Innovation and Production in the Global Economy – Online

Appendix

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Abstract

In this online Appendix we discuss a number of results and definitions that appear in the main paper. We make use and cite various definitions and equations from the main paper, which, for brevity, we do not reintroduce in this online appendix.

1 Proof of Lemma 7

Note the definitions \( m^i \equiv T^i_p L^c_i / L^p_i = T^i_p M_i / (f^c L^p_i) \), \( \tilde{A}_i = (T^p_i)^{1/(1-\rho)} / L^p_i \), \( m \equiv \sum_k M_k / \sum L^p_k \), \( l_i \equiv L^p_i / \sum_k L^p_k \), \( M \equiv \sum M_i \), \( L^p \equiv \sum L^p_i \). We prove the following lemma:

**Lemma 7** Under restricted entry, consider a world where \( \tilde{A}_i = \tilde{A}, T^c_i = T^c \) for all \( i \), and where \( \frac{m_i}{m} < \frac{(\theta+1)(\theta \sigma - \sigma + 1)}{(\theta-\sigma+1)} \) \( \forall i \), and assume \( \rho \rightarrow 1 \) for all \( i \). The ratio of the real wage under frictionless trade and MP to the real wage under free trade and no MP, \( \mathcal{W}_i \equiv W^*_i / W_i \), is given by the expression:

\[
(\mathcal{W}_i)^\theta = \frac{[(1-\eta) m + \eta m_i]^{v} m^{1-v}}{m^{1/(1+\theta)} \sum_k m_k^{1/(1+\theta)} L_k},
\]

where \( v = \theta / (\sigma - 1) - 1 \).
**Proof:** Since we focus on frictionless trade, imposing $T_i^e = T^e$ we have

$$\Psi_{in} = T^e \left\{ \sum_k \left[ T_k^p (\gamma_{ik} w_k)^{-\theta} \right]^{\frac{1}{1-\rho}} \right\}^{1-\rho} \equiv \Psi_i,$$  \hspace{1cm} (2)

$$\psi_{iln} = \left[ T^e T_l^p (\gamma_{il} w_l)^{-\theta} / \Psi_i \right]^{\frac{1}{1-\rho}} \equiv \psi_{il}^r,$$  \hspace{1cm} (3)

and

$$\lambda_i^E = \frac{M_i \Psi_i}{\sum M_j \Psi_j} \equiv \lambda_i^E.$$

Using the definition of $\lambda_i^T$ and imposing free trade we have

$$w_n = \left[ \frac{\lambda_n^T}{\sum_i \left( T^e T_n^p T_i^p / \Psi_i \right) \left( \gamma_{in}^{-\theta} / \Psi_i \right)} \right]^{-(1-\rho)/\theta} \lambda_i^E,$$

and given (2) and $\lambda_i^E = \lambda_i^E$, we can write this expression as

$$w_n = \left[ \frac{\lambda_n^T}{(T^e T_n^p)^{1/(1-\rho)} \sum_i \left( \gamma_{in}^{-\theta} / \Psi_i \right) \left( \gamma_{in}^{-\theta} / \Psi_i \right)} \right]^{-(1-\rho)/\theta} \lambda_i^E.$$  \hspace{1cm} (4)

We will use this expression below and consider separately the two cases: zero MP costs and infinite MP costs.

**Frictionless trade but no MP.** In this case, we additionally have $\lambda_i^T = \lambda_i^E$, and $\Psi_n = T^e T_n^p w_n^{-\theta}$ so that expression (4) yields

$$w_n = \left[ \frac{\lambda_n^T}{(T^e T_n^p)^{1/(1-\rho)} \sum_j \left( \gamma_{in}^{-\theta} / \Psi_j \right) \left( \gamma_{in}^{-\theta} / \Psi_j \right)} \right]^{-(1-\rho)/\theta},$$

whereas equation (85) with $\tau_{in} = 1$ for all $i, n$ and infinite MP costs becomes

$$\lambda_i^E = \lambda_i^T = \frac{M_i T^e T_i^p w_i^{-\theta}}{\sum_j M_j T^e T_j^p w_j^{-\theta}},$$  \hspace{1cm} (5)

which gives us

$$\left( \frac{w_i}{w_l} \right)^{-\theta} = \frac{\lambda_i^T}{M_l T_i^p} / \frac{\lambda_l^T}{M_i T_l^p}.$$  \hspace{1cm} (6)
No MP implies \( X_l = Y_l \), hence (81) implies

\[
 w_l L_l^p = (1 - \eta) \lambda_l^T \sum_n X_n,
\]

and thus

\[
 \frac{w_l}{w_i} = \frac{\lambda_l^T / \lambda_i^T}{L_l^p / L_i^p}.
\]

Combined with (6) this equation yields

\[
 \frac{w_l}{w_i} = \left( \frac{L_l^p M_l T_l^p}{L_i^p M_i T_i^p} \right)^{1/(1+\theta)}.
\] (7)

Using \( T_l^p = \tilde{A}^{1-\rho} \left( L_j^p \right)^{1-\rho} \) and setting \( w_n = 1 \) by choice of numeraire, we then have

\[
 w_i = \left( \frac{M_n}{L_n^p} \right)^{\rho/(1+\theta)} \left( \frac{M_i}{L_i^p} \right)^{1/(1+\theta)}.
\] (8)

Using expression (B.16), which gives the real wage under exogenous entry, and noting that with no MP we have \( X_{nln} = 0 \) except for \( l = n \), \( \lambda_{nn}^T = X_{nnn} / X_n \), and \( X_n = Y_n \), the expression for welfare is

\[
 W_n = \kappa_n (1 - \eta)^{\frac{\sigma-1-\theta}{\sigma(\sigma-1)}} \left( T_n^p M_n \right)^\frac{1}{\theta} \left( \lambda_{nn}^T \right)^{-\frac{1}{\theta}}.
\]

Using expression (5), we can write

\[
 W_n = \kappa_n (1 - \eta)^{\frac{\sigma-1-\theta}{\sigma(\sigma-1)}} \left( T_n^p M_n \right)^\frac{1}{\theta} \left( \frac{M_n T_n^p w_n^{-\theta}}{\sum_k M_k T_k^p w_k^{-\theta}} \right)^{-\frac{1}{\theta}}.
\]

Now we substitute (8) in the expression above, and also note that we have \( T_n^p = \tilde{A}_n^{1-\rho} \left( L_n^p \right)^{1-\rho} \).

After some simplifications and considering the limit \( \rho \to 1 \) we obtain

\[
 W_n = \kappa_n (1 - \eta)^{\frac{\sigma-1-\theta}{\sigma(\sigma-1)}} \left( T_n^p \right)^{1/\theta} \left( m_n \right)^{1/(1+\theta)} \left[ \sum_k m_k^{\frac{1}{(1+\theta)}} L_k^p \right]^\frac{1}{\theta}.
\] (9)

**Frictionless trade and MP.** Using (76) and (78) together with the definition of \( \psi_{iln} \) and im-
posing zero MP costs we get

\[ \lambda^T_i = \sum_k \left( \frac{T^p w^\theta_k}{\Psi_k} \right)^{1/\rho} \frac{M_k \Psi_k}{\sum_j M_j \Psi_j}. \]

But now we have \( \Psi_i = \Psi \equiv T^e \left[ \sum_k \left( T^p w^\theta_k \right)^{1/\rho} \right]^{1-\rho} \), hence

\[ \lambda^E_i = \frac{M_i}{\sum_j M_j}, \quad (10) \]

and

\[ \lambda^T_i = \left( \frac{T^e T^p w^\theta_i}{(T^p)^{1/(1-\rho)} w_i^{\theta/(1-\rho)}} \right)^{1/\rho}. \quad (11) \]

Therefore, relative trade shares are

\[ \frac{\lambda^T_i}{\lambda^T_j} = \left( \frac{M_i}{\sum_j M_j} \right) \left( \frac{T^e T^p w^\theta_i}{T^e T^p w^\theta_j} \right)^{1/\rho}. \quad (12) \]

Using (75) and noting that \( \sum \frac{X_n}{X_i} = \lambda^E_{nn} \) (from the definition of \( \lambda^E_{in} \)) and recalling that \( \lambda^E_{nn} = \lambda^E_n \), then equation (B.16) can be rewritten as

\[ W^*_n = \kappa_n \left( T^e T^p M_n \right)^{1/\theta} \left( \lambda^E_n \right)^{-\frac{1}{\theta}} \left( \psi_{nnn} \right)^{-\frac{1}{\theta}} \left( \frac{1}{\sigma} X_n + \frac{\theta - \sigma + 1}{\theta \sigma} \right)^{\frac{\sigma-1}{\theta(\sigma-1)}} \left\{ \frac{1}{\sigma} X_n + \frac{\theta - \sigma + 1}{\theta \sigma} \right\} \]

But \( \psi_{in} = \left( \frac{T^p (w_i)^{-\theta}}{(T^p_T w_k)^{-\theta}} \right)^{1/\rho} \psi_n, \lambda^E_n = M_n / \sum M_i \) and \( T^e_T = \tilde{A}^{1-\rho} \left( L^p \right)^{1-\rho} \) hence we have

\[ W^*_n = \kappa_n \left( T^e \tilde{A}^{1-\rho} \left( L^p \right)^{1-\rho} \right)^{1/\theta} \left( \sum_k M_k \right)^{-\frac{1}{\theta}} \left( \psi_n \right)^{-\frac{1}{\theta}} \left( \frac{1}{\sigma} X_n + \frac{\theta - \sigma + 1}{\theta \sigma} \right)^{\frac{\sigma-1}{\theta(\sigma-1)}}. \quad (13) \]

