + (\phi_x'\delta'^2)^2\kappa_x^{-1} + \big(\phi_x'\delta'(1-\delta')\big)^2\kappa_y^{-1}. \tag{C.19}$$

This implies that uncertainty is higher than disagreement, the opposite of what is observed in the data. Therefore, adding a semi-public signal does not explain the disparity between cross-sectional disagreement and uncertainty observed in the data

Note that because $E_i[\bar{p}]$ and $E_i\left[\bar{E}[\bar{p}]\right]$ have different loadings on y_i and x_i (two sources of cross-sectional variation), $E_i[\bar{p}]$ and $E_i\left[\bar{E}[\bar{p}]\right]$ are not perfectly correlated, which contrasts with the basic noisy-information model.

B. Response to information.

Including a semi-public signal introduces the realization of the public signal noise, ε, to the state space. Managers form expectations about the price level and the aggregate belief of the price level according to:

$$E_i\begin{bmatrix}\bar{p}\\ \bar{E}[\bar{p}]\end{bmatrix} = \begin{bmatrix}1 & (1-\phi_x')\\ 1 & (1-\phi_x'\delta')\end{bmatrix} E_i\begin{bmatrix}m\\ \varepsilon\end{bmatrix} \tag{C.18}$$

Denote

$$\Upsilon = \begin{bmatrix}\upsilon_1 & \upsilon_2\\ \upsilon_3 & \upsilon_4\end{bmatrix} = \begin{bmatrix}1 & (1-\phi_x')\\ 1 & (1-\phi_x'\delta')\end{bmatrix}. \tag{C.19}$$

Manager i's priors about the state space are formed as combinations of the private and semi-public signals.

$$E_i^{Pre}\begin{bmatrix}m\\ \varepsilon\end{bmatrix} = \begin{bmatrix}\delta' x_i + (1-\delta')y_i\\ \delta'(y_i - x_i)\end{bmatrix} = \begin{bmatrix}\delta' & (1-\delta')\\ -\delta' & \delta'\end{bmatrix}\begin{bmatrix}x_i\\ y_i\end{bmatrix} \tag{C.20}$$

with a priori uncertainty conditional on a realization of the public signal, y formed according to:

$$\Psi \equiv \begin{bmatrix}\psi_1 & \psi_2\\ \psi_3 & \psi_4\end{bmatrix} \equiv E\left\{\begin{bmatrix}E^{Pre}[m]-m\\ E^{Pre}[\varepsilon]-\varepsilon\end{bmatrix}\begin{bmatrix}E^{Pre}[m]-m\\ E^{Pre}[\varepsilon]-\varepsilon\end{bmatrix}'\right\}$$

$$= \begin{bmatrix}(1-\delta')(v_{i,2}+\varepsilon) + \delta' v_{i,1}\\ \delta'(v_{i,2}+\varepsilon - v_{i,1})\end{bmatrix}\begin{bmatrix}(1-\delta')(v_{i,2}+\varepsilon) + \delta' v_{i,1}\\ \delta'(v_{i,2}+\varepsilon - v_{i,1})\end{bmatrix}'$$

$$= \begin{bmatrix} (1-\delta')^2\kappa_z^{-1} + \delta'^2\kappa_x^{-1} & (1-\delta')\delta'\kappa_z^{-1} + \delta'^2\kappa_x^{-1} \\ (1-\delta')\delta'\kappa_z^{-1} + \delta'^2\kappa_x^{-1} & \delta'^2(\kappa_z^{-1} + \kappa_x^{-1}) \end{bmatrix} \tag{C.21}$$

Signals come of the form:

$$s = H\begin{bmatrix} m \\ \varepsilon \end{bmatrix} + \xi \tag{C.22}$$

Manager's update their expectations about the state space as a mixture of the signal and their priors:

$$E_i^{Post}\begin{bmatrix} m \\ \varepsilon \end{bmatrix} = (I - PH)E_i^{Pre}\begin{bmatrix} m \\ \varepsilon \end{bmatrix} + Ps \tag{C.23}$$

Manager's posterior first-order and higher-order expectations are therefore

$$E_i^{Post}\begin{bmatrix} \bar{p} \\ \bar{E}[\bar{p}] \end{bmatrix} = E_i\begin{bmatrix} \bar{p} \\ \bar{E}[\bar{p}] \end{bmatrix} = \Upsilon(I - PH)E_i^{Pre}\begin{bmatrix} m \\ \varepsilon \end{bmatrix} + \Upsilon Ps \tag{C.24}$$

C. Econometric specification

To better understand the structure of equation (C.24) and its econometric implementation, consider the case of one signal in s. In this case, H is a 1×2 matrix: $H = [h_1 \quad h_2]$.[25] The Kalman gain is a 2×1 vector and the variance-covariance matrix is a single variance term $K_\xi = \kappa_s^{-1}$. Hence,

$$P = \begin{bmatrix} p_1 \\ p_1 \end{bmatrix} = \frac{\Psi H\prime}{\kappa_s^{-1} + H\Psi H\prime} = \frac{1}{\Lambda}\begin{bmatrix} h_1\psi_1 + h_2\psi_2 \\ h_1\psi_3 + h_2\psi_4 \end{bmatrix} \tag{C.25}$$

where $\Lambda \equiv \kappa_s^{-1} + h_1^2\psi_1 + h_1h_2(\psi_2 + \psi_3) + h_2^2\psi_4$.

Hence we can re-write equation (C.24) as

$$\begin{aligned}
E_i^{Post}\begin{bmatrix} \bar{p} \\ \bar{E}[\bar{p}] \end{bmatrix} &= \Upsilon\begin{bmatrix} 1 - p_1h_1 & -p_1h_2 \\ -p_2h_1 & 1 - p_2h_2 \end{bmatrix} E_i^{Pre}\begin{bmatrix} m \\ \varepsilon \end{bmatrix} + \Upsilon Ps \\
&= \frac{1}{\Lambda}\Upsilon\begin{bmatrix} \kappa_s^{-1} + h_1h_2\psi_3 + h_2^2\psi_4 & -h_1h_2\psi_1 - h_2^2\psi_2 \\ -h_1^2\psi_3 - h_1h_2\psi_4 & \kappa_s^{-1} + h_1h_2\psi_2 + h_1^2\psi_1 \end{bmatrix} E_i^{Pre}\begin{bmatrix} m \\ \varepsilon \end{bmatrix} + \Upsilon Ps \\
&= \frac{1}{\Lambda}\Upsilon\left\{\kappa_s^{-1}I + \begin{bmatrix} h_1h_2\psi_3 + h_2^2\psi_4 & -h_1h_2\psi_1 - h_2^2\psi_2 \\ -h_1^2\psi_3 - h_1h_2\psi_4 & h_1h_2\psi_2 + h_1^2\psi_1 \end{bmatrix}\right\} E_i^{Pre}\begin{bmatrix} m \\ \varepsilon \end{bmatrix} + \Upsilon Ps \\
&= \frac{1}{\Lambda}\kappa_s^{-1}\Upsilon E_i^{Pre}\begin{bmatrix} m \\ \varepsilon \end{bmatrix} + \frac{1}{\Lambda}\Upsilon\begin{bmatrix} h_1h_2\psi_3 + h_2^2\psi_4 & -h_1h_2\psi_1 - h_2^2\psi_2 \\ -h_1^2\psi_3 - h_1h_2\psi_4 & h_1h_2\psi_2 + h_1^2\psi_1 \end{bmatrix} E_i^{Pre}\begin{bmatrix} m \\ \varepsilon \end{bmatrix} + \Upsilon Ps \\
&= \frac{1}{\Lambda}\kappa_s^{-1}E_i^{Pre}\begin{bmatrix} \bar{p} \\ \bar{E}[\bar{p}] \end{bmatrix} + \frac{1}{\Lambda}\Upsilon\begin{bmatrix} h_1h_2\psi_3 + h_2^2\psi_4 & -h_1h_2\psi_1 - h_2^2\psi_2 \\ -h_1^2\psi_3 - h_1h_2\psi_4 & h_1h_2\psi_2 + h_1^2\psi_1 \end{bmatrix} E_i^{Pre}\begin{bmatrix} m \\ \varepsilon \end{bmatrix} + \Upsilon Ps \\
&= \frac{1}{\Lambda}\kappa_s^{-1}E_i^{Pre}\begin{bmatrix} \bar{p} \\ \bar{E}[\bar{p}] \end{bmatrix} \\
&+ \frac{1}{\Lambda}\begin{bmatrix} \upsilon_1h_1h_2\psi_3 + \upsilon_1h_2^2\psi_4 - \upsilon_2h_1^2\psi_3 - \upsilon_2h_1h_2\psi_4 & -\upsilon_1h_1h_2\psi_1 - \upsilon_1h_2^2\psi_2 + \upsilon_2h_1h_2\psi_2 + \upsilon_2h_1^2\psi_1 \\ \upsilon_3h_1h_2\psi_3 + \upsilon_3h_2^2\psi_4 - \upsilon_4h_1^2\psi_3 - \upsilon_4h_1h_2\psi_4 & -\upsilon_3h_1h_2\psi_1 - \upsilon_3h_2^2\psi_2 + \upsilon_4h_1h_2\psi_2 + \upsilon_4h_1^2\psi_1 \end{bmatrix} E_i^{Pre}\begin{bmatrix} m \\ \varepsilon \end{bmatrix} \\
&+ \Upsilon Ps
\end{aligned} \tag{C.26}$$

Similar to equation (23) for the basic noisy-information model, equation (C.26) relates posterior beliefs on $\bar{p}$ to prior beliefs on $\bar{p}$, public signal y and the new information in the signal provided in a treatment (s). The coefficient on the prior beliefs is governed by the gain of the Kalman filter. At the same time, equation (C.26) features new terms that depend

[25] Below we show later what value H takes in our signals.

on the pre-treatment beliefs on fundamentals $E_i^{Pre}[\varepsilon]$ and $E_i^{Pre}[m]$. Because we do not observe these terms and $E_i^{Pre}[\varepsilon]$ and $E_i^{Pre}[m]$ are all correlated with $E_i^{Pre}[\bar{p}]$ and $E_i^{Pre}[\bar{E}[\bar{p}]]$, we may have biased estimates of the slope coefficient on prior beliefs about $\bar{p}$ when we regress posterior beliefs on prior beliefs and a constant (equation (1) in the paper).

In what follows, we sign and quantify these potential biases. We consider first signal B (that is, we provide firms with information on $E_i^{Pre}[\bar{p}]$) and then we consider signal C (that is, we provide firms with information on $E_i^{Post}[\bar{E}[\bar{p}]]$).

Signal B

Given equation (C.12), we can show that $h_1 = 1$ and $h_2 = (1 - \phi_x'\delta')$. Note also that $v_1 = v_3 = 1$. Given the expressions for v_2 and v_4 in Equation (C.19)

$$B_{21} \equiv v_3 h_1 h_2 \psi_3 + v_3 h_2^2 \psi_4 - v_4 h_1^2 \psi_3 - v_4 h_1 h_2 \psi_4 = (h_2 - v_4)\psi_3 + h_2(h_2 - v_4)\psi_4 = 0 \quad \text{(C.27)}$$

$$B_{22} \equiv -v_3 h_1 h_2 \psi_1 - v_3 h_2^2 \psi_2 + v_4 h_1 h_2 \psi_2 + v_4 h_1^2 \psi_1 = (v_4 - h_2)\psi_1 + h_2(v_4 - h_2)\psi_2 = 0 \quad \text{(C.28)}$$

Hence, the relationship between $E_i^{Post}[\bar{E}[\bar{p}]]$ and $E_i^{Pre}[\bar{E}[\bar{p}]]$ in equation (C.26) is not biased.

One can also show that

$$B_{11} \equiv v_1 h_1 h_2 \psi_3 + v_1 h_2^2 \psi_4 - v_2 h_1^2 \psi_3 - v_2 h_1 h_2 \psi_4 = (h_2 - v_2)\psi_3 + h_2(h_2 - v_2)\psi_4$$
$$= \phi_x'(1 - \delta')[\psi_3 + (1 - \phi_x'\delta')\psi_4] > 0 \quad \text{(C.29)}$$

$$B_{12} \equiv -v_1 h_1 h_2 \psi_1 - v_1 h_2^2 \psi_2 + v_2 h_1 h_2 \psi_2 + v_2 h_1^2 \psi_1 = (v_2 - h_2)\psi_1 + h_2(v_2 - h_2)\psi_2$$
$$= -\phi_x'(1 - \delta')[\psi_1 + (1 - \phi_x'\delta')\psi_2] < 0 \quad \text{(C.30)}$$

Equation (C.20) shows that both $E_i^{Pre}[m]$ and $E_i^{Pre}[\varepsilon]$ are composed of signals x_i and y_i. We can therefore construct the omitted term as omitted terms due to x_i and due to y_i:

$$O_i^{(B1;FO)} \equiv \frac{1}{\Lambda}[\delta'(B_{11} - B_{12})]x_i \quad \text{(C.31)}$$

$$O_i^{(B2;FO)} \equiv \frac{1}{\Lambda}[B_{11} - \delta'(B_{11} - B_{12})]y_i \quad \text{(C.32)}$$

From Equation (##), we see that

$$Cov\left(E_i^{Pre}[\bar{p}], O_i^{(B1;FO)}\right) = \frac{1}{\Lambda}\left[\phi_x'\delta'^2(B_{11} - B_{12})\right]\kappa_x^{-1} > 0$$

$$Cov\left(E_i^{Pre}[\bar{p}], O_i^{(B2;FO)}\right) = \frac{1}{\Lambda}(1 - \phi_x'\delta')[B_{11} - \delta'(B_{11} - B_{12})]\kappa_z^{-1} \lesseqgtr 0$$

The bias due to private signal terms, x_i, is

$$bias_1^{(B:FO)} \equiv \frac{Cov\left(E_i^{Pre}[\bar{p}], O_i^{(B1,FO)}\right)}{Var\left(E_i^{Pre}[\bar{p}]\right)} > 0$$

The bias due to private signal terms, x_i, is

$$bias_2^{(B:FO)} \equiv \frac{Cov\left(E_i^{Pre}[\bar{p}], O_i^{(B2,FO)}\right)}{Var\left(E_i^{Pre}[\bar{p}]\right)} \lesseqgtr 0$$

The overall bias may then go in either direction:

$$bias^{B:FO} \equiv bias_1^{B:FO} + bias_2^{B:FO} \lesseqgtr 0$$

Signal C

Given Equation (C.14), we can show that $h_1 = 1$ and $h_2 = \left(1 - \phi_x' \delta'^2\right)$. Using the values of υ_1, υ_2, υ_3 and υ_4 given in Equation (C.19), we can show:

$$B_{11} \equiv \upsilon_1 h_1 h_2 \psi_3 + \upsilon_1 h_2^2 \psi_4 - \upsilon_2 h_1^2 \psi_3 - \upsilon_2 h_1 h_2 \psi_4 = (h_2 - \upsilon_2)\psi_3 + h_2(h_2 - \upsilon_2)\psi_4$$

$$= \phi_x'\left(1 - \delta'^2\right)\left[\psi_3 + \left(1 - \phi_x'\delta'^2\right)\psi_4\right] > 0 \quad \text{(C.33)}$$

$$B_{12} \equiv -\upsilon_1 h_1 h_2 \psi_1 - \upsilon_1 h_2^2 \psi_2 + \upsilon_2 h_1 h_2 \psi_2 + \upsilon_2 h_1^2 \psi_1 = (\upsilon_2 - h_2)\psi_1 + h_2(\upsilon_2 - h_2)\psi_2$$

$$= -\phi_x'\left(1 - \delta'^2\right)\left[\psi_1 + \left(1 - \phi_x'\delta'^2\right)\psi_2\right] < 0 \quad \text{(C.34)}$$

Using the logic outlined in the discussion of Signal B, we can characterize the omission in terms of omitted terms due to x_i and due to y_i:

$$O_i^{(C1;FO)} \equiv \frac{1}{\Lambda}[\delta'(B_{11} - B_{12})]x_i \quad \text{(C.35)}$$

$$O_i^{(C2;FO)} \equiv \frac{1}{\Lambda}[B_{11} - \delta'(B_{11} - B_{12})]y_i \quad \text{(C.36)}$$

Using equation (C.20), we see that

$$Cov\left(E_i^{Pre}[\bar{p}], O_i^{(C1;FO)}\right) = \frac{1}{\Lambda}\left[\phi_x'\delta'^2(B_{11} - B_{12})\right]\kappa_x^{-1} > 0$$

$$Cov\left(E_i^{Pre}[\bar{p}], O_i^{(C2;FO)}\right) = \frac{1}{\Lambda}(1 - \phi_x'\delta')[B_{11} - \delta'(B_{11} - B_{12})]\kappa_z^{-1} \lesseqgtr 0$$

The bias due to private signal terms, x_i, is

$$bias_1^{(C:FO)} \equiv \frac{Cov\left(E_i^{Pre}[\bar{E}[\bar{p}]], O_i^{(C1,FO)}\right)}{Var\left(E_i^{Pre}[\bar{p}]\right)} > 0$$

The bias due to semi-public signal terms, x_i, is

$$bias_2^{(C:FO)} \equiv \frac{Cov\left(E_i^{Pre}[\bar{E}[\bar{p}]], O_i^{(C2,FO)}\right)}{Var\left(E_i^{Pre}[\bar{p}]\right)} \lesseqgtr 0$$

The overall bias may then go in either direction:

$$bias^{C:FO} \equiv bias_1^{C:FO} + bias_2^{C:FO} \lesseqgtr 0$$

The bias in the regression of higher-order posteriors in priors can be outlined similarly:

$$B_{21} \equiv \upsilon_3 h_1 h_2 \psi_3 + \upsilon_3 h_2^2 \psi_4 - \upsilon_4 h_1^2 \psi_3 - \upsilon_4 h_1 h_2 \psi_4 = (h_2 - \upsilon_4)\psi_3 + h_2(h_2 - \upsilon_4)\psi_4$$

$$= \phi_x'\delta'(1 - \delta')\left[\psi_3 + \left(1 - \phi_x'\delta'^2\right)\psi_4\right] > 0 \quad \text{(C.37)}$$

$$B_{22} \equiv -\upsilon_3 h_1 h_2 \psi_1 - \upsilon_3 h_2^2 \psi_2 + \upsilon_4 h_1 h_2 \psi_2 + \upsilon_4 h_1^2 \psi_1 = (\upsilon_4 - h_2)\psi_1 + h_2(\upsilon_4 - h_2)\psi_2$$

$$= -\phi_x'\delta'(1 - \delta')\left[\psi_1 + \left(1 - \phi_x'\delta'^2\right)\psi_2\right] < 0 \quad \text{(C.38)}$$

Using the logic outlined in the discussion of Signal B, we can characterize the omission in terms of omitted terms due to x_i and due to y_i:

$$O_i^{(C1;FO)} \equiv \frac{1}{\Lambda}[\delta'(B_{21} - B_{22})]x_i \quad \text{(C.39)}$$

$$O_i^{(C2;FO)} \equiv \frac{1}{\Lambda}[B_{21} - \delta'(B_{21} - B_{22})]y_i \quad \text{(C.40)}$$

From Equation (C.20), we see that

$$Cov\left(E_i^{Pre}\left[\bar{E}[\bar{p}]\right], O_i^{(C1;HO)}\right) = \frac{1}{\Lambda}\left[\phi_x'\delta'^3(B_{21} - B_{22})\right]\kappa_x^{-1} > 0$$

$$Cov\left(E_i^{Pre}\left[\bar{E}[\bar{p}]\right], O_i^{(C2;HO)}\right) = \frac{1}{\Lambda}\left(1 - \phi_x'\delta'^2\right)[B_{21} - \delta'(B_{21} - B_{22})]\kappa_z^{-1} \lesseqgtr 0$$

The bias due to private signal terms, x_i, is

$$bias_1^{(C:HO)} \equiv \frac{Cov\left(E_i^{Pre}[\bar{E}[\bar{p}]], O_i^{(C1,HO)}\right)}{Var\left(E_i^{Pre}[\bar{p}]\right)} > 0$$

The bias due to semi-public signal terms, x_i, is

$$bias_2^{(C:HO)} \equiv \frac{Cov\left(E_i^{Pre}[\bar{E}[\bar{p}]], O_i^{(C2,HO)}\right)}{Var\left(E_i^{Pre}[\bar{p}]\right)} \lesseqgtr 0$$

The overall bias may then go in either direction:

$$bias^{C:HO} \equiv bias_1^{C:HO} + bias_2^{C:HO} \lesseqgtr 0$$

We conclude that estimating equation (C.26) for signal C generally yields biased estimates. Although we cannot identify the model parameters from the moments collected in the survey, we experimented with various calibrations of the model and we found that biases are generally small.

Appendix D: Extension of the basic noisy-information model: Heterogenous long-run priors

A. Setup

As in equation (2), we allow managers to set prices as a linear combination of their expectations of the aggregate price level and a fundamental:

$$p_i = (1-\alpha)E_i[m] + \alpha E_i^*[\bar{p}] \quad \text{(D.1)}$$

Following Patton and Timmermann (2010), we allow expectations of the aggregate price level to be skewed by the manager's "long-run" prior, μ_i:

$$E_i^*[\bar{p}] = \omega\mu_i + (1-\omega)E_i[\bar{p}] = \omega\mu_i + (1-\omega)(1-\alpha)E_i\left[\bar{E}[m]\right] + (1-\omega)\alpha E_i\left[\bar{E}[\bar{p}]\right] \quad \text{(D.2)}$$

where $\mu_i \sim N\left(\bar{\mu}, \kappa_\mu^{-1}\right)$, $\omega = \frac{Var(E_i[\bar{p}])}{\gamma^2 + Var(E_i[\bar{p}])}$, and $\gamma^2 \geq 0$ is a parameter measuring the extent to which the managers prefer their own "long-run" priors. $\gamma^2 = 0$ means that the manager forms his entire expectation based on information contained in signals y and x.

The average expectation is:

$$\bar{E}[\bar{p}] = \omega\bar{\mu} + (1-\omega)(1-\alpha)\bar{E}^2[m] + (1-\omega)\alpha\bar{E}^2[\bar{p}] \quad \text{(D.3)}$$

and, given the skew towards the "long-run" prior, the individual higher-order expectation is:

$$E_i\left[\bar{E}[\bar{p}]\right] = \omega E_i[\bar{\mu}] + (1-\omega)(1-\alpha)E_i\left[\bar{E}^2[m]\right] + (1-\omega)\alpha E_i[\bar{E}^2[\bar{p}]] \quad \text{(D.4)}$$

Note that, by assumption, managers directly skew only their own (first-order) expectation of the price level and do not skew higher-order expectations. Following the same logic, we find further higher-order expectations:

$$\bar{E}^2[\bar{p}] = \omega\bar{E}[\bar{\mu}] + (1-\omega)(1-\alpha)\bar{E}^3[m] + (1-\omega)\alpha\bar{E}^3[\bar{p}] \quad \text{(D.5)}$$

$$E_i\left[\bar{E}^2[\bar{p}]\right] = \omega E_i\left[\bar{E}[\bar{\mu}]\right] + (1-\omega)(1-\alpha)E_i\left[\bar{E}^3[m]\right] + (1-\omega)\alpha E_i\left[\bar{E}^3[\bar{p}]\right] \quad \text{(D.6)}$$

The price is set as:

$$\begin{aligned} p_i &= (1-\alpha)E_i[m] + \alpha\omega\mu_i + \alpha(1-\omega)(1-\alpha)E_i\left[\bar{E}[m]\right] + \alpha^2(1-\omega)E_i\left[\bar{E}[\bar{p}]\right] \\ &= (1-\alpha)E_i[m] + \alpha\omega\mu_i + \alpha(1-\omega)(1-\alpha)E_i\left[\bar{E}[m]\right] \\ &\quad + \alpha^2(1-\omega)\omega E_i[\bar{\mu}] + \alpha^2(1-\omega)^2(1-\alpha)E_i\left[\bar{E}^2[m]\right] + \alpha^3(1-\omega)^2\omega E_i\left[\bar{E}[\bar{\mu}]\right] \\ &\quad + \alpha^3(1-\omega)^3(1-\alpha)E_i\left[\bar{E}^3[m]\right] + \alpha^4(1-\omega)^3E_i\left[\bar{E}^3[\bar{p}]\right] + \cdots \end{aligned} \quad \text{(D.7)}$$

We can rewrite equation (D.7) as:

$$\begin{aligned} p_i &= \alpha\omega\mu_i + \alpha\omega\sum_{k=0}^{\infty}\alpha^{k+1}(1-\omega)^{k+1}E_i\left[\bar{E}^k[\bar{\mu}]\right] \\ &\quad + (1-\alpha)\sum_{k=0}^{\infty}\alpha^k(1-\omega)^k\left[\left(1-\delta^{k+1}\right)y + \delta^{k+1}x_i\right] \end{aligned} \quad \text{(D.8)}$$

To make progress, we need to impose some structure on $E_i\left[\bar{E}^k[\bar{\mu}]\right]$. The optimal price depends on the individual's expectations of the average prior, $\bar{\mu}$. We allow this mean to be unknown, but let each manager observe a private signal of the mean: $\varsigma_i \sim N(\bar{\mu}, \kappa_\varsigma^{-1})$. We assume that the manager's own "long-run" prior skews his view of the aggregate prior (this assumption extends the effect of priors into higher-order expectations):

$$E_i[\bar{\mu}] = \omega'\mu_i + (1-\omega')\varsigma_i \quad \text{(D.9)}$$

where $\omega' = \frac{\kappa_\varsigma^{-1}}{(\gamma')^2+\kappa_\varsigma^{-1}}$ and $(\gamma')^2 \geq 0$ is a parameter measuring the extent to which the managers prefer their own "long-run" priors when he is forming beliefs about $\bar{\mu}$.

The average value of expectation $E_i[\bar{\mu}]$ in equation (D.9) is $\bar{E}[\bar{\mu}] = \bar{\mu}$. Therefore, $E_i[\bar{E}[\bar{\mu}]] = \omega'\mu_i + (1-\omega')\varsigma_i$ and $\bar{E}^2[\bar{\mu}] = \bar{\mu}$. By using repeated substitutions, we find that the expectation for all orders of expectations of the aggregate prior are the same, i.e. $\bar{E}^k[\bar{\mu}] = \bar{\mu}$ for any k. Using this insight, we find that the optimal price formula can be written as :

$$p_i = \alpha\omega\mu_i + \alpha\omega\sum_{k=0}^{\infty}\alpha^{k+1}(1-\omega)^{k+1}(\omega'\mu_i + (1-\omega')\varsigma_i) \\ +(1-\alpha)\sum_{k=0}^{\infty}\alpha^k(1-\omega)^k\left[\left(1-\delta^{k+1}\right)y + \delta^{k+1}x_i\right] \tag{D.10}$$

We can further rewrite this equation in terms of strategies:

$$p_i = \phi_\mu\mu_i + \phi_\varsigma\varsigma_i + \phi_x x_i + \phi_y y \tag{D.11}$$

where

$$\phi_\mu = \alpha\omega\left[1 + \frac{\alpha(1-\omega)}{1-\alpha(1-\omega)}\omega'\right] \tag{D.12}$$

$$\phi_\varsigma = \alpha\omega\left[\frac{\alpha(1-\omega)}{1-\alpha(1-\omega)}(1-\omega')\right] \tag{D.13}$$

$$\phi_x = \frac{\delta(1-\alpha)}{1-\alpha\delta(1-\omega)} \tag{D.14}$$

$$\phi_y = \frac{(1-\alpha)}{1-\alpha(1-\omega)} - \frac{\delta(1-\alpha)}{1-\alpha\delta(1-\omega)} \tag{D.15}$$

To simplify notation, define

$$\theta \equiv \phi_\mu + \phi_\varsigma = \frac{\alpha\omega}{1-\alpha(1-\omega)} \tag{D.16}$$

Note that

$$1-\theta = \phi_x + \phi_y = \frac{(1-\alpha)}{1-\alpha(1-\omega)} \tag{D.17}$$

If long-priors priors don't matter, $\omega = 0$, $\omega' = 0$, $\theta = 0$.

Given equation (D.11), the aggregate price level is:

$$\bar{p} = \theta\bar{\mu} + \phi_x m + \phi_y y. \tag{D.18}$$

The expectations are formed accordingly (with the weight assigned to the "long-run" prior):

$$E_i[\bar{p}] = \omega\mu_i + (1-\omega)[\theta E_i[\bar{\mu}] + (1-\theta)E_i(m)] \\ = \omega\mu_i + (1-\omega)\left[\theta E_i[\bar{\mu}] + \phi_x\delta x_i + \left((1-\phi_x\delta)-\theta\right)y\right] \\ = (\omega + (1-\omega)\theta\omega')\mu_i + (1-\omega)\theta(1-\omega')\varsigma_i + (1-\omega)\left[\phi_x\delta x_i + \left((1-\phi_x\delta)-\theta\right)y\right] \tag{D.19}$$

The average expected price is:

$$\bar{E}[\bar{p}] = (\omega + (1-\omega)\theta)\bar{\mu} + (1-\omega)\left[\phi_x\delta m + \left((1-\phi_x\delta)-\theta\right)y\right] \tag{D.20}$$

Manager i then expects other managers to believe:

$$E_i[\bar{E}[\bar{p}]] = (\omega + (1-\omega)\theta)E_i[\bar{\mu}] + (1-\omega)\left[\phi_x\delta E_i[m] + \left((1-\phi_x\delta)-\theta\right)y\right] \\ = (\omega + (1-\omega)\theta)\omega'\mu_i + (\omega + (1-\omega)\theta)(1-\omega')\varsigma_i \\ +(1-\omega)\left[\phi_x\delta^2 x_i + \left((1-\phi_x\delta^2)-\theta\right)y\right] \tag{D.21}$$

The average higher-order expectation is

$$\bar{E}^2[\bar{p}] = (\omega + (1-\omega)\theta)\bar{\mu} + (1-\omega)\left[\phi_x\delta^2 m + \left((1-\phi_x\delta^2) - \theta\right)y\right] \quad \text{(D.22)}$$

Note that the basic noisy-information model is nested in this when $\omega = 0$.