We want to find the expression for \( W^*_n \) when \( \rho \to 1 \). We first conjecture that under this limit wages equalize and we a) derive an expression for the last parenthetical term of the welfare expression; b) show that \( \psi_n \) tends to a constant, which is finite and bounded away from zero; and then c) show that the wage equalization conjecture is true. Combining these three results,
the limit of the expression (13) as \( \rho \to 1 \) is

\[
W^n_m = \kappa_n \left( T^e \right)^{\frac{1}{\sigma}} M^n \left( 1 - \eta \right)^{\frac{\theta - 1 - \sigma}{\theta \sigma}} \left[ \frac{m}{(1 - \eta) m + \eta m_i} \right]^{\frac{\theta - 1 - \sigma}{\theta \sigma}}.
\]  

(14)

a) We first postulate that wages equalize as \( \rho \to 1 \) and show that

\[
\frac{1}{\hat{\sigma}} Y_n + \frac{\theta - \sigma + 1}{\theta \sigma} = \frac{(1 - \eta) m}{(1 - \eta) m + \eta m_i}.
\]

From the current account balance condition (82) combined with the labor market clearing condition (81), using \( \sum \lambda_{in}^T X_n = Y_i \), we obtain

\[
X_i = \left( 1 - \eta - \frac{\theta - \sigma + 1}{\sigma \theta} \right) \sum \lambda_{in}^T X_n + \frac{\theta - \sigma + 1}{\sigma \theta} X_i + \eta \sum \lambda_{in}^E X_n \implies X_i \left( 1 - \frac{\theta - \sigma + 1}{\sigma \theta} \right) = \frac{1}{\hat{\sigma}} Y_i + \eta \sum \lambda_{in}^E X_n ,
\]

and given that \( \lambda_{in}^E = \lambda_i^E \),

\[
\frac{X_i}{\sum_k X_k} \left( 1 - \frac{\theta - \sigma + 1}{\sigma \theta} \right) = \frac{1}{\hat{\sigma}} \frac{Y_i}{\sum_k X_k} + \eta \lambda_i^E.
\]

(15)

Labor market clearing, equation (81) implies

\[
- \frac{1}{\hat{\sigma}} \frac{Y_i}{\sum_k X_k} = \frac{\theta - \sigma + 1}{\theta \sigma} \frac{X_i}{\sum_k X_k} - \frac{w_i L^p_i}{\sum_k X_k}.
\]

(16)

Note that \( \sum_k Y_k = \sum_k X_k \) combined with the labor market clearing condition implies \( \sum_k w_k L^p_k = (1 - \eta) \sum_k Y_k \). Then combine (15) and (16) together with \( \lambda_i^E = M_i / \sum_k M_k \), which yields

\[
\frac{X_i}{\sum_k X_k} \left( 1 - \frac{\theta - \sigma + 1}{\sigma \theta} \right) = \frac{1}{\hat{\sigma}} \frac{\theta - \sigma + 1}{\theta \sigma} \frac{X_i}{\sum_k X_k} - \frac{w_i L^p_i}{\sum_k X_k} + \eta \lambda_i^E \implies \frac{X_i}{\sum_k X_k} = \frac{w_i L^p_i}{\sum_k X_k} + \eta \lambda_i^E \implies \frac{X_i}{\sum_k X_k} = \frac{1}{(1 - \eta) \sum_k w_k L^p_k + \eta \sum_k M_k}.
\]

(17)

But (15) with \( \lambda_i^E = M_i / \sum_j M \) implies

\[
1 - \eta \frac{M_i}{\sum_k M_k} \frac{X_i}{X_i} = \frac{1}{\hat{\sigma}} \frac{Y_i}{X_i} + \frac{\theta - \sigma + 1}{\sigma \theta}.
\]
Using (17) we then get
\[
\frac{1}{\sigma} \frac{Y_i}{X_i} + \frac{\theta - \sigma + 1}{\sigma \theta} = 1 - \eta \frac{M_i}{\sum_k M_k (1 - \eta)} \frac{1}{\sum_k \frac{w_i L_i^p}{w_k L_k^p} + \eta \frac{M_i}{\sum_k M_k}}
\]
\[
= 1 - \frac{\eta}{(1 - \eta) \frac{w_i L_i^p}{\sum_k w_k L_k^p} \frac{M_i}{M} + \eta},
\]
If wages are equalized, using that \(M_i = L_i^c / f^c\) and wage equalization, then the RHS becomes
\[
1 - \frac{\eta}{(1 - \eta) \frac{w_i L_i^p}{\sum_k w_k L_k^p} \frac{M_i}{M} + \eta} = 1 - \frac{\eta}{(1 - \eta) \frac{m}{m_i} + \eta} \implies
\]
\[
= 1 - \frac{\eta m_i}{(1 - \eta) m + \eta m_i} \implies
\]
\[
= (1 - \eta) \frac{m}{(1 - \eta) m + \eta m_i},
\]
completing the derivation.

b) Now we want to show that under the condition in the proposition in this limit equilibrium all countries have, \(\psi_l > 0\). To show that we want to find the limit
\[
\lim \psi_l = \lim_{\rho \to 1} \left( \frac{T_i^p w_i^{-\theta}}{\sum_k \left( T_k^p w_k^{-\theta} \right)} \right)^{\frac{1}{1-\rho}}
\]
\[
= \lim_{\rho \to 1} \frac{1}{\sum_k \left( \frac{T_k^p w_k^{-\theta}}{T_i^p w_i^{-\theta}} \right)^{\frac{1}{1-\rho}}}
\]
\[
= \lim \lambda_i^T.
\]
Thus, we simply need to construct the trade shares in the case of wage equalization with \(\rho \to 1\). The equilibrium conditions in a frictionless equilibrium are the current account balance,
\[
X_i = w_i L_i^p + \eta \frac{M_i}{M} \sum_n X_n,
\]
and labor market clearing,
\[
\frac{\theta - \sigma + 1}{\sigma \theta} X_i + (1 - 1/\sigma) \lambda_i^T \sum_n X_n = w_i L_i^p,
\]
with \(\lambda_i^T\) given by (11), with \(T_i^p = \bar{A}^{1-\rho} \left( L_i^p \right)^{1-\rho}\). Adding up across the current account bal-
ance conditions implies
\[ \sum_k X_k = \frac{1}{1 - \eta} \sum_k w_k L_k^p. \]

Combining the current account balance with labor market clearing and using this last result together with the expression for \( \lambda_i^T \) implies
\[
\frac{\theta - \sigma + 1}{\sigma \theta} \left( w_i L_i^p + \frac{M_i}{M} \sum_k w_k L_k^p \right) + \left( 1 - 1/\sigma \right) \frac{\sum_k w_k L_k^p}{L_i^p} \frac{1}{1 - \eta} \sum_k w_k L_k^p = w_i L_i^p \implies \]
\[
\frac{\eta \theta - \sigma + 1}{\sigma \theta} \frac{M_i}{M} + \frac{1}{\bar{\sigma} \left( 1 - \eta \right)} \sum_k w_k L_k^p \frac{-\theta/(1-\rho)}{\theta} = \left( 1 - \frac{\theta - \sigma + 1}{\sigma \theta} \right) \frac{w_i L_i^p}{\sum_k w_k L_k^p} \quad (18)
\]
Together with a normalization for wages this is a system of \( N \) non-linear equations in \( N \) unknowns. If wages are equalized in the limit as \( \rho \to 1 \), we can let \( w_i = 1 \), then we have from (18) that
\[
\lambda_i^T = \lim_{\rho \to 1} \frac{w_i}{\sum_k w_k} \frac{l_i}{l_i^p} = \bar{\sigma} \left( 1 - \eta \right) \left( 1 - \frac{\theta - \sigma + 1}{\sigma \theta} \right) l_i - \bar{\sigma} \frac{\theta - \sigma + 1}{\sigma \theta} \frac{M_i}{M},
\]
and in order for that to be positive we need to assume that
\[
\frac{(\sigma \theta - \sigma + 1)}{(\theta - \sigma + 1)} (1 + \theta) > \frac{m_i}{m}. \quad (19)
\]
This is the condition required in the proposition for interior solution. Notice that, around symmetry, \( m_i = m \), since the LHS of this equation is always strictly greater than 1.

c) In the last step, we want to show that in the limit as \( \rho \to 1 \) equilibrium wages are equalized. Equation (18) can be rewritten as
\[
a_i/l_i + \left\{ \frac{w_i}{\sum_k w_k} \frac{l_i}{l_i^p} \right\}^{\theta/(1-\rho)} = bw_i. \quad (20)
\]
where
\[
a_i \equiv \bar{\sigma} \frac{\theta - \sigma + 1}{\sigma \theta} \frac{m_i}{m} l_i \quad \text{and} \quad b \equiv \bar{\sigma} \left( 1 - \eta \right) \left( 1 - \frac{\theta - \sigma + 1}{\sigma \theta} \right).
\]
Assumption (19) is then
\[
0 < b - a_i/l_i.
\]
Since, given the normalization of one wage to one, \( \max w_v \geq 1 \), so that letting \( j = \arg \max_v w_v \),
we then have $b \max w_v - a_j/l_j > 0$, and (20) implies

$$\frac{\max w_v}{\left[ \sum_k l_k w_k^{-\theta/(1-\rho)} \right]^{-(1-\rho)/\theta}} = \left( b \max w_v - a_j/l_j \right)^{- (1-\rho)/\theta}. \quad (21)$$

Note that

$$\lim_{\rho \to 1} \left\{ \left[ \sum_k l_k w_k^{-\theta/(1-\rho)} \right]^{-(1-\rho)/\theta} \right\} = \min w_k,$$

and thus the LHS of expression (21) when we take the limit is,

$$\lim_{\rho \to 1} \left\{ \frac{\max w_v}{\left[ \sum_k l_k w_k^{-\theta/(1-\rho)} \right]^{-(1-\rho)/\theta}} \right\} = \lim_{\rho \to 1} \frac{\max w_v}{\min w_v}.$$

But taking the limit on the RHS of expression (21)

$$\lim_{\rho \to 1} \left\{ \left( b \max w_v - a_j/l_j \right)^{- (1-\rho)/\theta} \right\} = 1,$$

since $b \max w_v - a_j/l_j$ is bounded away from zero and must be $b \max w_v - a_j/l_j \leq 1$ since the LHS of (21) is always greater or equal to 1. Hence, taking limits of (21) we have

$$\lim_{\rho \to 1} \left( \frac{\max w_v}{\min w_v} \right) = 1,$$

which means that wages equalize.