Difference in average higher-order and first-order expectations is given by:

$$\bar{E}^2[\bar{p}] - \bar{E}[\bar{p}] = (1-\omega)\phi_x\delta(1-\delta)(y-m) > 0 \quad \text{(D.23)}$$

Using equations (D.19)-(D.22), we can find expressions for cross-sectional disagreement and for forecast uncertainty.

$$Var\left[E_i[\bar{p}]\right] = (\omega + (1-\omega)\theta\omega')^2\kappa_\mu^{-1} + \left((1-\omega)\theta(1-\omega')\right)^2\kappa_\varsigma^{-1} + (1-\omega)^2(\phi_x\delta)^2\kappa_x^{-1} \quad \text{(D.24)}$$

$$Var\left[E_i\left[\bar{E}[\bar{p}]\right]\right] = \left((\omega + (1-\omega)\theta)\omega'\right)^2\kappa_\mu^{-1} + \left((\omega + (1-\omega)\theta)(1-\omega')\right)^2\kappa_\varsigma^{-1}$$
$$+(1-\omega)^2(\phi_x\delta^2)^2\kappa_x^{-1} \quad \text{(D.25)}$$

$$\Omega_{\{E_i[\bar{p}]|y\}} = \left((1-\omega)\theta(1-\omega')\right)^2\kappa_\varsigma^{-1} + (1-\omega)^2(\phi_x\delta)^2\kappa_x^{-1} \quad \text{(D.26)}$$

$$\Omega_{\{E_i[\bar{E}[\bar{p}]]|y\}} = \left((\omega + (1-\omega)\theta)(1-\omega')\right)^2\kappa_\varsigma^{-1} + (1-\omega)^2(\phi_x\delta^2)^2\kappa_x^{-1} \quad \text{(D.27)}$$

While deriving expressions for uncertainty in equations (D.26) and (D.27), we assume that each manager knows his own "long-run" prior with certainty and is not considering that his "long-run" prior differs from the aggregate prior.

Note that, unlike the basic noisy-information model where uncertainty about higher-order expectations is always lower than uncertainty in first-order expectations, the "long-run" prior modification of the basic model does not make a clear prediction, that is, the difference in uncertainty for higher- and first-order expectations is ambiguous and depends on parameter values (in particular on relative magnitudes of κ_ς^{-1} and κ_x^{-1}):

$$\Omega_{\{E_i[\bar{E}[\bar{p}]]|y\}} - \Omega_{\{E_i[\bar{p}]|y\}} = \left[\omega\left((\omega + (1-\omega)\theta) + \left((1-\omega)\theta\right)\right)\right](1-\omega')^2\kappa_\varsigma^{-1} + (1-\omega)^2(\phi_x\delta)^2(\delta^2-1)\kappa_x^{-1}.$$

But in any case, the model is capable of reproducing fact 4. In a similar spirit, the model is capable of reproducing fact 3.

To derive the slope for the regression of higher-order expectations on first-order expectations, we use equation (D.19) and (D.21):

$$\hat{b}^{OLS} = \frac{cov(E_i[\bar{p}], E_i\left[\bar{E}[\bar{p}]\right]}{var(E_i[\bar{p}])}$$

$$= \frac{(\omega + (1-\omega)\theta\omega')\left((\omega + (1-\omega)\theta)\omega'\right)\kappa_\mu^{-1} + \left((1-\omega)\theta(\omega + (1-\omega)\theta)(1-\omega')^2\right)\kappa_\varsigma^{-1} + (1-\omega)^2(\phi_x^2\delta^3)\kappa_x^{-1}}{(\omega + (1-\omega)\theta\omega')^2\kappa_\mu^{-1} + \left((1-\omega)\theta(1-\omega')\right)^2\kappa_\varsigma^{-1} + (1-\omega)^2(\phi_x\delta)^2\kappa_x^{-1}}$$

One can then show that

$$\hat{b}^{OLS} - 1 = \frac{\left[\begin{array}{c}(\omega + (1-\omega)\theta\omega')\left((\omega + (1-\omega)\theta)\omega' - 1\right)\kappa_\mu^{-1} + (1-\omega')^2(1-\omega)\theta\left((\omega + (1-\omega)\theta) - 1\right)\kappa_\varsigma^{-1} \\ +(1-\omega)^2(\phi_x^2\delta^2)(\delta-1)\kappa_x^{-1}\end{array}\right]}{(\omega + (1-\omega)\theta\omega')^2\kappa_\mu^{-1} + \left((1-\omega)\theta(1-\omega')\right)^2\kappa_\varsigma^{-1} + (1-\omega)^2(\phi_x\delta)^2\kappa_x^{-1}} < 0$$

because $\omega, \omega', \theta, \delta \in [0,1]$. Hence, the slope of the regression is less than one, which is consistent with the data.

B. Response to information.

The introduction of priors extends the state space to include $\bar{\mu}$. Managers form expectations in the following way:

$$E_i\begin{bmatrix}\bar{p} \\ \bar{E}[\bar{p}]\end{bmatrix} = \begin{bmatrix}\omega \\ 0\end{bmatrix}\mu_i + \begin{bmatrix}(1-\omega)\theta & (1-\omega)\phi_x \\ \omega + (1-\omega)\theta & (1-\omega)\phi_x\delta\end{bmatrix}E_i\begin{bmatrix}\bar{\mu} \\ m\end{bmatrix} + \begin{bmatrix}(1-\omega)(1-\phi_x-\theta) \\ (1-\omega)(1-\phi_x\delta-\theta)\end{bmatrix}y \quad \text{(D.28)}$$

Denote

$$\Upsilon \equiv \begin{bmatrix} \upsilon_1 & \upsilon_2 \\ \upsilon_3 & \upsilon_4 \end{bmatrix} \equiv \begin{bmatrix} (1-\omega)\theta & (1-\omega)\phi_x \\ \omega + (1-\omega)\theta & (1-\omega)\phi_x\delta \end{bmatrix}. \tag{D.29}$$

The prior expectations of the state space are:

$$E_i^{Pre}\begin{bmatrix} \bar{\mu} \\ m \end{bmatrix} = \begin{bmatrix} \omega' & (1-\omega') \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \mu_i \\ \varsigma_i \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \delta & (1-\delta) \end{bmatrix}\begin{bmatrix} x_i \\ y \end{bmatrix} \tag{D.30}$$

The prior uncertainty about the state variables is given by:

$$\Psi \equiv E\left\{\begin{bmatrix} (E_i[\bar{\mu}] - \bar{\mu}) \\ (E_i[m] - m) \end{bmatrix}\begin{bmatrix} (E_i[\bar{\mu}] - \bar{\mu}) & (E_i[m] - m) \end{bmatrix}\right\} \equiv \begin{bmatrix} \psi_1 & 0 \\ 0 & \psi_4 \end{bmatrix}$$

$$= \begin{bmatrix} (1-\omega')^2\kappa_\varsigma^{-1} & 0 \\ 0 & \delta^2\kappa^{-1} \end{bmatrix} \tag{D.31}$$

Signals come of the form:

$$s = H\begin{bmatrix} \bar{\mu} \\ m \end{bmatrix} + By + \xi \tag{D.32}$$

Because y is observed by the manager, he can subtract By to form the equivalent signal:

$$\tilde{s} = H\begin{bmatrix} \bar{\mu} \\ m \end{bmatrix} + \xi \tag{D.33}$$

Using this signal, he forms a posterior estimate of the state space.

$$E_i^{Post}\begin{bmatrix} \bar{\mu} \\ m \end{bmatrix} = [I - PH]E_i^{Pre}\begin{bmatrix} \bar{\mu} \\ m \end{bmatrix} + P\tilde{s} \tag{D.34}$$

where P is the gain from the Kalman filter and is of the form $P = \Psi H'\left(K_\xi + H\Psi H'\right)^{-1}$, K_ξ is the variance-covariance matrix of the vector of noise terms ξ.

We can substitute the prior and posterior expectations into the equation above to get prior and posterior estimates of first- and higher-order inflation expectations:

$$E_i^{Post}\begin{bmatrix} \bar{p} \\ \bar{E}[\bar{p}] \end{bmatrix} = \begin{bmatrix} \omega \\ 0 \end{bmatrix}\mu_i + \Upsilon E_i^{Post}\begin{bmatrix} \bar{\mu} \\ m \end{bmatrix} + \begin{bmatrix} (1-\omega)(1-\phi_x-\theta) \\ (1-\omega)(1-\phi_x\delta-\theta) \end{bmatrix}y$$

$$= \begin{bmatrix} \omega \\ 0 \end{bmatrix}\mu_i + \Upsilon\left([I - PH]E_i^{Pre}\begin{bmatrix} \bar{\mu} \\ m \end{bmatrix} + Ps\right) + \begin{bmatrix} (1-\omega)(1-\phi_x-\theta) \\ (1-\omega)(1-\phi_x\delta-\theta) \end{bmatrix}y \tag{D.35}$$

C. Econometric specification

To better understand the structure of equation (D.35) and its econometric implementation, consider the case of one signal in s. In this case, H is a 1×2 matrix: $H = [h_1 \quad h_2]$.[26] The Kalman gain is a 2×1 vector and the variance-covariance matrix is a single variance term $K_\xi = \kappa_s^{-1}$. Hence,

$$P = \begin{bmatrix} p_1 \\ p_1 \end{bmatrix} = \frac{\Psi H\prime}{\kappa_s^{-1} + H\Psi H\prime} = \frac{1}{\kappa_s^{-1} + h_1^2\psi_1 + h_2^2\psi_4}\begin{bmatrix} h_1\psi_1 \\ h_2\psi_4 \end{bmatrix} \tag{D.36}$$

We can write posteriors are follows:

$$E_i^{Post}\begin{bmatrix} \bar{p} \\ \bar{E}[\bar{p}] \end{bmatrix} = \begin{bmatrix} \omega \\ 0 \end{bmatrix}\mu_i + \Upsilon\begin{bmatrix} 1 - p_1h_1 & -p_1h_2 \\ -p_2h_1 & 1 - p_2h_2 \end{bmatrix}E_i^{Pre}\begin{bmatrix} \bar{\mu} \\ m \end{bmatrix} + \Upsilon P\tilde{s} + \begin{bmatrix} (1-\omega)(1-\phi_x-\theta) \\ (1-\omega)(1-\phi_x\delta-\theta) \end{bmatrix}y$$

[26] Below we show later what value H take in our signals.

$$= \begin{bmatrix}\omega\\0\end{bmatrix}\mu_i + \frac{1}{\kappa_s^{-1}+h_1^2\psi_1+h_2^2\psi_4}\Upsilon\begin{bmatrix}\kappa_s^{-1}+h_2^2\psi_4 & -h_1h_2\psi_1\\ -h_1h_2\psi_4 & \kappa_s^{-1}+h_1^2\psi_1\end{bmatrix}E_i^{Pre}\begin{bmatrix}\bar{\mu}\\m\end{bmatrix}$$

$$+\Upsilon P\tilde{s} + \begin{bmatrix}(1-\omega)(1-\phi_x-\theta)\\(1-\omega)(1-\phi_x\delta-\theta)\end{bmatrix}y$$

$$= \begin{bmatrix}\omega\\0\end{bmatrix}\mu_i$$

$$+\frac{1}{\kappa_s^{-1}+h_1^2\psi_1+h_2^2\psi_4}\begin{bmatrix}\upsilon_1\kappa_s^{-1}+\upsilon_1h_2^2\psi_4-\upsilon_2h_1h_2\psi_4 & -\upsilon_1h_1h_2\psi_1+\upsilon_2\kappa_s^{-1}+\upsilon_2h_1^2\psi_1\\ \upsilon_3\kappa_s^{-1}+\upsilon_3h_2^2\psi_4-\upsilon_4h_1h_2\psi_4 & -\upsilon_3h_1h_2\psi_1+\upsilon_4\kappa_s^{-1}+\upsilon_4h_1^2\psi_1\end{bmatrix}E_i^{Pre}\begin{bmatrix}\bar{\mu}\\m\end{bmatrix}$$

$$+\Upsilon P\tilde{s} + \begin{bmatrix}(1-\omega)(1-\phi_x-\theta)\\(1-\omega)(1-\phi_x\delta-\theta)\end{bmatrix}y$$

$$= \begin{bmatrix}\omega\\0\end{bmatrix}\mu_i$$

$$+\frac{1}{\kappa_s^{-1}+h_1^2\psi_1+h_2^2\psi_4}\begin{bmatrix}(\upsilon_1\kappa_s^{-1}+\upsilon_1h_2^2\psi_4-\upsilon_2h_1h_2\psi_4)E_i^{Pre}[\bar{\mu}]+(-\upsilon_1h_1h_2\psi_1+\upsilon_2\kappa_s^{-1}+\upsilon_2h_1^2\psi_1)E_i^{Pre}[m]\\(\upsilon_3\kappa_s^{-1}+\upsilon_3h_2^2\psi_4-\upsilon_4h_1h_2\psi_4)E_i^{Pre}[\bar{\mu}]+(-\upsilon_1h_1h_2\psi_1+\upsilon_4\kappa_s^{-1}+\upsilon_2h_1^2\psi_1)E_i^{Pre}[m]\end{bmatrix}$$

$$+\Upsilon P\tilde{s} + \begin{bmatrix}(1-\omega)(1-\phi_x-\theta)\\(1-\omega)(1-\phi_x\delta-\theta)\end{bmatrix}y$$

$$= \begin{bmatrix}\omega\\0\end{bmatrix}\mu_i$$

$$+\frac{1}{\kappa_s^{-1}+h_1^2\psi_1+h_2^2\psi_4}\left(\kappa_s^{-1}\begin{bmatrix}\upsilon_1 & \upsilon_2\\ \upsilon_3 & \upsilon_4\end{bmatrix}E_i^{Pre}\begin{bmatrix}\bar{\mu}\\m\end{bmatrix}+\begin{bmatrix}\psi_4(\upsilon_1h_2^2-\upsilon_2h_1h_2) & \psi_1(\upsilon_2h_1^2-\upsilon_1h_1h_2)\\ \psi_4(\upsilon_3h_2^2-\upsilon_4h_1h_2) & \psi_1(\upsilon_4h_1^2-\upsilon_3h_1h_2)\end{bmatrix}E_i^{Pre}\begin{bmatrix}\bar{\mu}\\m\end{bmatrix}\right)$$

$$+\Upsilon P\tilde{s} + \begin{bmatrix}(1-\omega)(1-\phi_x-\theta)\\(1-\omega)(1-\phi_x\delta-\theta)\end{bmatrix}y$$

$$= \begin{bmatrix}\omega\\0\end{bmatrix}\mu_i$$

$$+\frac{1}{\kappa_s^{-1}+h_1^2\psi_1+h_2^2\psi_4}\left(\kappa_s^{-1}\Upsilon E_i^{Pre}\begin{bmatrix}\bar{\mu}\\m\end{bmatrix}+\begin{bmatrix}\psi_4(\upsilon_1h_2^2-\upsilon_2h_1h_2) & \psi_1(\upsilon_2h_1^2-\upsilon_1h_1h_2)\\ \psi_4(\upsilon_3h_2^2-\upsilon_4h_1h_2) & \psi_1(\upsilon_4h_1^2-\upsilon_3h_1h_2)\end{bmatrix}E_i^{Pre}\begin{bmatrix}\bar{\mu}\\m\end{bmatrix}\right)$$

$$+\Upsilon P\tilde{s} + \begin{bmatrix}(1-\omega)(1-\phi_x-\theta)\\(1-\omega)(1-\phi_x\delta-\theta)\end{bmatrix}y$$

$$= \begin{bmatrix}\omega\\0\end{bmatrix}\mu_i$$

$$+\frac{\kappa_s^{-1}}{\kappa_s^{-1}+h_1^2\psi_1+h_2^2\psi_4}\left(\Upsilon E_i^{Pre}\begin{bmatrix}\bar{\mu}\\m\end{bmatrix}\right)$$

$$+\frac{1}{\kappa_s^{-1}+h_1^2\psi_1+h_2^2\psi_4}\begin{bmatrix}\psi_4(\upsilon_1h_2^2-\upsilon_2h_1h_2) & \psi_1(\upsilon_2h_1^2-\upsilon_1h_1h_2)\\ \psi_4(\upsilon_3h_2^2-\upsilon_4h_1h_2) & \psi_1(\upsilon_4h_1^2-\upsilon_3h_1h_2)\end{bmatrix}E_i^{Pre}\begin{bmatrix}\bar{\mu}\\m\end{bmatrix}$$

$$+\Upsilon P\tilde{s} + \begin{bmatrix}(1-\omega)(1-\phi_x-\theta)\\(1-\omega)(1-\phi_x\delta-\theta)\end{bmatrix}y$$

$$= \begin{bmatrix}\omega\\0\end{bmatrix}\mu_i$$

$$+\frac{\kappa_s^{-1}}{\kappa_s^{-1}+h_1^2\psi_1+h_2^2\psi_4}\left(\Upsilon E_i^{Pre}\begin{bmatrix}\bar{\mu}\\m\end{bmatrix}+\begin{bmatrix}\omega\\0\end{bmatrix}\mu_i+\begin{bmatrix}(1-\omega)(1-\phi_x-\theta)\\(1-\omega)(1-\phi_x\delta-\theta)\end{bmatrix}y-\begin{bmatrix}\omega\\0\end{bmatrix}\mu_i-\right.$$

$$\left.\begin{bmatrix}(1-\omega)(1-\phi_x-\theta)\\(1-\omega)(1-\phi_x\delta-\theta)\end{bmatrix}y\right)$$

$$+\frac{1}{\kappa_s^{-1}+h_1^2\psi_1+h_2^2\psi_4}\begin{bmatrix}\psi_4(\upsilon_1h_2^2-\upsilon_2h_1h_2) & \psi_1(\upsilon_2h_1^2-\upsilon_1h_1h_2)\\ \psi_4(\upsilon_3h_2^2-\upsilon_4h_1h_2) & \psi_1(\upsilon_4h_1^2-\upsilon_3h_1h_2)\end{bmatrix}E_i^{Pre}\begin{bmatrix}\bar{\mu}\\m\end{bmatrix}$$

$$+\Upsilon P\tilde{s}+\begin{bmatrix}(1-\omega)(1-\phi_x-\theta)\\(1-\omega)(1-\phi_x\delta-\theta)\end{bmatrix}y$$

$$=\frac{h_1^2\psi_1+h_2^2\psi_4}{\kappa_s^{-1}+h_1^2\psi_1+h_2^2\psi_4}\begin{bmatrix}\omega\\0\end{bmatrix}\mu_i+\left(\frac{\kappa_s^{-1}}{\kappa_s^{-1}+h_1^2\psi_1+h_2^2\psi_4}\right)E_i^{Pre}\begin{bmatrix}\bar{p}\\\bar{E}[\bar{p}]\end{bmatrix}$$

$$+\frac{1}{\kappa_s^{-1}+h_1^2\psi_1+h_2^2\psi_4}\begin{bmatrix}\psi_4(v_1h_2^2-v_2h_1h_2) & \psi_1(v_2h_1^2-v_1h_1h_2)\\\psi_4(v_3h_2^2-v_4h_1h_2) & \psi_1(v_4h_1^2-v_3h_1h_2)\end{bmatrix}E_i^{Pre}\begin{bmatrix}\bar{\mu}\\m\end{bmatrix}$$

$$+\Upsilon P\tilde{s}+\frac{h_1^2\psi_1+h_2^2\psi_4}{\kappa_s^{-1}+h_1^2\psi_1+h_2^2\psi_4}\begin{bmatrix}(1-\omega)(1-\phi_x-\theta)\\(1-\omega)(1-\phi_x\delta-\theta)\end{bmatrix}y$$

$$=\left(\frac{\kappa_s^{-1}}{\kappa_s^{-1}+h_1^2\psi_1+h_2^2\psi_4}\right)E_i^{Pre}\begin{bmatrix}\bar{p}\\\bar{E}[\bar{p}]\end{bmatrix}$$

$$\frac{h_1^2\psi_1+h_2^2\psi_4}{\kappa_s^{-1}+h_1^2\psi_1+h_2^2\psi_4}\begin{bmatrix}\omega\\0\end{bmatrix}\mu_i+\frac{1}{\kappa_s^{-1}+h_1^2\psi_1+h_2^2\psi_4}\begin{bmatrix}\psi_4(v_1h_2^2-v_2h_1h_2) & \psi_1(v_2h_1^2-v_1h_1h_2)\\\psi_4(v_3h_2^2-v_4h_1h_2) & \psi_1(v_4h_1^2-v_3h_1h_2)\end{bmatrix}E_i^{Pre}\begin{bmatrix}\bar{\mu}\\m\end{bmatrix}$$

$$+\Upsilon P\tilde{s}+\frac{h_1^2\psi_1+h_2^2\psi_4}{\kappa_s^{-1}+h_1^2\psi_1+h_2^2\psi_4}\begin{bmatrix}(1-\omega)(1-\phi_x-\theta)\\(1-\omega)(1-\phi_x\delta-\theta)\end{bmatrix}y \quad \text{(D.37)}$$

Similar to equation (23) for the basic noisy-information model, equation (D.37) relates posterior beliefs on $\bar{p}$ to prior beliefs on $\bar{p}$, public signal y and the new information in the signal provided in a treatment ($\tilde{s}$). The coefficient on the prior beliefs is governed by the gain of the Kalman filter. At the same time, equation (D.37) features new terms that depend on the "long-run" prior μ_i and pre-treatment beliefs on fundamentals $E_i^{Pre}[\bar{\mu}]$ and $E_i^{Pre}[m]$. Because we do not observe these terms and $E_i^{Pre}[\bar{\mu}], E_i^{Pre}[m], \mu_i$ are all correlated with $E_i^{Pre}[\bar{p}]$ and $E_i^{Pre}[\bar{E}[\bar{p}]]$, we may have biased estimates of the slope coefficient on prior beliefs about $\bar{p}$ when we regress posterior beliefs on prior beliefs and a constant (equation (1) in the paper).

In what follows, we sign and quantify these potential biases. We consider first signal B (that is, we provide firms with information on $E_i^{Pre}[\bar{p}]$) and then we consider signal C (that is, we provide firms with information on $E_i^{Post}[\bar{E}[\bar{p}]]$).

Signal B

Given equation (D.20), one can show that in this case $h_1=\omega+(1-\omega)\theta$ and $h_2=(1-\omega)(\phi_x\delta)$. Given expressions for $v_1,\ldots,v_4$ in equation (D.29), one can find that

$$B_{21}\equiv v_3h_2^2-v_4h_1h_2=(\omega+(1-\omega)\theta)(1-\omega)^2(\phi_x\delta)^2-(1-\omega)^2\phi_x^2\delta^2(\omega+(1-\omega)\theta)=0 \quad \text{(D.38)}$$

$$B_{22}\equiv v_4h_1^2-v_3h_1h_2=(1-\omega)\phi_x\delta[\omega+(1-\omega)\theta]^2-(\omega+(1-\omega)\theta)^2(1-\omega)(\phi_x\delta)=0 \quad \text{(D.39)}$$

Hence, the relationship between $E_i^{Post}[\bar{E}[\bar{p}]]$ and $E_i^{Pre}[\bar{E}[\bar{p}]]$ in equation (D.37) is not influenced by $E_i^{Pre}[\bar{\mu}], E_i^{Pre}[m], \mu_i$ (the coefficient on μ_i is equal to zero) and the estimated slope is not biased.

One can also show that

$$B_{11}\equiv v_1h_2^2-v_2h_1h_2=-(1-\omega)^2\phi_x^2\delta[\theta(1-\delta)(1-\omega)+\omega]\leq 0 \quad \text{(D.40)}$$

$$B_{12}\equiv v_2h_1^2-v_1h_1h_2=(1-\omega)\phi_x(\omega+(1-\omega)\theta)[\omega+(1-\omega)\theta(1-\delta)]\geq 0 \quad \text{(D.41)}$$

Because $E_i^{Pre}[\bar{p}]$ is positively correlated with $E_i^{Pre}[\bar{\mu}]$ and $E_i^{Pre}[m]$ (see equation (D.19)), the sign of the bias depends on the relative strength of two opposing forces: omitted $E_i^{Pre}[\bar{\mu}]$ biases the estimated slope down, while omitted $E_i^{Pre}[m]$ biases the estimated slope up. Also notice that μ_i is positively correlated with $E_i^{Pre}[\bar{p}]$ (thus biasing the estimated slope up) and $E_i^{Pre}[\bar{\mu}]$ (see equation (D.9)), which makes signing the bias a more challenging (in terms of algebra) task. If $\omega = 0$, $\upsilon_1 h_2^2 - \upsilon_2 h_1 h_2 = 0$ and $\upsilon_2 h_1^2 - \upsilon_1 h_1 h_2 = 0$ and the coefficient on μ_i is zero so that the estimated slope is unbiased in this case.

Because signals ς_i and x_i and "long-run" prior μ_i are uncorrelated, we can separate the bias-signing exercise into two parts: one due to ς_i and μ_i and one due to x_i. Using equation (D.9), one can find that the omitted term due to ς_i and μ_i in equation (D.37) is

$$O_i^{(B1;FO)} \equiv \left[\frac{h_1^2\psi_1 + h_2^2\psi_4}{\kappa_S^{-1} + h_1^2\psi_1 + h_2^2\psi_4}\omega + \frac{\psi_4 B_{11}}{\kappa_S^{-1} + h_1^2\psi_1 + h_2^2\psi_4}\omega'\right]\mu_i + \frac{\psi_4 B_{11}}{\kappa_S^{-1} + h_1^2\psi_1 + h_2^2\psi_4}(1-\omega')\varsigma_i \tag{D.42}$$

From equation (D.19), we know that $E_i[\bar{p}]$ loads on ς_i and μ_i with weights $(1-\omega)\theta(1-\omega')$ and $(\omega + (1-\omega)\theta\omega')$ respectively. Hence, the covariance of $E_i[\bar{p}]$ and $O_i^{(B1;FO)}$ is

$$Cov\left(E_i^{Pre}[\bar{p}], O_i^{(B1;FO)}\right) = \left[\frac{h_1^2\psi_1 + h_2^2\psi_4}{\kappa_S^{-1} + h_1^2\psi_1 + h_2^2\psi_4}\omega + \frac{\psi_4 B_{11}}{\kappa_S^{-1} + h_1^2\psi_1 + h_2^2\psi_4}\omega'\right](\omega + (1-\omega)\theta\omega')\kappa_\mu^{-1}$$
$$+ \frac{\psi_4 B_{11}}{\kappa_S^{-1} + h_1^2\psi_1 + h_2^2\psi_4}(1-\omega')^2(1-\omega)\theta\kappa_\varsigma^{-1} \tag{D.43}$$

Given that $E_i[m]$ loads on x_i with weight δ,[27] the omitted term due to $E_i[m]$ is

$$O_i^{(B2;FO)} \equiv \frac{\psi_1 B_{12}\delta}{\kappa_S^{-1} + h_1^2\psi_1 + h_2^2\psi_4}x_i \tag{D.44}$$

We also know from equation (D.19), that $E_i^{Pre}[\bar{p}]$ loads on x_i with weight $(1-\omega)\phi_x\delta$. The covariance of $O_i^{(B2:FO)}$ and $E_i^{Pre}[\bar{p}]$ is then

$$Cov\left(E_i^{Pre}[\bar{p}], O_i^{(B2;FO)}\right) = \frac{\psi_1 B_{12}(1-\omega)\phi_x\delta^2}{\kappa_S^{-1} + h_1^2\psi_1 + h_2^2\psi_4}\kappa_x^{-1} > 0 \tag{D.45}$$

We use equation (D.19) to compute the cross-sectional variance of $E_i^{Pre}[\bar{p}]$. By combining equations (D.19) and (D.43), we compute the bias due to "long-run" priors via μ_i and ς_i

$$bias_1^{(B:FO)} \equiv \frac{Cov\left(E_i^{Pre}[\bar{p}], O_i^{(B1;FO)}\right)}{Var\left(E_i^{Pre}[\bar{p}]\right)} \lesseqgtr 0 \tag{D.46}$$

By combining equations (D.19) and (D.45), we compute the bias due to long-run priors via x_i:

$$bias_2^{(B:FO)} \equiv \frac{Cov\left(E_i^{Pre}[\bar{p}], O_i^{(B2;FO)}\right)}{Var\left(E_i^{Pre}[\bar{p}]\right)} > 0 \tag{D.47}$$

Equations (D.46) and (D.47) demonstrate that the sign of the bias depends on parameter values.

Importantly, this analysis suggests that the estimated slope coefficient on the priors in specification (1) for first-order expectations do not have a one-to-one mapping to the gain of the Kalman filter. Instead, it is the gain plus a bias:

$$slope_B^{FO} = \left(\frac{\kappa_B^{-1}}{\kappa_B^{-1} + h_1^2\psi_1 + h_2^2\psi_4}\right) + bias_1^{(B:FO)} + bias_2^{(B:FO)}, \tag{D.48}$$

which potentially provides with another moment to match. In contrast, the estimated slope coefficient on the priors in specification (1) for higher-order expectations continues to have a one-to-one mapping to the gain of the Kalman filter:

$$slope_B^{HO} = \left(\frac{\kappa_B^{-1}}{\kappa_B^{-1} + h_1^2\psi_1 + h_2^2\psi_4}\right). \tag{D.49}$$

---

[27] Recall that we do not use y in this context because y is a common signal and so it does not generate cross-sectional variation.