We have completed the derivations of the two formulas for no MP and frictionless MP. From the two formulas (9) and (14) we obtain

$$\mathcal{W}_n = \frac{\kappa_n (T^c)^{1/\bar{\sigma}} M^{1/\bar{\sigma}} (1 - \eta)^{\sigma - 1 - \theta} \left[ \frac{m}{(1-\eta)m+\eta m_i} \right]^{\frac{\sigma-1-\theta}{\bar{\sigma}(\sigma-1)}}}{\kappa_n (T^c)^{1/\bar{\sigma}} (1 - \eta)^{\sigma - 1 - \theta} (m_n)^{1/(1+\theta)} \left[ \sum_k m_k^{1/(1+\theta)} L_k^{\rho} \right]} \quad \Rightarrow$$
\begin{align*}
\mathcal{W}_n &= \frac{M^\frac{1}{\theta}}{L^\frac{1}{\theta}} \left[ \frac{m}{(1-\eta)m+\eta m_i} \right]^{-\frac{\sigma-1}{\sigma}} \left( m_n \right)^{1/(1+\theta)} \left[ \sum_k m_k^{1/(1+\theta)} l_k \right]^{\frac{1}{\theta}} \\
\mathcal{W}_n &= \frac{m^{\frac{1}{\theta}}}{(1-\eta)m+\eta m_i} \left[ \sum_k m_k^{1/(1+\theta)} l_k \right]^{\frac{1}{\theta}} \left( m_n \right)^{1/(1+\theta)} \left[ \sum_k m_k^{1/(1+\theta)} l_k \right]^{\frac{1}{\theta}} \\
(\mathcal{W}_n)^\theta &= \frac{m^{1-\nu} \left[ (1-\eta) m + \eta m_i \right]^{\theta}}{(m_n)^{\theta/(1+\theta)} \sum_k m_k^{1/(1+\theta)} l_k}
\end{align*}

which is expression (B.20), with \( \nu \equiv \theta / (\sigma - 1) - 1 \) where notice that under symmetry \( \mathcal{W}_n = 1 \). This last derivation completes the proof of the Lemma.

2 Unilateral MP liberalization

**Proposition 1** Consider a two-country world under endogenous entry and frictionless trade. Assume that \( A_1 = A_2, T_1 = T_2, \) and \( L_1 = L_2 \) and that countries are not fully specialized in innovation or production. Then:

i) If \( \gamma_{12} < \gamma_{21} \) then \( r_1 > \eta > r_2 \).

ii) Let \( \gamma^* \equiv (2 \theta - 1)^{(1-\rho)/\theta} \) and assume that \( \gamma_{21} < \gamma^*, \gamma_{12} \leq \gamma_{21} \). Then country 1 gains when \( \gamma_{21} \) increases.

**Proof:** We proceed by proving each of two parts. **Part (i)** In country 1 labor market clearing, equation (81), can be written as

\[ X_1 \frac{1 + \theta \sigma - 1}{\theta} \frac{\sigma - 1}{\sigma} = \frac{\sigma - 1}{\sigma} \left( \lambda_1^E \psi_{111} + \lambda_2^E \psi_{211} \right) (X_1 + X_2) + X_1 r_1. \]

Free entry implies for country \( i \)

\[ X_i r_i = \eta \lambda_i^E (X_1 + X_2), \quad (22) \]

so we can write the labor market clearing as

\[ X_1 \frac{1 + \theta \sigma - 1}{\theta} \frac{\sigma - 1}{\sigma} = X_1 r_1 (\theta \psi_{111} + 1) + \theta X_2 r_2 \psi_{211}, \quad (23) \]
\[ X_2 \frac{1 + \theta \sigma - 1}{\theta} \frac{\sigma - 1}{\sigma} = X_2 r_2 (\theta \psi_{222} + 1) + \theta X_1 r_1 \psi_{122}. \quad (24) \]
The second equation implies

\[ r_1 = \frac{X_2}{\theta X_1 \psi_{122}} \left[ \frac{1 + \theta \sigma - 1}{\sigma} - r_2 \left( \frac{\theta \psi_{222} + 1}{\theta \psi_{122}} \right) \right], \tag{25} \]

and substituting in (23) yields

\[ X_1 \frac{1 + \theta \sigma - 1}{\sigma} = (\theta \psi_{111} + 1) \frac{X_2}{\theta \psi_{122}} \frac{1 + \theta \sigma - 1}{\sigma} + X_2 r_2 \left[ \theta \psi_{211} - (\theta \psi_{111} + 1) \left( \frac{\theta \psi_{222} + 1}{\theta \psi_{122}} \right) \right] \implies \]

\[ r_2 = (1 + \theta) \eta \frac{X_1 - (\theta \psi_{111} + 1) \frac{X_2}{\theta \psi_{122}} \left[ \theta \psi_{211} - (\theta \psi_{111} + 1) \left( \frac{\theta \psi_{222} + 1}{\theta \psi_{122}} \right) \right]}{X_2}, \]

and back into (25),

\[ r_1 = (1 + \theta) \eta \frac{X_2 - X_1 \left( \frac{\theta \psi_{222} + 1}{\theta \psi_{122}} \right)}{X_1 \left[ \theta \psi_{211} - (\theta \psi_{111} + 1) \left( \frac{\theta \psi_{222} + 1}{\theta \psi_{122}} \right) \right]}, \]

or

\[ r_1 = (1 + \theta) \eta \frac{\theta \psi_{122} - \omega (\theta \psi_{222} + 1)}{\omega \left[ \theta^2 \psi_{211} \psi_{122} - (\theta \psi_{111} + 1)(\theta \psi_{222} + 1) \right]}, \]

\[ = (1 + \theta) \eta \frac{\omega (\theta \psi_{222} + 1) - \theta \psi_{122}}{\omega \left[ \theta (\psi_{111} + \psi_{222}) + 1 \right]}, \tag{26} \]

where \( \Psi_1 / \Psi_2 = X_1 / X_2 \) in this case so that

\[ \omega \equiv w_1 / w_2 = \Psi_1 / \Psi_2 = X_1 / X_2. \tag{27} \]

We use the definition of \( \psi_{iln} \) to substitute and obtain

\[ r_1 = (1 + \theta) \eta \frac{\omega \left( \theta - \frac{1}{\omega \gamma_{21} - \frac{\theta}{\frac{1}{1 - p} + 1}} \right) - \theta - \frac{(\gamma_{21})^{-\frac{\theta}{1 - p}}}{\omega^{-\frac{\theta}{1 - p} + (\gamma_{21})^{-\frac{\theta}{1 - p}}}}}{\omega \left[ \theta \left( \omega^{-\frac{\theta}{1 - p} + (\gamma_{21})^{-\frac{\theta}{1 - p}}} + \frac{1}{(\omega \gamma_{21})^{-\frac{\theta}{1 - p} + 1}} \right) + 1 \right]}, \]

and using (27),

\[ \omega = \left( \frac{\omega^{-\frac{\theta}{1 - p} + (\gamma_{21})^{-\frac{\theta}{1 - p}}}}{(\omega \gamma_{21})^{-\frac{\theta}{1 - p} + 1}} \right)^{1 - \rho} \implies \omega^{-\frac{\theta}{1 - p} + (\gamma_{21})^{-\frac{\theta}{1 - p}} + 1} = \frac{\omega^{-\frac{\theta}{1 - p} + (\gamma_{21})^{-\frac{\theta}{1 - p}}}}{\omega^{1/(1 - \rho)}}, \tag{28} \]
so that

\[
r_1 = (1 + \theta) \eta \left\{ \frac{\theta \omega^{1/(1-\rho)}}{\omega^{1-\rho} + (\gamma_{12})^{1-\rho}} + 1 - \frac{\theta}{\omega} \left( \frac{\gamma_{12}}{\omega^{1-\rho} + (\gamma_{12})^{1-\rho}} \right)^{-\frac{\theta}{1-\rho}} \right\} + 1,
\]

\[
= (1 + \theta) \eta \left\{ \frac{\theta \omega^{\theta/(1-\rho)} + 1 + \left( \frac{\omega}{\gamma_{12}} \right)^{\theta/(1-\rho)} - \frac{\theta}{\omega} \left( \frac{\omega}{\gamma_{12}} \right)^{\theta/(1-\rho)} + \theta}{\theta \omega^{\theta/(1-\rho)} + 1 + \left( \frac{\omega}{\gamma_{12}} \right)^{\theta/(1-\rho)} + \theta} \right\}.
\]

\[
= (1 + \theta) \eta \left[ 1 - \theta \left( \frac{1 + \frac{1}{\omega} \left( \frac{\omega}{\gamma_{12}} \right)^{\theta/(1-\rho)} + \theta}{\theta \omega^{\theta/(1-\rho)} + 1 + \left( \frac{\omega}{\gamma_{12}} \right)^{\theta/(1-\rho)} + \theta} \right) \right].
\]

(29)

The proof for the first part is completed by showing that \( r_1 > \eta \), since using (22) for \( i = 1, 2 \) and adding up we obtain

\[
\omega r_1 + r_2 = \eta (\omega + 1) \implies r_2 = \eta + \omega (\eta - r_1),
\]

so that \( r_1 > \eta \implies r_2 < \eta \).