Signal C

Using equation (D.21), one can show that in this case $h_1 = \omega + (1-\omega)\theta$ and $h_2 = (1-\omega)(\phi_x\delta^2)$. Given the logic of the previous subsection, one can find that

$$B_{21} \equiv \upsilon_3 h_2^2 - \upsilon_4 h_1 h_2 = (\omega + (1-\omega)\theta)(1-\omega)^2(\phi_x\delta^2)^2 - (1-\omega)^2\phi_x^2\delta^3(\omega + (1-\omega)\theta)$$
$$= -(1-\omega)^2\phi_x^2\delta^3(\omega + (1-\omega)\theta)(1-\delta) \le 0 \quad \text{(D.50)}$$
$$B_{22} \equiv \upsilon_4 h_1^2 - \upsilon_3 h_1 h_2 = (1-\omega)\phi_x\delta[\omega + (1-\omega)\theta]^2 - (\omega + (1-\omega)\theta)^2(1-\omega)(\phi_x\delta^2)$$
$$= (1-\omega)\phi_x\delta[\omega + (1-\omega)\theta]^2(1-\delta) \ge 0 \quad \text{(D.51)}$$

These results suggest that in contrast to the case of signal B, estimated slope in regression (1) using higher-order expectations may be biased.

Following the logic of our derivations for signal B, we find that

$$B_{11} \equiv \upsilon_1 h_2^2 - \upsilon_2 h_1 h_2 = -(1-\omega)^2\phi_x^2\delta^2[\theta(1-\delta^2)(1-\omega) + \omega] \le 0 \quad \text{(D.52)}$$
$$B_{12} \equiv \upsilon_2 h_1^2 - \upsilon_1 h_1 h_2 = (1-\omega)\phi_x(\omega + (1-\omega)\theta)[\omega + (1-\omega)\theta(1-\delta^2)] \ge 0 \quad \text{(D.53)}$$

These results suggest that, for the slope of regression (1) using first-order beliefs, the signs of the biases for signal C are similar to the signs of the biases for signal B. Again, if $\omega = 0$, $B_{11} = 0$, $B_{12} = 0$, $B_{21} = 0$ and $B_{22} = 0$ and the coefficient on μ_i is zero so that the estimated slope is unbiased in regression (1) using either first-order beliefs or higher-order beliefs.

Note that, for first-order beliefs, the structure of updating for signal C is identical to the structure of updating for signal B and the differences are only the precision of the signal and slightly different expressions for B_{11} and B_{12}. Hence, the expression for $bias^{(C:FO)}$ (i.e., the bias in the regression (1) for signal C using first-order beliefs) is similar to the expression for $bias^{(B:FO)}$ but we plug in different values of B_{11}, B_{12}, κ_s. As a result, we can't sign the $bias^{(C:FO)}$ without specifying the parameters.

For the higher-order beliefs, we need to derive new expressions to quantify the bias. One can see that the omitted term for to ς_i and μ_i in equation (D.37) for signal C is

$$O_i^{(C1;HO)} \equiv \frac{\psi_4 B_{21}}{\kappa_s^{-1} + h_1^2\psi_1 + h_2^2\psi_4}\omega'\mu_i + \frac{\psi_4 B_{21}}{\kappa_s^{-1} + h_1^2\psi_1 + h_2^2\psi_4}(1-\omega')\varsigma_i \quad \text{(D.54)}$$

From equation (D.21), we know that $E_i^{Pre}[\bar{E}[\bar{p}]]$ loads on ς_i and μ_i with weights $(\omega + (1-\omega)\theta)(1-\omega')$ and $(\omega + (1-\omega)\theta)\omega'$. Hence, the covariance of $E_i^{Pre}[\bar{E}[\bar{p}]]$ and $O_i^{(C1;HO)}$ is

$$Cov\left(E_i^{Pre}[\bar{E}[\bar{p}]], O_i^{(C1;HO)}\right) = \frac{\psi_4 B_{21}}{\kappa_s^{-1} + h_1^2\psi_1 + h_2^2\psi_4}(\omega')^2(\omega + (1-\omega)\theta)\kappa_\mu^{-1}$$
$$+ \frac{\psi_4 B_{21}}{\kappa_s^{-1} + h_1^2\psi_1 + h_2^2\psi_4}(1-\omega')^2(\omega + (1-\omega)\theta)\kappa_\varsigma^{-1} < 0 \quad \text{(D.55)}$$

Given that $E_i[m]$ loads on x_i with weight δ, the omitted term due to $E_i[m]$ is

$$O_i^{(C2;HO)} \equiv \frac{\psi_1 B_{22}\delta}{\kappa_s^{-1} + h_1^2\psi_1 + h_2^2\psi_4}x_i \quad \text{(D.56)}$$

We also know from equation (D.21), that $E_i^{Pre}[\bar{E}[\bar{p}]]$ loads on x_i with weight $(1-\omega)\phi_x\delta^2$. The covariance of $O_i^{(C2;HO)}$ and $E_i^{Pre}[\bar{E}[\bar{p}]]$ is then

$$Cov\left(E_i^{Pre}[\bar{E}[\bar{p}]], O_i^{(B2;FO)}\right) = \frac{\psi_1 B_{22}(1-\omega)\phi_x\delta^3}{\kappa_s^{-1} + h_1^2\psi_1 + h_2^2\psi_4}\kappa_x^{-1} > 0 \quad \text{(D.57)}$$

We use equation (D.21) to compute the cross-sectional variance of $E_i^{Pre}\left[\bar{E}[\bar{p}]\right]$. By combining equations (D.55) and (D.21), we compute the bias due to long-run priors via μ_i and ς_i

$$bias_1^{(C:HO)} \equiv \frac{Cov\left(E_i^{Pre}[\bar{E}[\bar{p}]], O_i^{(C1;HO)}\right)}{Var\left(E_i^{Pre}[\bar{E}[\bar{p}]]\right)} < 0 \tag{D.58}$$

By combining equations (D.55) and (D.56), we compute the bias due to long-run priors via x_i:

$$bias_2^{(C:HO)} \equiv \frac{Cov\left(E_i^{Pre}[\bar{E}[\bar{p}]], O_i^{(C2;HO)}\right)}{Var\left(E_i^{Pre}[\bar{E}[\bar{p}]]\right)} > 0 \tag{D.59}$$

Hence,

$$bias^{(C:HO)} = bias_1^{(C:HO)} + bias_2^{(C:HO)} \lesseqgtr 0 \tag{D.60}$$

While the bias $bias^{(C:HO)}$ is ambiguous, one can show that the expression for the bias can be simplified to

$$bias^{(C:HO)} = \frac{(1-\omega)^2\phi_x^2\delta^3(\omega+(1-\omega)\theta)^2(1-\delta)}{\kappa_C^{-1}+h_1^2\psi_1+h_2^2\psi_4}\kappa^{-1}\left[(1-\omega')^2(1-\delta^2)\kappa_\varsigma^{-1}-\delta^2(\omega')^2\kappa_\mu^{-1}\right]$$

and hence the bias is negative if $(\omega')^2\delta^2\kappa_\mu^{-1} > (1-\omega')^2(1-\delta^2)\kappa_\varsigma^{-1}$, that is, the dispersion of the long-run priors (κ_μ^{-1}, "disagreement") has to be sufficiently large relative to the dispersion in private signals (κ_ς^{-1}, also equal to uncertainty) about the average value of the about the average value of the long-run bias $\bar{\mu}$.

In summary, we know that the estimated slopes for treatment C are biased:

$$slope_C^{FO} = \left(\frac{\kappa_C^{-1}}{\kappa_C^{-1}+h_1^2\psi_1+h_2^2\psi_4}\right) + bias_1^{(C:FO)} + bias_2^{(C:FO)}, \tag{D.61}$$

$$slope_C^{HO} = \left(\frac{\kappa_C^{-1}}{\kappa_C^{-1}+h_1^2\psi_1+h_2^2\psi_4}\right) + bias_1^{(C:HO)} + bias^{(C:HO)}. \tag{D.62}$$

Analysis of biases in the estimated slopes

This section is aimed to better understand the nature of the biases. To keep the discussion focused, we will omit terms that are not central to the main points. For example, we can "simplify" equation (D.28) to

$$E_i\begin{bmatrix}\bar{p}\\ \bar{E}[\bar{p}]\end{bmatrix} = \begin{bmatrix}\upsilon_1 & \upsilon_2\\ \upsilon_3 & \upsilon_4\end{bmatrix} E_i\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + \{other\ terms\} \tag{D.63}$$

where as in (D.29) we have

$$\Upsilon \equiv \begin{bmatrix}\upsilon_1 & \upsilon_2\\ \upsilon_3 & \upsilon_4\end{bmatrix} \equiv \begin{bmatrix}(1-\omega)\theta & (1-\omega)\phi_x\\ \omega+(1-\omega)\theta & (1-\omega)\phi_x\delta\end{bmatrix}$$

We can also write signals B and C as

$$\tilde{s}_B = \bar{E}[\bar{p}] + noise = h_1^B\bar{\mu} + h_2^B m + noise = \{\omega+(1-\omega)\theta\}\bar{\mu} + \{(1-\omega)\phi_x\delta\}m + noise \tag{D.64}$$

$$\tilde{s}_C = \bar{E}^2[\bar{p}] + noise = h_1^C\bar{\mu} + h_2^C m + noise = \{\omega+(1-\omega)\theta\}\bar{\mu} + \{(1-\omega)\phi_x\delta^2\}m + noise \tag{D.65}$$

Note that, by taking an average of first-order expectations, signal B effectively aggregates first-order expectations of managers into a second-order expectation. In a similar spirit, signal C effectively aggregates second-order expectations of managers into a third-order expectation.

We can also focus on the "bias" terms in equation (D.37):

$$\frac{1}{\kappa_S^{-1}+h_1^2\psi_1+h_2^2\psi_4}\begin{bmatrix}\psi_4 h_2(\upsilon_1 h_2-\upsilon_2 h_1) & \psi_1 h_1(\upsilon_2 h_1-\upsilon_1 h_2)\\ \psi_4 h_2(\upsilon_3 h_2-\upsilon_4 h_1) & \psi_1 h_1(\upsilon_4 h_1-\upsilon_3 h_2)\end{bmatrix} E_i^{Pre}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} \tag{D.66}$$

Given that $v_2 h_1 \geq v_1 h_2$ and $v_4 h_1 \geq v_3 h_2$, we know that the first column of the matrix multiplying $E_i^{Pre}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix}$ is (weakly) negative and the second column of the matrix is (weakly) positive. Furthermore, from our discussion above, we know that $v_4 h_1^B = v_3 h_2^B$ for the response of higher-order expectations to signal B. For all other expectations and signals we have strict inequalities. Inequalities $v_2 h_1 \geq v_1 h_2$ and $v_4 h_1 \geq v_3 h_2$ can be rearranged as $\frac{v_2}{v_1} \geq \frac{h_2}{h_1}$ and $\frac{v_4}{v_3} \geq \frac{h_2}{h_1}$, or in words

$$\frac{weig\quad of\ m\ in\ FO(HO)\ expectation\ \bar{p}}{weight\ of\ \bar{\mu}\ in\ FO(HO)\ expectation\ \bar{p}} \geq \frac{weight\ of\ m\ in\ signal\ s}{weigh\quad of\ \bar{\mu}\ in\ signal\ s} \tag{D.67}$$

and this weak inequality turns into equality only for the response of higher-order expectations to signal B. More specifically, equation (D.63) shows that expectations of the price level $\bar{p}$ are linear combinations of two "fundamentals" $\bar{\mu}$ and m with the corresponding weights v_1 and v_2 for first-order (FO) expectations and v_3 and v_4 for higher-order (HO) expectations. The signals in (D.64)-(D.65) are also linear combinations of these two "fundamentals" but the weights on the "fundamentals" are different. The relative weight on m in expectations of the price level $\bar{p}$ is weakly greater than the relative weight on m in the signals. We have the equality of the relative weights only when we have the response of second-order expectations in response to signal B. To appreciate the significance of this equality, we let K be the factor of proportionality for the weights (i.e., $[v_3 \quad v_4] = K[h_1^B \quad h_2^B]$) and examine whether we can express observed posterior second-order expectations of $\bar{p}$ as a function of observed prior second-order expectations of $\bar{p}$ and signal B:

$$\begin{aligned}
E_i^{post}\left[\bar{E}[\bar{p}]\right] &= [v_3 \quad v_4] E_i^{Post}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + \{other\ terms\}\\
&= K[h_1^B \quad h_2^B]\left\{(I_{2\times 2} - PH)E_i^{Pre}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + P\tilde{s}^B\right\} + \{other\ terms\}\\
&= KH\left\{(I_{2\times 2} - PH)E_i^{Pre}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix}\right\} + KHP\tilde{s}^B\\
&= KH(I_{2\times 2} - PH)E_i^{Pre}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + KHP\tilde{s}^B\\
&= K(H - HPH)E_i^{Pre}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + KHP\tilde{s}^B\\
&= K(I_{1\times 1} - HP)HE_i^{Pre}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + KHP\tilde{s}^B\\
&= (I_{1\times 1} - HP)KHE_i^{Pre}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + KHP\tilde{s}^B\\
&= (I_{1\times 1} - HP)[v_3 \quad v_4]E_i^{Pre}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + KHP\tilde{s}^B\\
&= (I_{1\times 1} - HP)E_i^{prior}\left[\bar{E}[\bar{p}]\right] + KHP\tilde{s}^B
\end{aligned} \tag{D.68}$$

This derivation shows that because the observable $E_i\left[\bar{E}[\bar{p}]\right]$ has the same relative weights on $E_i\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix}$ as the signal B on $\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix}$, we can reduce the two-dimensional state-space of $E_i\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix}$ to one dimension; that is, it is "as if" we can construct a synthetic state for $E_i\left[\bar{E}[\bar{p}]\right]$ that collapses two state variables in one. Notice that this case applies only when we feed a second-order expectations (signal B) to second-order expectations. In other cases, we feed a higher-order expectation to measure the reaction in terms of a lower-order expectations. For example, signal C feeds a third-order expectation to measure the reaction of first-order expectations $E_i[\bar{p}]$ or second-order expectations $E_i\left[\bar{E}[\bar{p}]\right]$. In a similar spirit, we have a discrepancy in relative weights when we feed a second-order expectation (signal B) to measure the response of first-order expectations $E_i[\bar{p}]$.