To show that \( r_1 > \eta \), given (29), it suffices to show that

\[
\left( 1 - \theta \frac{1 + \frac{1}{\omega} \left( \frac{\omega}{\gamma_{12}} \right)^{\theta/(1-\rho)} + \theta}{\theta \omega^{\theta/(1-\rho)} + 1 + \left( \frac{\omega}{\gamma_{12}} \right)^{\theta/(1-\rho)} + \theta} \right) (1 + \theta) > 1 \implies
\]

\[
\left[ 1 + \frac{1}{\omega} \left( \frac{\omega}{\gamma_{12}} \right)^{\theta/(1-\rho)} \right] (1 + \theta) < \theta \omega^{\theta/(1-\rho)} + 1 + \left( \frac{\omega}{\gamma_{12}} \right)^{\theta/(1-\rho)} + \theta \implies
\]

\[
0 < \theta \omega^{\theta/(1-\rho)} \left[ 1 - \frac{1}{\omega} \left( \frac{1}{\gamma_{12}} \right)^{\theta/(1-\rho)} \omega^{1-\rho} \right] + \left( \frac{\omega}{\gamma_{12}} \right)^{\theta/(1-\rho)} \left( 1 - \frac{1}{\omega} \right)
\]

which holds true as long as \( \omega > 1 \). We know that if \( \gamma_{21} = \gamma_{12} \implies \omega = 1 \), so that to complete the proof we need to confirm that \( \partial \omega / \partial \gamma_{21} > 0 \). Working with (28) and totally differentiating we have

\[
\frac{\partial \omega}{\partial \gamma_{21}} = \frac{\omega \theta}{\gamma_{21} (1 - \rho)} \frac{1}{\Gamma(1-\rho)^{\theta/(1-\rho)}} \left[ \left( \omega \gamma_{21} \right)^{\theta/(1-\rho)} + 1 \right] + \frac{\theta}{\Gamma(1-\rho)^{\theta/(1-\rho)}} \left[ 1 - \omega^{\theta/(1-\rho)} (\gamma_{21})^{\theta/(1-\rho)} \right],
\]
and for this to be positive, it suffices to show that
\[ \omega^{\frac{1}{1 - \rho}} (\gamma_{21})^{-\frac{\theta}{1 - \rho}} \leq 1, \]
and using (28) we note that this is equivalent to
\[
\frac{\omega^{\frac{1}{1 - \rho}} + (\gamma_{12})^{\frac{1}{1 - \rho}}}{(\omega\gamma_{21})^{\frac{1}{1 - \rho}} + 1} \leq (\gamma_{21})^{\frac{\theta}{1 - \rho}} \implies \omega^{\frac{1}{1 - \rho}} + (\gamma_{12})^{\frac{1}{1 - \rho}} \leq (\gamma_{21})^{\frac{\theta}{1 - \rho}} \left[(\omega\gamma_{21})^{\frac{1}{1 - \rho}} + 1\right] \implies (\gamma_{12})^{\frac{1}{1 - \rho}} \leq (\gamma_{21})^{\frac{\theta}{1 - \rho}}
\]
which holds always true completing the proof of part (i).

**Part (ii)** We will derive an expression for the real wage in country 1, \( w_1/P_1 \), and then use that expression to prove part (ii). Notice that given that \( \omega = w_1/w_2 \) we can normalize the wage of country 2, \( w_2 = 1 \), so that \( \omega = w_1 \). Also, frictionless trade implies that the price index is the same across the two countries, \( P_1 = P_2 \equiv P \).

We first compute the price index. Irrespective of MP costs, under frictionless trade profits in country \( i \) are \( \eta L (w_1 + w_2) \lambda^E_i \), where \( L \equiv L_1 = L_2 \). But frictionless trade also implies that
\[
\lambda^E_i = \frac{M_i \Psi_i}{M_1 \Psi_1 + M_2 \Psi_2},
\]
hence free entry requires
\[
w_i = \eta L \frac{L}{f^e} (w_1 + w_2) \frac{\Psi_i}{M_1 \Psi_1 + M_2 \Psi_2}.
\]
Adding up wages we get
\[
\omega_1 + \omega_2 = \eta L \frac{L}{f^e} (w_1 + w_2) \frac{\Psi_1 + \Psi_2}{M_1 \Psi_1 + M_2 \Psi_2},
\]
and hence
\[
M_1 \Psi_1 + M_2 \Psi_2 = \eta L \frac{L}{f^e} (\Psi_1 + \Psi_2). \tag{30}
\]
Using (30) and the definition of the price index, equation (B.5), we have that
\[
P = \zeta^{-1} \left[ \left( \frac{M_1 \Psi_1}{L} \right)^{(\theta - \sigma + 1)/(1 - \sigma)} \frac{L}{f^e} \right]^{-1/\theta} \left( \Psi_1 + \Psi_2 \right)^{-1/\theta}. \tag{31}
\]
By the definition of $\Psi_i$ we have

$$\Psi_1 + \Psi_2 = T \left[ \omega^{-\frac{\theta}{\rho}} + (\gamma_{12})^{-\frac{\theta}{\rho}} \right]^{1-\rho} + T \left[ (\omega \gamma_{21})^{-\frac{\theta}{\rho}} + 1 \right]^{1-\rho}.$$  (32)

Using the definition of $\omega = \Psi_1 / \Psi_2$ we can show that $\frac{\partial \ln \omega}{\partial \ln \gamma_{11}} > 0$ (see part i). Also, using again (28) we can rewrite (32) as

$$\Psi_1 + \Psi_2 = T \left[ \omega^{-\frac{\theta}{\rho}} + (\gamma_{12})^{-\frac{\theta}{\rho}} \right]^{1-\rho} + T \left[ \omega^{-\frac{\theta}{\rho}} + (\gamma_{12})^{-\frac{\theta}{\rho}} \right]^{1-\rho} / \omega$$  (33)

$$= T \left[ \omega^{-\frac{\theta}{\rho}} + (\gamma_{12})^{-\frac{\theta}{\rho}} \right]^{1-\rho} \left( 1 + \frac{1}{\omega} \right)$$  (34)

We can now write the real wage of country 1, using (31), as

$$\frac{w_1}{P} = \frac{\omega}{\zeta^{-1} \left[ \left( \frac{E}{L} \right)^{(\theta-\sigma+1)/(1-\sigma)} \eta^{L_F} \right]^{-1/\theta} \left\{ T \left[ \omega^{-\frac{\theta}{\rho}} + (\gamma_{12})^{-\frac{\theta}{\rho}} \right]^{1-\rho} \left( 1 + \frac{1}{\omega} \right) \right\}^{-1/\theta}}$$

$$= \frac{\left\{ \frac{\omega^{-\frac{\theta}{\rho}} \omega^{\frac{\theta}{\rho}} + \omega^{\frac{\theta}{\rho}} (\gamma_{12})^{-\frac{\theta}{\rho}}}{\left[ \left( \frac{E}{L} \right)^{(\theta-\sigma+1)/(1-\sigma)} \eta^{L_F} \right]^{-1/\theta} T^{-1/\theta}} \right\}^{1/\theta}}{\frac{\left[ 1 + \omega^{\frac{\theta}{\rho}} (\gamma_{12})^{-\frac{\theta}{\rho}} \right]^{1-\rho} \left( 1 + \frac{1}{\omega} \right)}{\zeta^{-1} \left[ \left( \frac{E}{L} \right)^{(\theta-\sigma+1)/(1-\sigma)} \eta^{L_F} \right]^{-1/\theta} T^{-1/\theta}}}$$

Thus, changes in the welfare in country 1 are given by

$$\frac{\partial (w_1/P)}{\partial \gamma_1} = \frac{\partial}{\partial \omega} \left\{ \left[ 1 + \omega^{\frac{\theta}{\rho}} (\gamma_{12})^{-\frac{\theta}{\rho}} \right]^{1-\rho} \left( 1 + \frac{1}{\omega} \right) \right\} \frac{\partial \omega}{\partial \gamma_1}.$$  

Since, from part (i) we know that $\partial \omega / \partial \gamma_1 > 0$, it suffices to show that the first term is positive. But the sign of this first term is determined by the sign of

$$\theta \frac{\omega^{\frac{\theta}{\rho}} (\gamma_{12})^{-\frac{\theta}{\rho}}}{1 + \omega^{\frac{\theta}{\rho}} (\gamma_{12})^{-\frac{\theta}{\rho}}} - \frac{1}{\omega + 1}.$$  (35)

Notice that (35) is increasing in $\omega$, thus if we show that it is positive for $\gamma_{12} = \gamma_{21} \implies \omega = 1$
it will be satisfied for any $\gamma_{21} > \gamma_{12} \implies \omega > 1$. We will show that if $\gamma_2$ satisfies a certain condition, this expression is positive and thus $\frac{\partial (\omega_1 / P)}{\partial \gamma_{21}} > 0$. Notice that for (35) to be positive we need

$$\theta \omega^{\frac{\theta}{1-\rho}} (\gamma_{12})^{-\frac{\theta}{1-\rho}} (\omega + 1) > 1 + \omega^{\frac{\theta}{1-\rho}} (\gamma_{12})^{-\frac{\theta}{1-\rho}} \implies \omega^{\frac{\theta}{1-\rho}} [\theta (\omega + 1) - 1] > (\gamma_{12})^{\frac{\theta}{1-\rho}}$$

For $\gamma_{12} = \gamma_{21} \implies \omega = 1$ and as long as $\gamma_{12} < \gamma^*$,

$$\gamma^* \equiv (2\theta - 1)^{(1-\rho)/\theta} .$$

the inequality is satisfied, proving the second part of the proposition; QED.