What are the consequences of such a discrepancy? Consider e.g. the response of higher-order expectations for signal C. Given $\frac{v_4}{v_3} > \frac{h_2}{h_1}$, we express $[v_3 \quad v_4] = K[h_1^C \quad h_2^C] + [0 \quad R] = K[h_1^C \quad h_2^C] + \boldsymbol{R} = KH + \boldsymbol{R}$ and $R > 0$ where as before K is a factor of proportionality. Then we can express the posterior second-order expectation for observable $E_i\left[\bar{E}[\bar{p}]\right]$ as:

$$
\begin{aligned}
E_i^{post}\left[\bar{E}[\bar{p}]\right] &= [v_3 \quad v_4] E_i^{Post}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + \{other\ terms\} \\
&= \{K[h_1^C \quad h_2^C] + \boldsymbol{R}\}\left\{(I_{2\times 2} - PH)E_i^{Prior}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + P\tilde{s}^C\right\} \\
&= K[h_1^C \quad h_2^C]\left\{(I_{2\times 2} - PH)E_i^{Prior}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + P\tilde{s}^C\right\} + \boldsymbol{R}\left\{(I_{2\times 2} - PH)E_i^{Prior}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + P\tilde{s}^B\right\} \\
&= K\left\{H(I_{2\times 2} - PH)E_i^{Prior}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + HP\tilde{s}^C\right\} + \boldsymbol{R}\left\{(I_{2\times 2} - PH)E_i^{Prior}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + P\tilde{s}^B\right\} \\
&= K\left\{(I_{1\times 1} - HP)HE_i^{Prior}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + HP\tilde{s}^C\right\} + \boldsymbol{R}\left\{(I_{2\times 2} - PH)E_i^{Prior}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + P\tilde{s}^B\right\} \\
&= \left\{(I_{1\times 1} - HP)KHE_i^{Prior}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + HP\tilde{s}^C\right\} + \boldsymbol{R}\left\{(I_{2\times 2} - PH)E_i^{Prior}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + P\tilde{s}^B\right\} \\
&= \left\{(I_{1\times 1} - HP)(KH + \boldsymbol{R} - \boldsymbol{R})E_i^{Prior}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + HP\tilde{s}^C\right\} + \boldsymbol{R}\left\{(I_{2\times 2} - PH)E_i^{Prior}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + P\tilde{s}^B\right\} \\
&= \left\{(I_{1\times 1} - HP)([v_3 \quad v_4] - \boldsymbol{R})E_i^{Prior}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + HP\tilde{s}^C\right\} + \boldsymbol{R}\left\{(I_{2\times 2} - PH)E_i^{Prior}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + P\tilde{s}^B\right\} \\
&= \left\{(I_{1\times 1} - HP)[v_3 \quad v_4]E_i^{Prior}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} - (I_{1\times 1} - PH)\boldsymbol{R}E_i^{Prior}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + HP\tilde{s}^C\right\} \\
&\quad + \boldsymbol{R}\left\{(I_{2\times 2} - PH)E_i^{Prior}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + P\tilde{s}^B\right\} \\
&= \left\{(I_{1\times 1} - HP)E_i^{Prior}\left[\bar{E}[\bar{p}]\right] - (I_{1\times 1} - PH)\boldsymbol{R}E_i^{Prior}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + HP\tilde{s}^C\right\} + \boldsymbol{R}\left\{(I_{2\times 2} - PH)E_i^{Prior}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} + P\tilde{s}^B\right\} \\
&= (I_{1\times 1} - HP)E_i^{Prior}\left[\bar{E}[\bar{p}]\right] + \{KHP + \boldsymbol{R}P\}\tilde{s}^C + \{\boldsymbol{R}(I_{2\times 2} - PH) - (I_{1\times 1} - PH)\boldsymbol{R}\}E_i^{Prior}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} \\
&= (I_{1\times 1} - HP)E_i^{Prior}\left[\bar{E}[\bar{p}]\right] + \{KH + \boldsymbol{R}\}P\tilde{s}^C + \{\boldsymbol{R}(I_{2\times 2} - PH) - (I_{1\times 1} - PH)\boldsymbol{R}\}E_i^{Prior}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} \\
&= (I_{1\times 1} - HP)E_i^{Prior}\left[\bar{E}[\bar{p}]\right] + \{KH + \boldsymbol{R}\}P\tilde{s}^C \\
&\quad + \left\{[0 \quad R]\begin{bmatrix}1 - p_1h_1^C & -p_1h_2^C\\ -p_2h_1^C & 1 - p_2h_2^C\end{bmatrix} - \left(1 - p_1h_1^C - p_2h_2^C\right)[0 \quad R]\right\}E_i^{Prior}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} \\
&= (I_{1\times 1} - HP)E_i^{Prior}\left[\bar{E}[\bar{p}]\right] + \{KH + \boldsymbol{R}\}P\tilde{s}^C \\
&\quad + \left[R\left(-p_1h_2^C\right) \quad R\left(1 - p_2h_2^C\right) - R(1 - p_1h_1^C - p_2h_2^C)\right]E_i^{Prior}\begin{bmatrix}\bar{\mu}\\ m\end{bmatrix} \\
&= (I_{1\times 1} - HP)E_i^{Prior}\left[\bar{E}[\bar{p}]\right] + \{KH + \boldsymbol{R}\}P\tilde{s}^C - Rp_1h_2^CE_i^{Prior}[\bar{\mu}] + Rp_1h_1^CE_i^{Prior}[m] \qquad \text{(D.69)}
\end{aligned}
$$

Intuitively, we study the reaction of lower-order expectations (which have a relatively high weight on m) to a signal about higher-order expectations (which have a lower relative weight on m). Because of these differences in the weights, the posterior observable $E_i^{post}\left[\bar{E}[\bar{p}]\right]$ cannot be expressed only as a function of the prior observable $E_i^{Prior}\left[\bar{E}[\bar{p}]\right]$ and signal $\tilde{s}^C$ and so regression $E_i^{post}\left[\bar{E}[\bar{p}]\right]$ on $E_i^{Prior}\left[\bar{E}[\bar{p}]\right]$ and $\tilde{s}^C$ is misspecified. Because $\rho\left(E_i^{Prior}\left[\bar{E}[\bar{p}]\right], E_i^{Prior}[\bar{\mu}]\right) > 0$ and $\rho\left(E_i^{Prior}\left[\bar{E}[\bar{p}]\right], E_i^{Prior}[m]\right) > 0$, it follows that the regressor $E_i^{Prior}\left[\bar{E}[\bar{p}]\right]$ is negatively correlated with omitted term $\left\{-Rp_1h_2^CE_i^{Prior}[\bar{\mu}]\right\}$ thus creating a negative bias and positively correlated with omitted term $\left\{Rp_1h_1^CE_i^{Prior}[m]\right\}$ thus creating a positive bias. Equation (D.69) makes clear that to have a large negative bias in the estimated slope of specification (1), one needs large variation in long-run priors μ_i and/or a large value of $Rp_1 h_2^C$. This sign pattern of biases also holds for the response of first-order expectations to signals B and C.

Intuitively, as we increase the order of expectations, the weight on m is shrinking because, while thinking about the expectations of others, managers put increasingly more weight on the common public signal y. On the other hand, the weight on $\bar{\mu}$ does not shrink because there is no common public signal about the average prior $\bar{\mu}$. Hence, as we increase the order of expectations in signals, signals are increasingly skewed (in relative terms) toward $\bar{\mu}$. As a result, when we extrapolate the posterior beliefs for a given order of expectations from the prior beliefs for that order in response to a signal that measures a higher order of expectations, this extrapolation overstate the contribution due to $\bar{\mu}$ and understate the contribution due to m.

D. Identification and estimation of structural parameters

We have six structural parameters in the model with long-run priors: κ_x, κ_y, κ_μ, κ_ς, ω, ω'. In addition, there is strategic complementarity parameter α that we identify using external data (as in Afrouzi (2018)).

On the other hand, we have four independent moments from the data (equations (D.24)-(D.27)) that we used before in calibrating the basic noisy-information model: two moments on disagreement $Var\left[E_i[\bar{p}]\right]$ and $Var\left[E_i\left[\bar{E}[\bar{p}]\right]\right]$; two moments on uncertainty $\Omega_{\{E_i[\bar{p}]|y\}}$ and $\Omega_{\{E_i[\bar{E}[\bar{p}]]|y\}}$.[28] In addition, we have now more information in slope coefficients estimated in specification (1) for signals B and C: additional moments given by (D.48), (D.49), (D.61), (D.62). At the same time, we note that using the estimated slopes potentially "consumes" two degrees of freedom since we now need to calibrate κ_C and κ_B. We can impose a restriction that $\kappa_B = \kappa_C$. Also note that equation (D.49) does not bring in an independent moment in general because it can be expressed as a function of other moments (i.e., we have over-identification here) as long as there is no bias.

The system is still under-identified (the order condition is not satisfied) and so we need to introduce external information (more moments) and/or impose additional constraints. For example, we make the following assumption (A1): $\gamma = \gamma'$. Additionally, we use data from CGK (*another* survey), we can identify ω from regressing short-term (one-year-ahead) inflation expectations on long-run (5-year-ahead) inflation expectations, which proxies long-run priors μ_i. This regression corresponds to equation (41) in the paper. This regression is valid under the assumption that long-run priors are not affected by short-run ("business cycle") fluctuations. This assumption corresponds to the assumption in the model that long-run priors μ_i are uncorrelated with other shocks in the model. We find that $\hat{\omega} = 0.79$. If we had long-run and short-run higher-order expectations for inflation, we would have been able to recover ω' from the slope coefficient in a similar regression using higher-order expectations, thus giving us another moment.

[28] $Cov\left(E_i[\bar{p}], E_i\left[\bar{E}[\bar{p}]\right]\right)$ is not an independent moment because this covariance may be expressed as a product of $Var\left[E_i[\bar{p}]\right] - \Omega_{\{E_i[\bar{p}]|y\}}$ and $Var\left[E_i\left[\bar{E}[\bar{p}]\right]\right] - \Omega_{\{E_i[\bar{E}[\bar{p}]]|y\}}$.

Since we do not have enough information to identify all structural parameters, we are interested in whether the noisy-information model with long-run priors can in principle reproduce the patterns observed in the data. Specifically, we do a moment matching exercises where empirical moments are $\widehat{Var[E_i[\bar{p}]]}$, $\widehat{Var\left[E_i\left[\bar{E}[\bar{p}]\right]\right]}$, $\widehat{\Omega}_{\{E_i[\bar{p}]|y\}}$, $\widehat{\Omega}_{\{E_i[\bar{E}[\bar{p}]]|y\}}$, $\widehat{slope}_B^{FO}$, $\widehat{slope}_B^{HO}$, $\widehat{slope}_C^{FO}$, $\widehat{slope}_C^{FO}$ and the theoretical predictions for these moments are given by (D.23)-(D.27) and (D.48), (D.49), (D.61), (D.62). We find that if $\kappa_B = \kappa_C$, the noisy-information model with long-run can qualitatively reproduce facts 1 through 6 but it will struggle with matching the difference between $\widehat{slope}_B^{FO} \approx \widehat{slope}_B^{HO} \approx 0.5$ and $\widehat{slope}_C^{FO} \approx \widehat{slope}_C^{FO} \approx 0.1$. In other words, the model can generate $slope_B^{FO} \approx slope_B^{HO} > slope_C^{FO} \approx slope_C^{FO}$ but the difference between $slope_B^{FO}$ and $slope_C^{FO}$ would be smaller than the difference between $\widehat{slope}_B^{FO}$ and $\widehat{slope}_C^{FO}$. This issue can be fixed if we allow for small difference in the precision of signals B and C, that is, $\kappa_C \neq \kappa_B$. For example, if $\kappa_C = 2\kappa_B$ (and we keep assumption (A1)), the model can hit $\widehat{slope}_B^{FO} \approx \widehat{slope}_B^{HO} \approx 0.5$ and $\widehat{slope}_C^{FO} \approx \widehat{slope}_C^{FO} \approx 0.1$. Note that allowing $\kappa_C \neq \kappa_B$ can also help the basic noisy-information model to match $\widehat{slope}_B^{FO} \approx \widehat{slope}_B^{HO} \approx 0.5$ and $\widehat{slope}_C^{FO} \approx \widehat{slope}_C^{FO} \approx 0.1$, but in this case κ_C must be an order of magnitude larger than κ_B. Thus, relative to the basic noisy-information model, the noisy-information model with long-run priors can match the data with modest (rather than radical) variation in the precision of signals in treatments B and C.

Appendix E:
Extension of the basic noisy-information model: Overconfidence

In this extension, we allow managers to hold beliefs about signal precision that differ from the truth. In order to generate the same patterns seen in the data, managers must overestimate the precision of the private signal, κ_x, as in Daniel, Hirshleifer and Subrahmanyam (1998).

Specifically, managers continue to receive public and private signals:

$$x_i = m + \nu_{i,1} \tag{E.1}$$

$$y = m + \varepsilon \tag{E.2}$$

where $\nu_{i,1} \sim N(0, \kappa_x^{-1})$, and $\varepsilon \sim N\left(0, \kappa_y^{-1}\right)$. However, we allow them to overestimate the precision of the private signal such that $E_i[\kappa_x] > \kappa_x$. The manager now overestimates the relative precision of the private signal: $\tilde{\delta} = \frac{E[\kappa_x]}{\kappa_y + E[\kappa_x]} > \frac{\kappa_x}{\kappa_y + \kappa_x} = \delta$. His expectation of m is formed as follows:

$$E_i[m] = \tilde{\delta} x_i + \left(1 - \tilde{\delta}\right) y \tag{E.3}$$

so the agent's perceived level of the fundamental is overly sensitive to the private signal but insufficiently sensitive to the public signal. The pricing strategies follow from the over-precise value of δ: $\tilde{\phi}_x = \frac{(1-\alpha)\tilde{\delta}}{(1-\alpha)\tilde{\delta} + (1-\tilde{\delta})}$ and $\tilde{\phi}_y = \frac{1-\tilde{\delta}}{(1-\alpha)\tilde{\delta} + (1-\tilde{\delta})}$. The aggregate price level is therefore

$$\overline{p} \equiv \int_0^N p_j dj = \tilde{\phi}_y y + \tilde{\phi}_x m. \tag{E.4}$$

Expectations follow in the same manner as in the basic model:

$$E_i[\overline{p}] = \tilde{\phi}_x \tilde{\delta} x_i + \left(1 - \tilde{\phi}_x \tilde{\delta}\right) y \tag{E.5}$$

except that the price response to the private (public) signal is higher (lower) than in the basic noisy-informational model since $\tilde{\phi}_x \tilde{\delta} > \phi_x \delta$. Following the logic of Section 3.1, an individual manager's higher-order expectation is:

$$E_i\left[\overline{E}[\overline{p}]\right] = \tilde{\phi}_x \tilde{\delta}^2 x_i + \left(1 - \tilde{\phi}_x \tilde{\delta}^2\right) y. \tag{E.6}$$

The cross-sectional disagreement in first-order and higher-order expectations depend on the true value of κ_x since this determines the actual distribution of private signals:

$$Var\left[E_i[\bar{p}]\right] = \left(\tilde{\phi}_x \tilde{\delta}\right)^2 \kappa_x^{-1}, \tag{E.7}$$

$$Var\left[E_i\left[\bar{E}[\bar{p}]\right]\right] = \left(\tilde{\phi}_x \tilde{\delta}^2\right)^2 \kappa_x^{-1}. \tag{E.8}$$

We assume that managers know their own prior with certainty and do not consider the dispersion of priors around the mean. Thus, their first-order and higher-order uncertainty about the price level takes the form:

$$\Omega_{\{E_i[\bar{p}]|y\}} = \left(\tilde{\phi}_x \tilde{\delta}\right)^2 E_i[\kappa_x]^{-1}, \tag{E.9}$$

$$\Omega_{\{E_i[\bar{E}[\bar{p}]]|y\}} = \left(\tilde{\phi}_x \tilde{\delta}^2\right)^2 E_i[\kappa_x]^{-1}. \tag{E.10}$$

When agents perceive the private signal as being more precise than the public signal, it follows that $Var\left[E_i[\bar{p}]\right] > \Omega_{\{E_i[\bar{p}]|y\}}$ and $Var\left[E_i\left[\bar{E}[\bar{p}]\right]\right] > \Omega_{\{E_i[\bar{E}[\bar{p}]]|y\}}$: the uncertainty in beliefs about the price level must be lower than the cross-sectional dispersion in beliefs as seen in the data.