### 3 Quasi Home Market Effects in a Two-Sector Trade Model

Consider two countries the Home and the Foreign, denoted by $H$ and $F$, respectively. Labor is the only factor of production and $L_H < L_F$. There are two goods and consumers in the two countries have symmetric Cobb-Douglas preferences over the goods, with $\alpha$ of total expenditure devoted to the ek-good, and a share $1 - \alpha$ of expenditure devoted to the k-good.

Each country can potentially produce both goods. The ek good, is produced under perfect competition and is composed of a continuum of varieties $\omega_{ek} \in [0, 1]$ that are aggregated in a CES fashion with an elasticity of substitution $\sigma_{ek} > 1$. These varieties are produced in country $i = H, F$ with a linear technology and productivity $z_i(\omega)$. There is an iceberg cost of shipping the ek-good $\tau_{in} \geq 1$, for $i \neq n$ and $\tau_{ii} = 1$. Productivity is drawn from a Fréchet distribution of the form $F_i = e^{T_i z^{-\theta}}$, with $T_i > 0$ and $\theta > 1$. We assume that $T_H/L_H = T_F/L_F = 1$. The technology in this sector is identical to the technology postulated in the perfect competition setup of Eaton and Kortum (2002).

The k-good is produced using a continuum of differentiated varieties, aggregated CES with elasticity of substitution $\sigma_k > 1$. Each variety in country $i = H, F$ is produced with a linear production function using labor and with identical productivity across countries. In order to produce a firm has to incur a fixed cost of entry $f^e$. There is an iceberg cost of shipping the k-good, $\gamma_{in} \geq 1$, for $i \neq n$, and $\gamma_{ii} = 1$. Firms are homogeneous and compete monopolistically and there is free entry into this sector. The (endogenous) number of firms is denoted by $M_i$, $i = H, F$. The technology in this sector is identical to the technology postulated in the monopolistic competition setup of Krugman (1980).
The equilibrium is defined as firm entry, \( M_i \), and wages, \( w_i \), for the two countries \( i = H, F \) such that labor markets clear and free entry drives profits to zero in both countries.

We prove the following Lemma:

**Lemma 1** Assume that \( \gamma_{HF} = \gamma_{FH} = 1 \), \( \tau_{HF} = \tau_{FH} > 1 \), and \( L_H > L_F \). Then the smaller country specializes in the Krugman sector.

**Proof**: First, notice the free entry condition implies that the equilibrium entry in the Krugman sector is given by

\[
M_i = \frac{r_i}{\sigma_k f} L_i, \quad i = H, F. \tag{36}
\]

Thus, the market shares for each of the countries in the two sectors can be written as

\[
\lambda_{in}^{ek} = \frac{T_i (w_i \tau_{in})^{-\theta}}{\sum_k T_k (w_k \tau_{kn})^{-\theta}} = \frac{L_i (w_i \tau_{in})^{-\theta}}{\sum_k L_k (w_k \tau_{kn})^{-\theta}},
\]

\[
\lambda_{in}^k = \frac{M_i (w_i \gamma_{in})^{1-\sigma}}{\sum_k M_k (w_k \gamma_{kn})^{1-\sigma}} = \frac{r_i L_i (w_i \gamma_{in})^{1-\sigma}}{\sum_k r_k L_k (w_k \gamma_{kn})^{1-\sigma}}.
\]

Using those expressions we now have to solve for equilibrium wages, \( w_i \), and entry \( r_i \), using the three of the four labor market clearing conditions for the two sectors and one wage normalization, say \( w_H = 1 \). Therefore, the equilibrium conditions for \( r_H, r_L \) and \( w_H \) and \( w_F \) are \( w_H = 1 \) and

\[
w_F (1 - r_F) L_F = \alpha \sum_n \lambda_{Hn}^{ek} w_n L_n, \tag{37}
\]

\[
w_i r_i L_i = (1 - \alpha) \sum_n \lambda_{in}^k w_n L_n \text{ for } i = H, F. \tag{38}
\]

We can now prove the lemma. We start by proving that wages equalize in both countries when \( \gamma_{in} = 1 \). Using the equilibrium condition for the k-sector,

\[
(w_i)^\sigma = (1 - \alpha) \frac{\sum_n w_n L_n}{\sum_k r_k L_k (w_k)^{1-\sigma}} \text{ for } i = H, F,
\]

which implies that \( w_H = w_F \). Imposing the normalization and using the equation above we obtain a condition

\[
\sum_k r_k L_k = (1 - \alpha) \sum_n L_n, \tag{39}
\]

15
which combined with (37) can be used, in turn, to solve for \( r_i, i = H, F, \) and thus

\[
  r_F = 1 - \alpha \sum_n \frac{(\tau_{Fn})^{-\theta} L_n}{\sum_k L_k (\tau_{kn})^{-\theta}}. \tag{40}
\]

We substitute (40) into (39) to obtain

\[
  r_H L_H + \left(1 - \alpha \sum_n \frac{(\tau_{Fn})^{-\theta} L_n}{\sum_k L_k (\tau_{kn})^{-\theta}}\right) L_F = (1 - \alpha) \sum_n L_n \implies \tag{41}
  \frac{L_F}{L_H} = \left(1 - \alpha + \frac{\sum_k L_k (\tau_{kF})^{-\theta}}{\sum_k L_k (\tau_{kH})^{-\theta}}\right)
\]

\[
  r_H = (1 - \alpha) + \alpha \frac{L_F}{L_H} \left(1 + \frac{1}{\sum_k \frac{L_k}{L_F} (\tau_{kF})^{-\theta}} + \frac{(\tau_{FH})^{-\theta}}{\sum_k \frac{L_k}{L_H} (\tau_{kH})^{-\theta}}\right) \tag{42}
\]

\[
  r_H = (1 - \alpha) + \alpha \frac{L_F}{L_H} \left(1 + \frac{L_H (\tau_{HF})^{-\theta}}{L_F (\tau_{HF})^{-\theta}} + \frac{(\tau_{FH})^{-\theta}}{1 + \frac{L_H}{L_H} (\tau_{FH})^{-\theta}}\right) \tag{43}
\]

We show that \( L_H > L_F \) implies \( r_H < 1 - \alpha \), under symmetry of trade costs. Notice that this term is negative iff

\[
  (\tau_{FH})^{-\theta} \left(1 + \frac{L_H}{L_F} (\tau_{HF})^{-\theta}\right) < \frac{L_H}{L_F} (\tau_{HF})^{-\theta} \left(1 + \frac{L_F}{L_H} (\tau_{FH})^{-\theta}\right) \implies \tag{44}
  \frac{L_H}{L_F} \left((\tau_{HF})^{-\theta} (\tau_{FH})^{-\theta} - (\tau_{HF})^{-\theta}\right) > 1,
\]

and under symmetry of trade costs, the last parenthetical term is negative iff \( L_H > L_F \), proving the result. It is straightforward to prove that \( r_F > 1 - \alpha \) using equation (39), \textit{QED}.

\section{Isomorphism}

In this section we present a formal isomorphism of our model where the firm has a multivariate Pareto productivity to one where each firm’s productivity in a location is the product of a core productivity and a location-specific efficiency shock.

\subsection{Environment}

The basic environment is the same as in the main paper. There are \( i = 1, ..., N \) countries. Each country has a continuum of potential entrants, \( M_i \), each producing a differentiated variety.
Consumers have Dixit-Stiglitz preferences over varieties, with elasticity of substitution $\sigma$. Denote by $i$ the firm’s country of origin, and $l$ the country of production for each destination country $n$ where the firm sells its product. The rest of the notation of aggregate variables is the same as in the main text.

Firms operate labor-only constant returns to scale technologies, with the marginal product of labor being firm specific and location specific. In particular, a firm is distinguished by two productivity variables: a “core productivity” parameter $\phi$ and a vector of location specific productivity adjustment parameters, $z = (z_1, z_2, ..., z_N)$. A firm with productivity variables $\phi$ and $z$ producing in country $l$ has labor productivity $\phi z_l$.

A firm from country $i$ can serve country $n$ by exporting to country $n$ or by opening an affiliate in country $l \neq i, n$ and exporting from there to country $n$, or by opening an affiliate in $n$ and selling the good domestically. If $l \neq n$ there is a transportation cost incurred of $\geq 1$ with $\tau_{nn} = 1$ and the triangular inequality.

These assumptions imply that a firm with productivity $\phi$ and $z$ producing in location $l$ to serve market $n$ has a unit cost given by $w_l \tau_{ln} / (\phi z_l)$. Such a firm chooses to serve country $n$ from the cheapest production location $l$, charging a price

$$p_{in} = \min \left\{ p_{iln} \right\} = \frac{\sigma}{\sigma - 1} \min_l \left\{ \frac{\gamma_{il} w_l \tau_{ln}}{\phi z_l} \right\}.$$ 

We assume that $\phi$ is drawn from a Pareto distribution,

$$\phi \sim F_i(\phi) = 1 - \left( \frac{b_i}{\phi} \right)^\kappa,$$

where $\kappa + 1 - \sigma > 0$, while $z_l$ for firms from $i$ are drawn i.i.d from a Fréchet distribution with parameters $\theta$ and $T_{il}$,

$$z_l \sim e^{-T_{il} z^{-\theta}}.$$ 

Finally, we again assume that firms incur a destination-specific fixed cost $w_n F_{in}$ in order to have the possibility of serving market $n$. In addition, we assume that these fixed costs are paid before the vector of location-specific efficiency shocks $z$ is observed, but knowing the firm’s core productivity $\phi$. We assume that firms have to decide ex-ante how many markets they might end up serving; once, the vector $z$ is observed, the set of possible markets to serve is given. Hence, firms choose to pay the entry cost to the destination market $n$ if expected profits are larger than entry costs. Firms make this calculation market by market.
4.2 Firm’s Problem

The entry decision into each market $n$ gives us a threshold productivity level $\phi_{in}^*$ for which firms from country $i$ with $\phi \geq \phi_{in}^*$ pay the fixed cost $w_n F_{in}$ and serve $n$ and firms with $\phi < \phi_{in}^*$ do not. To derive $\phi_{in}^*$, imagine first that a firm from $i$ with productivity vector $(\phi, z)$ is forced to supply a country $n$ from $l$. The revenues of this firm would be $(P_n / p_{iln})^{\sigma-1} X_n$.

Let $v_{iln} \equiv \frac{\sigma-1}{\sigma} \phi p_{iln}$. Expected profits (gross of the fixed marketing cost) for such a firm are

$$P_n^{\sigma-1} \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma} \phi^{\sigma-1} (X_n / \sigma) E (v_{iln}^{1-\sigma}).$$

Letting $G(x; T) \equiv 1 - e^{-T x}$, then we know that $v_{iln}$ is distributed according to $G(v_{iln}; (\gamma_{il} w_l \tau_{ln})^{-\theta} T_{il})$.

But notice that for every $c \geq 1$ we have

$$[E(v_{iln}^c)]^{1/c} = \left[ \int_0^\infty v^c dG(v; \left( \frac{\sigma}{\sigma-1} \gamma_{il} w_l \tau_{ln} \right)^{-\theta} T_{il}) \right]^{1/c} = \Gamma \left( \frac{\theta + c}{\theta} \right) \left[ (\gamma_{il} w_l \tau_{ln})^{-\theta} T_{il} \right]^{1/(\theta c)} = \Gamma \left( \frac{\theta + c}{\theta} \right) \left[ (\gamma_{il} w_l \tau_{ln})^{-\theta} T_{il} \right]^{1/(\theta c)},$$

where we denote $\gamma_c = \Gamma \left( \frac{\theta + c}{\theta} \right) ^{1/c}$.

This result implies that expected profits in this case are

$$P_n^{\sigma-1} \phi^{\sigma-1} (X_n / \sigma) E (v_{iln}^{1-\sigma}) = P_n^{\sigma-1} \phi^{\sigma-1} (X_n / \sigma) \hat{\gamma} \left[ (\gamma_{il} w_l \tau_{ln})^{-\theta} T_{il} \right]^{(\sigma-1)/\theta},$$

where

$$\hat{\gamma} = \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma} \Gamma \left( \frac{\theta + 1 - \sigma}{\theta} \right).$$

But in fact firms in $i$ are not forced to supply a country $n$ from $l$, they will choose the production location that minimizes delivery cost, hence

$$E \left[ \max_l P_n^{\sigma-1} \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma} \phi^{\sigma-1} (X_n / \sigma) (v_{iln}^{1-\sigma}) \right] = P_n^{\sigma-1} \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma} \phi^{\sigma-1} (X_n / \sigma) E (\min_l v_{iln})^{1-\sigma}.$$

But $v_{iln} \equiv \min_l v_{iln}$ is distributed according to $G(v_{iln}; \Psi_{in})$, where

$$\Psi_{in} \equiv \sum_j (\gamma_{ij} w_j \tau_{jn})^{-\theta} T_{ij}.$$

We finally have that the expected profits we are interested in are

$$P_n^{\sigma-1} \phi^{\sigma-1} (X_n / \sigma) B_{in}.$$
where

\[ B_{in} \equiv E(\min_l v_{iln})^{1-\sigma} = \gamma_{1-\sigma} \Psi_{in}^{(\sigma-1)/\theta}. \]  

(46)

This definition implies that \( P_n^{\sigma-1} (\phi^*_n)^{\sigma-1} (X_n/\sigma) B_{in} = w_n F_{in} \), and hence

\[ \phi^*_n = \left[ \frac{\sigma w_n F_{in}}{X_n B_{in}} \right]^{1/\sigma - 1} \frac{1}{P_n}. \]  

(47)

Given that the expected marginal cost of producing and shipping the good from country \( i \) to \( n \), using production location \( l \) is

\[ c^*_n = E \left( \min_l \left\{ \frac{\sigma \gamma_{il} w_{iln}}{z_l \phi^*_n} \right\} \right) \]

\[ = E \left( \min_l \left\{ \frac{\gamma_{il} w_{iln}}{z_l} \right\} \right) \sigma \left[ \frac{w_n F_{in}}{X_n B_{in}} \right]^{1/\sigma - 1} P_n \]

\[ = \Psi_{in}^{-1/\theta} \sigma^{1/\sigma - 1} \left[ \frac{w_n F_{in}}{X_n} \right]^{1/\sigma} \Psi_{in}^{1/\theta} P_n \]

\[ = \gamma_{1-\sigma}^{1/\sigma - 1} \sigma^{1/\sigma} \left[ \frac{w_n F_{in}}{X_n} \right]^{1/\sigma} P_n. \]

where we have made use of definition (48). Notice that this expected cutoff cost, is the same as the realized cutoff cost, up to a constant, in the main paper.

We can also calculate the share of firms from \( i \) serving in \( n \) that choose to do so from location \( l \), \( \phi_{iln} \). Since \( v_{iln} \) is distributed \( G(v_{iln}; (\gamma_{il} w_{iln})^{-\theta} T_{il}) \) and \( l = \arg \min_l v_{iln} \), then standard results with the Fréchet distribution imply that \( \phi_{iln} \) (which is also the share of sales by firms in \( i \) in \( n \) that is produced in location \( l \)) imply that

\[ \psi_{iln} = \frac{(\gamma_{il} w_{iln})^{-\theta} T_{il}}{\Psi_{in}}. \]  

(48)

4.3 Price index

To calculate the price index \( P_n \), note that since \( p_{iln} = \frac{\sigma}{\sigma-1} v_{iln} / \phi \) and the measure of firms from country \( i \) with \( \phi \) is \( M_i dF_i(\phi) \) then

\[ P_n^{1-\sigma} = \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma} \sum M_i \int_{\phi^*_n}^{\infty} E \left( \min_l v_{iln} \right)^{1-\sigma} \phi^{\sigma-1} dF_i(\phi) \]
and hence
\[ P_n^{1-\sigma} = \gamma_1 \sum M_i \Psi_{in}^{(\sigma-1)/\theta} b_i^\kappa (\phi_{in}^*)^{-(\kappa+1-\sigma)}, \]
where
\[ c_1 \equiv \hat{\gamma} \frac{\kappa}{\kappa + 1 - \sigma}. \]
Plugging in from (47) we get (after some simplifications)
\[ P_n = c_2 \left( \frac{w_n}{X_n} \right)^{(\kappa+1-\sigma)/\kappa(\sigma-1)} \left[ \sum_i b_i^\kappa M_i F_{in}^{-(\kappa+1-\sigma)/(\sigma-1)} \Psi_{in}^{\kappa/\theta} \right]^{-1/\kappa}, \]
where
\[ c_2 \equiv c_1^{-1/(\kappa(\sigma/\hat{\gamma})^{(\kappa+1-\sigma)/\kappa(\sigma-1)}). \]

### 4.4 Market Shares

We will construct and make use of three market shares that are relevant for our model and potentially empirically relevant. To introduce these shares, let \( X_{iln} \) denote the sales to market \( n \) by firms originating in country \( i \) that are produced in country \( l \). Notice that \( \sum_{i,l} X_{iln} = X_n \) and \( \sum_{i,n} X_{iln} = Y_l \). The share of spending that is done in market \( n \) for goods produced from market \( i \) firms is \( \lambda_{in}^E = \sum_i X_{iln} / \sum_{i,l} X_{iln} \). The share of spending that is done in market \( n \) for goods produced in country \( l \) is \( \lambda_{in}^T = \sum_i X_{iln} / \sum_{i,n} X_{iln} \). Finally, the share of production in country \( l \) done by firms from country \( i \) is \( \lambda_{il}^M = \sum_n X_{iln} / \sum_{i,n} X_{iln} \).