With respect to information treatments, managers in the basic model should respond to signals according to equation (23) where prior beliefs are multiplied by $(1 - PH)$ with P being the gain of the filter and H being a coefficient mapping the state m into the signal s. As Signals B and C are constructed from aggregate expectations, H_B and H_C correspond to the relative weight that managers assign to their private signal, x_i, in forming their first-order and second-order expectations, respectively. The coefficient $(1 - PH) = \frac{\kappa_s^{-1}}{\kappa_s^{-1} + H^2 \delta \kappa^{-1}}$ differs across signals. The structure of the basic model is preserved in this setting, but with perceived signal precisions (and associated response parameters) in lieu of actual signal precisions. This continues to yield $H_B > H_C$, such that managers should respond more strongly to Signal B (first-order treatment) than Signal C (higher-order treatment) which is counterfactual.

However, if one is willing to consider models of overconfidence in which managers mis-ascertain the precisions of the signals they receive, one might also consider it reasonable for agents to potentially be overconfident about the quality of the signals introduced in the information experiment. Suppose for example that managers assign more weight to the signal than the signal fundamentals merit such that the posterior expectations are formed according to:

$$E_i^{Post}\begin{bmatrix}\overline{p}\\ \overline{E}[\overline{p}]\end{bmatrix} = PH\begin{bmatrix}\phi_y\\ \phi_y + \phi_x(1-\delta)\end{bmatrix} y + (1-(1+\Theta)PH)E_i^{Pre}\begin{bmatrix}\overline{p}\\ \overline{E}[\overline{p}]\end{bmatrix} + \begin{bmatrix}\phi_x\\ \delta\phi_x\end{bmatrix}P\tilde{s} \qquad \text{(E.10)}$$

where $\Theta > 0$ indicates the overweighting of new information towards representative states. If we hold the signal noise κ_s^{-1} constant and allow Θ to vary across signals, we can match the pattern in our data if $\Theta_C > \Theta_B$, or if managers distort their beliefs towards representative states more after observing Signal C than after observing Signal B. This would be broadly consistent with the diagnostic expectations of Bordalo et al. (2020).

To reconcile the data, this approach requires managers to have differential over-confidence in signals B and C and thus it resembles the basic noisy-information model in requiring differential precisions for signals B and C, that is, both models require an extra degree of freedom to rationalize the observed reactions to signals. Data collected in the survey do not permit us to establish whether varying over-confidence or varying precision is behind the smaller weight on the prior in response to signal C relative to signal B. Also note that over-confidence alone continues to predict perfect correlation between higher-order and first-order beliefs because x_i continues to be the only source of cross-sectional variation as in the basic noisy-information model. Consequently, one should introduce a semi-public signal or another device to create an addition source of cross-sectional variation in beliefs.

Appendix F:
Sampling Frame and Response Rate

Appendix Table F.1: Number of Firms by Sector and Size in NZ, 2016

	Number of Firms 6-9 Workers	10-19 Workers	20-49 Workers	50-99 Workers	100+ Workers	> 6 Workers
Manufacturing	1737	1791	1248	420	312	5508
Rental, Hiring and Real Estate	528	330	153	15	36	1062
Professional, Technical, Scientific Services & Administrative Support Services	2595	2016	1188	357	336	6492
Financial and Insurance Services	267	159	96	42	69	633
Construction	2487	1821	837	204	93	5442
Wholesale Trade	1284	1107	657	222	120	3390
Retail Trade	2172	1704	678	258	315	5127
Accommodation and Food Services	2601	2511	1230	201	108	6651
Transport, Postal, Warehousing & Information Media	744	681	438	171	156	2190
Total	14415	12120	6525	1890	1545	36495

Source: Statistics New Zealand

Appendix Table F.2: Percentage of Firms by Sector and Size in NZ, 2016

	Percentage of Firms 6-9 Workers (%)	10-19 Workers (%)	20-49 Workers (%)	50-99 Workers (%)	100+ Workers (%)	> 6 Workers (%)
Manufacturing	31.54	32.52	22.66	7.63	5.66	100
Rental, Hiring and Real Estate	49.72	31.07	14.41	1.41	3.39	100
Professional, Technical, Scientific Services & Administrative Support Services	39.97	31.05	18.30	5.50	5.18	100
Financial and Insurance Services	42.18	25.12	15.17	6.64	10.90	100
Construction	45.70	33.46	15.38	3.75	1.71	100
Wholesale Trade	37.88	32.65	19.38	6.55	3.54	100
Retail Trade	42.36	33.24	13.22	5.03	6.14	100
Accommodation and Food Services	39.11	37.75	18.49	3.02	1.62	100
Transport, Postal, Warehousing & Information Media	33.97	31.10	20.00	7.81	7.12	100

Source: Statistics New Zealand

Appendix Table F.3: Number of Firms by Sector and Size in the Population of our Survey, 2017

	Number of Firms 6-9 Workers	10-19 Workers	20-49 Workers	50-99 Workers	100+ Workers	> 6 Workers
Manufacturing	946	975	680	420	312	3333
Rental, Hiring and Real Estate	200	125	58	15	36	433
Professional, Technical, Scientific Services & Administrative Support Services	868	674	397	357	336	2633
Financial and Insurance Services	80	47	29	42	69	267
Construction	241	177	81	204	93	796
Wholesale Trade	65	56	33	222	120	496
Retail Trade	84	66	26	258	315	750
Accommodation and Food Services	272	263	129	201	108	973
Transport, Postal, Warehousing & Information Media	20	32	48	164	156	420
Total	2776	2415	1481	1883	1545	10100

Appendix Table F.4: Percentage of Firms by Sector and Size in the Population of our Survey, 2017

	Percentage of Firms 6-9 Workers (%)	10-19 Workers (%)	20-49 Workers (%)	50-99 Workers (%)	100+ Workers (%)	> 6 Workers (%)
Manufacturing	28	29	20	13	9	100
Rental, Hiring and Real Estate	46	29	13	3	8	100
Professional, Technical, Scientific Services & Administrative Support Services	33	26	15	14	13	100
Financial and Insurance Services	30	18	11	16	26	100
Construction	30	22	10	26	12	100
Wholesale Trade	13	11	7	45	24	100
Retail Trade	11	9	4	34	42	100
Accommodation and Food Services	28	27	13	21	11	100
Transport, Postal, Warehousing & Information Media	5	8	11	39	37	100

Appendix Table F.5: Survey Framework of Main Wave, Number of Firms According to Employment Size Group

	6-9 Workers			10-19 Workers			20-49 Workers			50-99 Workers			100+ Workers		
	Stats NZ Records	Firms Approached	Response	Stats NZ Records	Firms Approached	Response	Stats NZ Records	Firms Approached	Response	Stats NZ Records	Firms Approached	Response	Stats NZ Records	Firms Approached	Response
Manufacturing	1737	946	73	1791	975	94	1248	680	83	420	420	44	312	312	25
Rental, Hiring and Real Estate	528	200	14	330	125	13	153	58	13	15	15	9	36	36	0
Professional, Technical, Scientific Services & Administrative Support Services	2595	868	41	2016	674	46	1188	397	66	357	357	36	336	336	5
Financial and Insurance Services	267	80	21	159	47	17	96	29	29	42	42	10	69	69	4
Construction	2487	241	18	1821	177	19	837	81	24	204	204	16	93	93	3
Wholesale Trade	1284	65	12	1107	56	14	657	33	17	222	222	11	120	120	2
Retail Trade	2172	84	32	1704	66	27	678	26	35	258	258	14	315	315	15
Accommodation and Food Services	2601	272	9	2511	263	12	1230	129	14	201	201	5	108	108	1
Transport, Postal, Warehousing & Information Media	744	20	13	681	32	23	438	48	33	171	164	12	156	156	8

Appendix Table F.6: Survey Framework of Main Wave, Percentage of Firms According to Employment Size Group

	6-9 Workers			10-19 Workers			20-49 Workers			50-99 Workers			100+ Workers		
	Stats NZ Records (%)	Firms Approached (%)	Response (%)	Stats NZ Records (%)	Firms Approached (%)	Response (%)	Stats NZ Records (%)	Firms Approached (%)	Response (%)	Stats NZ Records (%)	Firms Approached (%)	Response (%)	Stats NZ Records (%)	Firms Approached (%)	Response (%)
Manufacturing	32	28	23	33	29	29	23	20	26	8	13	14	6	9	8
Rental, Hiring and Real Estate	50	46	29	31	29	27	14	13	27	1	3	18	3	8	0
Professional, Technical, Scientific Services & Administrative Support Services	40	33	21	31	26	24	18	15	34	5	14	19	5	13	3
Financial and Insurance Services	42	30	26	25	18	21	15	11	36	7	16	12	11	26	5
Construction	46	30	23	33	22	24	15	10	30	4	26	20	2	12	4
Wholesale Trade	38	13	21	33	11	25	19	7	30	7	45	20	4	24	4
Retail Trade	42	11	26	33	9	22	13	4	28	5	34	11	6	42	12
Accommodation and Food Services	39	28	22	38	27	29	18	13	34	3	21	12	2	11	2
Transport, Postal, Warehousing & Information Media	34	5	15	31	8	26	20	11	37	8	39	13	7	37	9

Appendix Table F.7: Survey Framework of Main Wave, Total Firms

	Number of Firms			Percentage of Firms		
	Stats NZ Records (#)	Firms Approached (#)	Response (#)	Stats NZ Records (%)	Firms Approached (%)	Response (%)
Manufacturing	5508	3333	319	100	61	10
Rental, Hiring and Real Estate	1062	433	49	100	41	11
Professional, Technical, Scientific Services & Administrative Support Services	6492	2633	194	100	41	7
Financial and Insurance Services	633	267	81	100	42	30
Construction	5442	796	80	100	15	10
Wholesale Trade	3390	496	56	100	15	11
Retail Trade	5127	750	123	100	15	16
Accommodation and Food Services	6651	973	41	100	15	4
Transport, Postal, Warehousing & Information Media	2190	420	89	100	19	21
Total	36495	10100	1032	100	27.64	10.22

Appendix Table F.8: Survey Framework of Follow-up Wave, Number of Firms

	6-9 Workers		10-19 Workers		20-49 Workers		50-99 Workers		100+ Workers		Totals	
	Firms Approached	Response	Firms Approached	Response	Firms Approached	Response	Firms Approached	Response	Firms Approached	Response	Firms Approached	Response
Manufacturing	73	36	94	43	83	42	44	26	25	10	319	157
Rental, Hiring and Real Estate	14	6	13	8	13	4	9	2	0	0	49	20
Professional, Technical, Scientific Services & Administrative Support Services	41	22	46	22	66	38	36	17	5	0	194	99
Financial and Insurance Services	21	10	17	10	29	15	10	4	4	2	81	41
Construction	18	6	19	11	24	13	16	7	3	2	80	39
Wholesale Trade	12	7	14	6	17	9	11	3	2	1	56	26
Retail Trade	32	15	27	14	35	14	14	11	15	10	123	64
Accommodation and Food Services	9	5	12	6	14	8	5	2	1	0	41	21
Transport, Postal, Warehousing & Information Media	13	6	23	13	33	18	12	6	8	5	89	48
Total	233	113	265	133	314	161	157	78	63	30	1032	515

Appendix Table F.9: Survey Framework of Follow-up Wave, Response Rates

	6-9 Workers Response Rates	**10-19 Workers** Response Rates	**20-49 Workers** Response Rates	**50-99 Workers** Response Rates	**100+ Workers** Response Rates
Manufacturing	49	46	51	59	40
Rental, Hiring and Real Estate	43	62	31	22	0
Professional, Technical, Scientific Services & Administrative Support Services	54	48	58	47	0
Financial and Insurance Services	48	59	52	40	50
Construction	33	58	54	44	67
Wholesale Trade	58	43	53	27	50
Retail Trade	47	52	40	79	67
Accommodation and Food Services	56	50	57	40	0
Transport, Postal, Warehousing & Information Media	46	57	55	50	63

Appendix G:
The effects of expectations on firm decisions

CGK and Coibion, Gorodnichenko and Ropele (2020) document that information treatments lead not only to revisions of inflation expectations but also to changes in firms' behavior. Armantier et al. (2015) provide some evidence of this type for households. Treatments in these earlier studies provide firms with information about the inflation target of the central bank, professional forecasts, or past inflation. Little is known about how firms react to treatments that involve information about higher-order beliefs. While we find that revisions of beliefs are similar for first- and higher-order inflation expectations, a priori one may observe considerable heterogeneity in employment/investment/etc. responses across these information treatments. In this appendix, we try to establish that firms act upon their self-reported expectations.

To estimate the effect of changes in inflation expectations on the choices of firms, our approach follows CGK. Specifically, before firms were treated in the first wave, they were asked about their three-month-ahead plans for future employment, investment, wages, and prices. Three months after the initial wave, we surveyed firms again and asked them to report changes in these four variables over the preceding three months. Using this information, we compute forecast error for each variable. The key advantage of using forecast errors is that they effectively difference out firm-fixed effects and thus reduce the size of idiosyncratic variation in the data.

In the next step, we regress forecast errors on changes in inflation expectations:

$$FE_i(X) = constant + b \times \left(E_i^{posterior}(\pi) - E_i^{prior}(\pi)\right) + error_i \qquad \text{(G.1)}$$

where $FE_i(X)$ is the forecast error for variable X, $E_i^{prior}(\pi)$ is the pre-treatment expected inflation, $E_i^{posterior}(\pi)$ is the post-treatment expected inflation. For $E_i^{posterior}(\pi)$, we use beliefs of firms measured immediately after the treatment. The revision in expectations following an information treatment $\left(E_i^{posterior}(\pi) - E_i^{prior}(\pi)\right)$ should be proportional to the difference between the signal and the expected value of the signal, that is, the surprise induced by a treatment (see section 4.3 for a formal derivation). Because we know pre-treatment values of $E_i(\pi)$, $E_i(\bar{E}(\pi))$ and $E_i(\pi_{t-1})$, we calculate the surprise and use it as an instrument for $\left(E_i^{posterior}(\pi) - E_i^{prior}(\pi)\right)$ as in Coibion, Gorodnichenko and Ropele (2020) and Coibion et al. (2019). Note that for the control group the surprise is zero because firms in this group are not provided with any information. For Group D, which receives both the first- and higher-order expectations, we construct the average surprise in expectations. This instrumental variable approach ensures that estimated b has a causal interpretation. This is important because we observe mean reversion in reported beliefs in the control group and therefore some variation in the difference between posteriors and priors is potentially endogenous. When we estimate specification (G.1), we do it on data combining the control group and a given treatment group.

Appendix Table G1 reports estimates of b for various treatments using the revisions in first-order inflation expectations on the right-hand side of equation (G.1). While treatments vary in their ability to move inflation expectations, the results in Appendix Table G1 suggest that, conditional on moving inflation expectations a given amount, the reaction of firms to a given change in expectations is largely similar across treatments. Consistent with CGK, we find that raising inflation expectations by one percentage point generates an approximately 0.4 percentage point increase in employment (column 1), an approximately 0.2 percentage point increase in fixed assets (column 2), and no effect on firms' prices (column 3) or wages (column 4) over the three months following the treatment. The IV estimates of the effects are approximately double the OLS estimates (Appendix Table G4). We also find similar results when we replace first-order inflation expectations as the regressor in equation (G.1) with higher-order inflation expectations (Appendix Table G5).

The survey also collects information about 6-month-ahead plans for firm-specific outcomes in the initial survey and 3-month-ahead plans for the same outcomes in the follow-up survey. This design allows us to also study the response of *revisions* in plans to information treatments (that is, the outcome variable in specification (G.1) is 3-month-ahead plan in the follow-up wave minus the 6-month-ahead plan in the initial wave). We find that while information treatments tend to increase planned investment, these treatments have no statistically significant effect on plans for employment, prices, and wages (see Appendix Table G3).

Note that these causal estimates measure the "total" effects of the information treatment, that is, the combined influence of a treatment on both first- and higher-order inflation expectations as well as other expectations. Since Treatments B, C, and E have only one signal, we cannot separately identify the contribution of first- and higher-order beliefs on firms' actions. Treatment D contains two signals (two instruments) and thus offers us an opportunity to run a horserace regression with first- and higher-order expectations included in specification (G.1). We find (Appendix Table G2) that none of the expectations systematically dominates the other and, more generally, few estimates are statistically significantly different from zero. These inconclusive results likely reflect the strong correlation in revisions of first- and higher-order expectations, which limits our ability to identify the independent effects of various orders of expectations.

Appendix Table G1. Effect of Information Treatment on Actions.

Treatment effect (relative to control group)	Percent change in: Workers	Fixed Assets	Price of Main Product	Wages
	(1)	(2)	(3)	(4)
Treatment B, $\overline{E}[\pi_t]$				
$\left(E_i^{posterior}(\pi) - E_i^{prior}(\pi)\right)$	0.407***	0.342***	0.141	0.003
	(0.152)	(0.125)	(0.132)	(0.015)
Observations	245	245	245	245
R^2	-0.038	-0.050	0.028	0.001
1st stage F-stat	149.6	149.6	149.6	149.6
Treatment C, $\overline{E}^2[\pi_t]$				
$\left(E_i^{posterior}(\pi) - E_i^{prior}(\pi)\right)$	0.493*	0.141**	-0.078	0.043*
	(0.260)	(0.063)	(0.072)	(0.024)
Observations	252	252	252	252
R^2	-0.097	0.103	-0.043	-0.198
1st stage F-stat	15.47	15.47	15.47	15.47
Treatment D, $\overline{E}[\pi_t]$ and $\overline{E}^2[\pi_t]$				
$\left(E_i^{posterior}(\pi) - E_i^{prior}(\pi)\right)$	-0.264	0.214***	0.019	0.016
	(0.184)	(0.060)	(0.062)	(0.018)
Observations	253	253	253	253
R^2	0.004	0.066	0.002	0.010
1st stage F-stat	318.8	318.8	318.8	318.8
Treatment E, π_{t-1}				
$\left(E_i^{posterior}(\pi) - E_i^{prior}(\pi)\right)$	0.352***	0.251***	0.096	0.021
	(0.095)	(0.096)	(0.094)	(0.013)
Observations	251	251	251	251
R^2	0.049	-0.028	-0.005	-0.000
1st stage F-stat	49.19	49.19	49.19	49.19
Memorandum: Pooled treatment				
$\left(E_i^{posterior}(\pi) - E_i^{prior}(\pi)\right)$	0.146*	0.229***	0.045	0.016**
	(0.085)	(0.043)	(0.046)	(0.008)
Observations	515	515	515	515
R^2	0.007	0.101	0.007	0.005
1st stage F-stat	221.1	221.1	221.1	221.1

Notes: The table reports the coefficient on the revision of a firm's first-order inflation expectation in specification (G.1). The regressand in each column is the forecast error for a given firm-specific outcome indicated in the second row of the table. The regressor is instrumented with surprise component in the provided signal, that is, the difference between information provided in a treatment and pre-treatment expectation for the variable provided in the treatment. *1st stage F-stat* reports the first-stage F-statistic. The last panel (pooled treatment) uses surprises pooled across treatments as an instrument for the revision of beliefs. Robust standard errors are reported in parentheses. ***, **, and * indicate significance at the 0.01, 0.05 and 0.10 percent levels, respectively.

Appendix Table G2. Horserace regressions.

Panel A. Second-stage regression

Dependent variable: percent change in:

Regressors	Workers	Fixed Assets	Price of Main Product	Wages
	(1)	(2)	(3)	(4)
$E_i^{posterior}\pi - E_i^{prior}\pi$	-0.086	0.168**	0.036	0.028
	(0.222)	(0.083)	(0.053)	(0.021)
$E_i^{posterior}[\bar{E}(\pi)] - E_i^{prior}[\bar{E}(\pi)]$	-0.239	0.062	-0.023	-0.016
	(0.206)	(0.071)	(0.077)	(0.016)
Observations	253	253	253	253
R^2	0.002	0.100	-0.001	0.005

Panel B. First-stage regression

Dependent variable:

	$E_i^{posterior}\pi - E_i^{prior}\pi$	$E_i^{posterior}[\bar{E}(\pi)] - E_i^{prior}[\bar{E}(\pi)]$
	(1)	(2)
$s_B - E_i^{prior}\pi$	0.906***	-0.044**
	(0.037)	(0.019)
$s_C - E_i^{prior}[\bar{E}(\pi)]$	-0.034	0.953***
	(0.030)	(0.027)
Observations	253	253
R^2	0.656	0.679
1st stage F-stat	501.9	655.4

Notes: Panel A of the table reports the coefficient on the revision firms' first-order inflation expectations and the revision of their higher-order inflation expectation in specification (G.1). The regressand in each column is the forecast error for a given firm-specific outcome indicated in the second row of the table. The regressors are instrumented with surprise component in the provided signals, that is, the difference between information provided in a treatment and pre-treatment expectation for the variable provided in the treatment. The first-stage regression is reported in Panel B. *1st stage F-stat* reports the first-stage F-statistic. Robust standard errors are reported in parentheses. ***, **, and * indicate significance at the 0.01, 0.05 and 0.10 percent levels, respectively.

Appendix Table G3. Effect of Information Treatment on Future Plans.

Treatment effect (relative to control group)	Percent change in: Workers	Fixed Assets	Price of Main Product	Wages
	(1)	(2)	(3)	(4)
Treatment B, $\overline{E}[\pi_t]$				
$\left(E_i^{posterior}(\pi) - E_i^{prior}(\pi)\right)$	0.004	0.491**	0.396*	-0.015
	(0.338)	(0.250)	(0.218)	(0.036)
Observations	245	245	245	245
R^2	-0.000	-0.011	0.016	-0.001
1st stage F-stat	149.6	149.6	149.6	149.6
Treatment C, $\overline{E}^2[\pi_t]$				
$\left(E_i^{posterior}(\pi) - E_i^{prior}(\pi)\right)$	0.017	0.027	-0.140	0.098
	(0.367)	(0.138)	(0.231)	(0.083)
Observations	252	252	252	252
R^2	-0.000	0.005	-0.033	-0.021
1st stage F-stat	15.47	15.47	15.47	15.47
Treatment D, $\overline{E}[\pi_t]$ and $\overline{E}^2[\pi_t]$				
$\left(E_i^{posterior}(\pi) - E_i^{prior}(\pi)\right)$	-0.220	0.219**	0.106	0.036
	(0.187)	(0.093)	(0.125)	(0.055)
Observations	253	253	253	253
R^2	0.008	0.006	0.002	0.009
1st stage F-stat	318.8	318.8	318.8	318.8
Treatment E, π_{t-1}				
$\left(E_i^{posterior}(\pi) - E_i^{prior}(\pi)\right)$	0.109	0.098	-0.202	-0.012
	(0.204)	(0.121)	(0.148)	(0.030)
Observations	251	251	251	251
R^2	0.001	0.007	-0.024	-0.004
1st stage F-stat	49.19	49.19	49.19	49.19
Memorandum: Pooled treatment				
$\left(E_i^{posterior}(\pi) - E_i^{prior}(\pi)\right)$	-0.051	0.200***	0.060	0.019
	(0.133)	(0.071)	(0.085)	(0.024)
Observations	515	515	515	515
R^2	-0.001	0.010	0.005	0.004
1st stage F-stat	221.1	221.1	221.1	221.1

Notes: The table reports the coefficient on revision of own inflation expectations in specification (G.1). The regressand in each column is revision in plans for a given firm-specific outcome indicated in the second row of the table; that is, the outcome variable in specification (2) is 3-month-ahead plan in the follow-up wave minus the 6-month-ahead plan in the initial wave. The regressor is instrumented with surprise component in the provided signal, that is, the difference between information provided in a treatment and pre-treatment expectation for the variable provided in the treatment. *1st stage F-stat* reports the first-stage F-statistic. The last panel (pooled treatment) uses surprises pooled across treatments as an instrument for the revision of beliefs. Robust standard errors are reported in parentheses. ***, **, and * indicate significance at the 0.01, 0.05 and 0.10 percent levels, respectively.

Appendix Table G4. Effect of Information Treatment on Actions, OLS.

Treatment effect (relative to control group)	Percent change in: Workers	Fixed Assets	Price of Main Product	Wages
	(1)	(2)	(3)	(4)
Treatment B, $\overline{E}[\pi_t]$				
$\left(E_i^{posterior}(\pi) - E_i^{prior}(\pi)\right)$	0.169**	0.145**	0.146**	0.003
	(0.072)	(0.057)	(0.074)	(0.005)
Observations	245	245	245	245
R^2	0.038	0.061	0.028	0.001
Treatment C, $\overline{E}^2[\pi_t]$				
$\left(E_i^{posterior}(\pi) - E_i^{prior}(\pi)\right)$	0.155*	0.127***	0.010	0.008
	(0.089)	(0.046)	(0.029)	(0.005)
Observations	252	252	252	252
R^2	0.025	0.105	0.001	0.013
Treatment D, $\overline{E}[\pi_t]$ and $\overline{E}^2[\pi_t]$				
$\left(E_i^{posterior}(\pi) - E_i^{prior}(\pi)\right)$	-0.149	0.134***	0.023	0.013
	(0.129)	(0.047)	(0.036)	(0.011)
Observations	253	253	253	253
R^2	0.010	0.103	0.002	0.010
Treatment E, π_{t-1}				
$\left(E_i^{posterior}(\pi) - E_i^{prior}(\pi)\right)$	0.207***	0.117***	0.042	0.010**
	(0.066)	(0.036)	(0.030)	(0.005)
Observations	251	251	251	251
R^2	0.096	0.093	0.009	0.024
Memorandum: Pooled treatment				
$\left(E_i^{posterior}(\pi) - E_i^{prior}(\pi)\right)$	0.106*	0.163***	0.046*	0.010**
	(0.062)	(0.029)	(0.025)	(0.005)
Observations	515	515	515	515
R^2	0.009	0.121	0.007	0.010

Notes: The table reports the OLS coefficient on revision of own inflation expectations in specification (G.1). The regressand in each column is forecast error for a given firm-specific outcome indicated in the second row of the table. Robust standard errors are reported in parentheses. ***, **, and * indicate significance at the 0.01, 0.05 and 0.10 percent levels, respectively.

Appendix Table G5. Effect of Information Treatment on Actions, higher-order expectations on the RHS.

Treatment effect (relative to control group)	Percent change in: Workers	Fixed Assets	Price of Main Product	Wages
	(1)	(2)	(3)	(4)
Treatment B, $\bar{E}[\pi_t]$				
$E_i^{posterior}[\bar{E}(\pi)] - E_i^{prior}[\bar{E}(\pi)]$	0.644***	0.540***	0.223	0.005
	(0.245)	(0.205)	(0.215)	(0.023)
Observations	245	245	245	245
R^2	-0.144	-0.294	-0.028	-0.003
1st stage F-stat	60.64	60.64	60.64	60.64
Treatment C, $\bar{E}^2[\pi_t]$				
$E_i^{posterior}[\bar{E}(\pi)] - E_i^{prior}[\bar{E}(\pi)]$	0.326**	0.093**	-0.052	0.028**
	(0.162)	(0.043)	(0.046)	(0.014)
Observations	252	252	252	252
R^2	0.039	0.022	-0.009	0.047
1st stage F-stat	561.8	561.8	561.8	561.8
Treatment D, $\bar{E}[\pi_t]$ and $\bar{E}^2[\pi_t]$				
$E_i^{posterior}[\bar{E}(\pi)] - E_i^{prior}[\bar{E}(\pi)]$	-0.355	0.288***	0.026	0.021
	(0.244)	(0.084)	(0.084)	(0.024)
Observations	253	253	253	253
R^2	-0.010	-0.010	0.003	-0.016
1st stage F-stat	182.9	182.9	182.9	182.9
Treatment E, π_{t-1}				
$E_i^{posterior}[\bar{E}(\pi)] - E_i^{prior}[\bar{E}(\pi)]$	0.311***	0.222**	0.085	0.018
	(0.095)	(0.089)	(0.085)	(0.012)
Observations	251	251	251	251
R^2	0.035	0.033	-0.012	0.000
1st stage F-stat	83.65	83.65	83.65	83.65
Memorandum: Pooled treatment				
$E_i^{posterior}[\bar{E}(\pi)] - E_i^{prior}[\bar{E}(\pi)]$	0.153*	0.240***	0.048	0.017**
	(0.089)	(0.048)	(0.049)	(0.008)
Observations	515	515	515	515
R^2	0.005	0.012	-0.005	-0.001
1st stage F-stat	365.6	365.6	365.6	365.6

Notes: The table reports the coefficient on revision of higher-order inflation expectations in specification (G.1). The regressand in each column is forecast error for a given firm-specific outcome indicated in the second row of the table. The regressor is instrumented with surprise component in the provided signal, that is, the difference between information provided in a treatment and pre-treatment expectation for the variable provided in the treatment. *1st stage F-stat* reports the first-stage F-statistic. The last panel (pooled treatment) uses surprises pooled across treatments as an instrument for the revision of beliefs. Robust standard errors are reported in parentheses. ***, **, and * indicate significance at the 0.01, 0.05 and 0.10 percent levels, respectively.