We can use standard arguments about price indices in this kind of environment to get
\[ \lambda_{in}^E = \frac{M_i b_i^\kappa F_{in}^{-(\kappa+1-\sigma)/(\sigma-1)} \Psi_{in}^{\kappa/\theta}}{\sum_j M_j b_j^\kappa F_{jn}^{-(\kappa+1-\sigma)/(\sigma-1)} \Psi_{jn}^{\kappa/\theta}}. \]

In relation to the main paper, this expression is ‘distorted’ through the term \( F_{in} \) which, of course, dissapears if \( F_{in} = F_n \). Notice that the term \( \Psi_{in} \) is slightly different from that paper, but very closely related.

To get trade shares, recall that firms from \( i \) that sell in \( n \) produce a share \( \psi_{iln} \) in country \( n \) (equation 74). Hence the import share by \( n \) from \( l \) is the weighted average of these production shares across \( i \) weighed by the \( \lambda_{in}^E \),
\[ \lambda_{in}^T = \sum_i \psi_{iln} \lambda_{in}^E. \]
to $n$ is $\psi_{iln}\lambda_{in}^E X_n$, hence the total value of goods produced in $l$ by firms from $i$ is $\sum_n \psi_{iln}\lambda_{in}^E X_n$, and so

$$\lambda_{il}^M = \frac{\sum_n \psi_{iln}\lambda_{in}^E X_n}{\sum_i \psi_{iln}\lambda_{in}^E X_n}. \tag{52}$$

### 4.5 Profits

Let us now compute total profits made by firms from $i$ from their production in country $l$, $\Pi_{il}$. We know that total variable profits made by firms from $i$ in country $l$ are $\sum_n \psi_{iln}\lambda_{in}^E X_n / \sigma$. What are the fixed costs paid by these firms? The measure of firms in country $i$ that serve country $n$ through location $l$ is $\psi_{iln} M_i M_i^\gamma X_n$, hence

$$\Pi_{il} = \sum_n \left( \psi_{iln}\lambda_{in}^E X_n / \sigma - \psi_{iln} M_i \sum_j \sum_n \psi_{jn}\lambda_{jn}^E X_n / \sigma \right) \xi_{in} X_n M_i \sum_n \psi_{jn}\lambda_{jn}^E X_n / \sigma.$$

To proceed, note that from (47) and (46) we have

$$b_i^\kappa (\phi_{in}^*)^{-\kappa} = \frac{b_i^\kappa (\tilde{\gamma})^{\kappa/(\sigma-1)} \Psi_{in}^{\kappa/\theta}}{\left[ \frac{\sigma \xi_{in} X_n}{X_n} \right]^{\frac{\kappa}{\sigma-1}} \frac{1}{\sigma-1}}.$$

whereas from (49) and (76) we have

$$P_n^{-\kappa} = c_2^{-\kappa} \left( \frac{\xi_{in}}{X_n} \right)^{-\frac{(\kappa+1-\sigma)}{(\sigma-1)}} \frac{M_i b_i^\kappa F_{in}^{\frac{\kappa}{(\kappa+1-\sigma)/(\sigma-1)} \Psi_{in}^{\kappa/\theta}} / \lambda_{in}^E$.}

hence

$$b_i^\kappa (\phi_{in}^*)^{-\kappa} = \frac{\psi_{iln} M_i}{\left[ \sigma \xi_{in} X_n \right]^{\frac{\kappa}{\sigma-1}} \left( \frac{\xi_{in}}{X_n} \right)^{-\frac{(\kappa+1-\sigma)}{(\sigma-1)}} \frac{M_i b_i^\kappa F_{in}^{\frac{\kappa}{(\kappa+1-\sigma)/(\sigma-1)} \Psi_{in}^{\kappa/\theta}} / \lambda_{in}^E}}.$$

Using expression (53) and the above definitions we have that

$$\Pi_{il} = \sum_n \psi_{iln}\lambda_{in}^E X_n / \sigma - \sum_n \psi_{iln} c_2^\kappa (\tilde{\gamma}/\sigma)^{\kappa/(\sigma-1)} \frac{\lambda_{in}^E X_n}{\xi_{in} X_n} \frac{\lambda_{in}^E X_n}{\xi_{in} X_n} \sum_n \psi_{jn}\lambda_{jn}^E X_n$$

$$= \left( 1 / \sigma - c_2^\kappa (\tilde{\gamma}/\sigma)^{\kappa/(\sigma-1)} \right) \sum_n \psi_{iln}\lambda_{in}^E X_n.$$
so

\[ \Pi_{il} = \eta \lambda_{il}^{MY} Y_{il}, \]  

where

\[ \eta = \frac{\sigma - 1}{\sigma \kappa}, \]

with the interesting result the realized share of profits is constant but depends on \( \kappa \).

### 4.6 Equilibrium

Wages, spending, and number of firms, \( w_i, X_i \) and \( M_i \) solve the exact same system of equation in Section 10.3 in the paper, provided that \( \lambda_{in}^E \) and \( \psi_{iln} \) are the same. Given that we proceed to characterize welfare in the model.

### 4.7 Welfare

To look at welfare, we start with

\[ \lambda_{nn}^T = \sum_i \frac{w_i^{-\theta} T_{in}}{\sum_k (w_k \tau_{kn})^{-\theta} T_{ik}} \lambda_{in}^E. \]

Using \( \Psi_{in} \equiv \sum_l (w_l / \tau_{ln})^{-\theta} T_{il} \) this implies

\[ w_n = \left( \frac{\lambda_{nn}^T}{\sum_i \lambda_{in}^E T_{in} / \Psi_{in}} \right)^{-1/\theta}. \]

We also have

\[ \lambda_{in}^E = \frac{\gamma b_i^v M_i \Psi_{in}^{(\sigma-1)/\theta} (\phi_{in}^*)^{-(\kappa+1-\sigma)}}{p_n^{1-\sigma}} \]

Plugging in for \( \phi_{in}^* = \left( \sigma \frac{w_n F_{in}}{X_n B_{in}} \right)^{1/(\sigma-1)} \frac{1}{p_n} \) and \( B_{in} = \Gamma \Psi_{in}^{(\sigma-1)/\theta} \) we then get

\[ p_n^{1-\sigma} = \frac{c_1 b_i^v M_i \Psi_{in}^{\kappa/\theta} \left( \frac{w_n F_{in}}{X_n} \right)^{-(\kappa+1-\sigma)/(\sigma-1)} p_n^{\kappa+1-\sigma}}{\lambda_{nn}^E} \]

hence

\[ p_n = c_1 b_n^{-1} M_n^{-1/\kappa} \Psi_{nn}^{-1/\theta} \left( \frac{w_n F_{nn}}{X_n} \right)^{(\kappa+1-\sigma)/\kappa(\sigma-1)} \left( \lambda_{nn}^E \right)^{1/\kappa}. \]
and real wage is

\[
\frac{w_n}{P_n} = \left( \frac{\lambda T_{nn}}{\lambda_{nn}} \right)^{-1/\theta} \left( \lambda_{nn}^E \right)^{-1/\kappa} \left( \sum_i \lambda_{in} T_{in} \Psi_{nn}/\Psi_{in} \right)^{1/\theta} b_n M_n^{1/\kappa} c_1 \left( \frac{w_n F_{nn}}{X_n} \right)^{(\kappa+1-\sigma)/\kappa(\sigma-1)}
\]

Let \( \varepsilon_{iln} \equiv X_{iln}/\sum_j X_{jln} \), then we can show that

\[
\sum_i T_{in} \Psi_{nn} \Psi_{in}^{-1} \pi_{in} = \frac{T_{nn} E_{nn}}{\varepsilon_{nnn}},
\]

so finally we have

\[
\frac{X_n}{P_n} = \left( L_n/(1-\eta) \right)^{\kappa+1-\sigma}/\kappa \left( \frac{X_n}{Y_n} \right)^{\kappa+1-\sigma}/\kappa \left( \lambda_{nn}^T \right)^{-1/\theta} \left( \varepsilon_{nnn} \right)^{-1/\theta} \left( \lambda_{nn}^E \right)^{1/\theta-1/\kappa} \left( T_{nn} \right)^{1/\theta} b_n M_n^{1/\kappa},
\]

so the gains from openness are

\[
GO = \left( \frac{X_n}{Y_n} \right)^{\kappa+1-\sigma}/\kappa \left( \lambda_{nn}^T \right)^{-1/\theta} \left( \varepsilon_{nnn} \right)^{-1/\theta} \left( \lambda_{nn}^E \right)^{1/\theta-1/\kappa} \left( \frac{X_{nnn}}{X_n} \right)^{1/\theta-1/\kappa} \left( \sum_i X_{nln} \right)^{1/\theta-1/\kappa}
\]

where we used the definitions of \( \lambda_{nn}^T, \varepsilon_{nnn} \), and \( \lambda_{nn}^E \). This expression is the same as the main paper as long as \( 1/\theta = (1-\rho)/\tilde{\theta} \) and \( 1/\kappa - 1/\theta = \rho/\tilde{\theta} \) so that

\[
1/\kappa = 1/\tilde{\theta}, 1/\theta = (1-\rho)/\tilde{\theta}.
\]

Notice that the case of \( \rho = 0 \) in the main paper corresponds to the case \( \kappa = \theta \) here.

### 4.8 Formally Connecting to the models.

We finally formally connect this model to the model in the main text. The variables with tildes correspond to the variables of the model in the main text.

i) Parameters \( \gamma, \tau, T \) and \( L \) are the same,

ii) We set \( F_{in} = F_n = \tilde{F}_n \)

iii) Set \( 1/\kappa = 1/\tilde{\theta}, 1/\theta = (1-\rho)/\tilde{\theta} \implies \tilde{\theta} = \kappa, \frac{\tilde{\theta}}{1-\rho} = \theta \)
iv) Set wages, \( w_i = \tilde{w}_i \), spending, \( X_i = \tilde{X}_i \), and entry, \( M_i = \tilde{M}_i \), the same.

v) Given those, we can define \( \Psi_{in} = \tilde{\Psi}_{in} \), \( \psi_{iln} = \tilde{\psi}_{iln} \) and \( \lambda_{E} = \tilde{\lambda}_{E} \) since

\[
\Psi_{in} \equiv \sum_j \left( \gamma_{ij} \tilde{w}_j \tilde{T}_{jn} \right)^{-\theta} T_{ij} = \sum_j \left( \tilde{\gamma}_{ij} \tilde{w}_j \tilde{T}_{jn} \right)^{-\frac{1}{1-\rho}} T_{ij} = \tilde{\Psi}_{in}^{1/(1-\rho)},
\]

and

\[
\lambda_{E} = \frac{M_i b_i^\theta F_{in}^{-(\kappa+1-\sigma)/(\sigma-1)} \Psi_{in}^{\kappa/\theta}}{\sum_j M_j b_j^\kappa F_{jn}^{-(\kappa+1-\sigma)/(\sigma-1)} \Psi_{jn}^{\kappa/\theta}} = \tilde{\lambda}_{E} = \frac{\tilde{\Psi}_{in}}{\sum_j \tilde{M}_j \tilde{b}_j \tilde{\Psi}_{jn}} = \tilde{\lambda}_{E}
\]

and also

\[
\psi_{iln} = \left( \gamma_{il} \tilde{w}_l \tilde{T}_{ln} \right)^{-\theta} \frac{T_{il}}{\Psi_{in}} = \frac{\left( \tilde{\gamma}_{il} \tilde{w}_l \tilde{T}_{ln} \right)^{-\frac{1}{1-\rho}} \tilde{T}_{il}}{\tilde{\Psi}_{in}^{1/(1-\rho)}} = \tilde{\psi}_{iln}
\]

completing the isomorphism of all the variables in the two models. Notice that these derivations also imply \( \lambda_{T} = \tilde{\lambda}_{T} \) and \( \lambda_{il}^{M} = \tilde{\lambda}_{il}^{M} \).

vi) We finally show that the price index is the same up to a constant. Using (49) we can write the price index as

\[
P_n = \gamma_2 \left( \frac{w_n}{X_n} \right)^{\frac{\kappa+1-\sigma}{\kappa(\sigma-1)}} \left[ \frac{M_n b_n^\theta F_{nn}^{-(\kappa+1-\sigma)/(\sigma-1)} \Psi_{nn}^{\kappa/\theta}}{\lambda_{nn}^E} \right]^{-1/\kappa}
\]

and using the definitions above

\[
P_n = c \left( \frac{\tilde{w}_n}{\tilde{X}_n} \right)^{\frac{\kappa+1-\sigma}{\kappa(\sigma-1)}} \left[ \frac{\tilde{M}_n \tilde{b}_n^\kappa F_{nn}^{-(\kappa+1-\sigma)/(\sigma-1)} \tilde{\Psi}_{nn}^{1/(1-\rho)}}{\tilde{\lambda}_{nn}^E} \right]^{-1/\tilde{\lambda}_{nn}^E}
\]

given that \( \lambda_{nn}^E = \tilde{\lambda}_{nn}^E \), so that this would imply \( P_n = \tilde{P}_n \).

vii) The final step is to show that the models solve the same equilibrium conditions. Given
the above definitions and the discussion in Subsection 4.6 this holds completing the derivation of the isomorphism between the two setups.

5 Alternative Tariff Weights

Tariffs vary substantially across industries by country and the model provides no guidance on the proper method of aggregating tariffs across industries. Our benchmark estimates were based on a measure computed as the simple average across tariffs by country. Here, we assess the robustness of our trade elasticity estimates by re-estimating the restricted and unrestricted gravity equations using tariff measures are calculated using the value of global trade in that industry divided by the value of total global trade as the common weights. As with the simple average, this weighting scheme avoids using (endogenous) country trade shares in the aggregation. The underlying tariff data is from the World Integrated Trade Solution data provided by the World Bank and calculated at the H.S. six-digit level for the year closest to 1999 for which data were available. The elasticity estimates, which obtained via OLS, are shown in the table below.

<table>
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<th>R-sq.</th>
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<tr>
<td></td>
<td>(1.9)</td>
<td>(0.3)</td>
<td>(0.3)</td>
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The elasticities are very similar to those obtained from the simple averages and are presented in Table 1 in the paper. Although the absolute value of the magnitudes of the trade elasticities for both the restricted and unrestricted gravity equations are modestly larger, the ratios are virtually unchanged.

6 Calibration

We illustrate a formal mapping between the three models that we use in our approach, the linear model, the exogenous entry model and the Roy model.

All three models can be described using a unified approach, as the following system of equations (which we repeat from the main text for convenience):

Labor market clearing, equation (81) in the main paper,
\[
L^p_i = \frac{1}{w_i} \left[ \frac{\theta - \sigma + 1}{\sigma \theta} X_i + (1 - 1/\sigma) \sum_n \lambda^T_{in} X_n \right], \tag{57}
\]
current account balance, expressed generally as,
\[
X_i = w_i \left( L^p_i + \omega_i L^e_i \right) + \Delta_i, \tag{58}
\]
the free entry condition, equation (83) in the main paper,
\[
\frac{n}{w_i} \sum_n \lambda^E_{in}(w,M) X_n = M_i \omega_i f^e_i, \tag{59}
\]
and labor market clearing in innovation,
\[
L^e_i = M_i f^e_i, \tag{60}
\]
which, with the use of additional conditions that we lay out for each model, can be used to determine \(w, X, \omega_i, L^e, L^p,\) and \(M\). First, in the exogenous entry case we have that \(L^e\) is fixed and we set \(L_i = L^e_i + L^p_i\), which trivially determines \(M, L^p\), using also (60). Substituting out (59) and (60) into (58) we end up with a system of two sets of equations and two set of unknowns, \(w, X\), exactly as described in Section 10.3. After solving that system we can go back in (59) to determine \(\omega_i\). Second, in the linear model (with endogenous entry) we have that \(L_i = L^e_i + L^p_i\), and also \(\omega_i = 1\), due to full labor mobility. Using these conditions, and a number of straightforward substitutions, equations (57)-(60) can be reduced to the system of three equations and three unknowns described in Section (10.3). Finally, in the Roy model we need an additional function that determines \(L^e, L^p\) as a function of relative wages, equations (B.27) and (B.28) of the Appendix in the paper, which are functions of \(\omega_i\) and a productivity parameter \(A^e_i\). Plugging these equations to the equilibrium system, (57)-(60), we have a system of four equations and four unknowns, \(w, X, \omega_i,\) and \(M\) that we can solve for.

We will now show a formal mapping between the linear model and the exogenous entry model. Formally, given a set of parameters \([f^e, T^e, T^p, \Delta, \tau, \gamma, \theta, \sigma]\) for the linear model, and the resulting equilibrium, this equilibrium can be trivially replicated for the exogenous entry model and the Roy model by keeping the same \([T^e, T^p, \Delta, \tau, \gamma, \theta, \sigma]\) and only varying \(f^e\) and \(A^e\), if appropriate.

**Lemma 2** Consider a vector of parameters \([f^e, T^e, T^p, \Delta, \tau, \gamma, \theta, \sigma]\), and the resulting \(w, X, L^e, and \)
M from the equilibrium of the linear model. For the same parameters, the same allocations are generated from the equilibrium of the exogenous cost model with $\omega_i = 1$ in both cases.

**Proof:** The proof is trivial since by setting $\bar{L}_i - M_i f^e_i = L^p_i$ in the exogenous cost model, the two models yield exactly the same equilibrium with $\omega_i = 1$, as in the linear model.

We now also prove that we can go from any version of the endogenous entry model to a version of the same model with different $\kappa$ by simply choosing a new set of parameters $f^e, A^e$ and leaving all the rest of the parameters constant. One way we can think of this exercise is as if we go from the calibration of the linear model, $\kappa! \infty$, to a related calibration of the Roy model, $\kappa! (1, +\infty)$ by changing only the parameters $f^e, A^e$.

**Lemma 3** Consider a vector of parameters $[f^e, A^e, T^e, T^p, \Delta, \tau, \gamma, \kappa, \theta, \sigma]$ and the resulting $w, \omega, X, M$ from the equilibrium system (57)-(60). Assume $\bar{\kappa} \neq \kappa$, and let $\bar{\omega}_i$ and $\bar{A}_i^e, \bar{f}_i^e$, be defined implicitly by:

$$L^p_i(\bar{\omega}_i; \bar{\kappa}, \bar{A}_i^e) = L^p_i(\omega_i; \kappa, A_i^e), \quad (61)$$

and

$$\bar{\omega}_i L^e_i(\bar{\omega}_i; \bar{\kappa}, \bar{A}_i^e) = \omega_i L^e_i(\omega_i; \kappa, A_i^e) \quad (62)$$

and let

$$\bar{f}_i^e \equiv \omega_i f^e_i / \bar{\omega}_i. \quad (63)$$

The equilibrium with parameters $\Phi = [\bar{f}, \bar{A}, T^e, T^p, \Delta, \tau, \gamma, \bar{\kappa}, \theta, \sigma]$ results in the same endogenous variables $w, X, M$ as the equilibrium with parameters $\Phi = [f^e, A^e, T^e, T^p, \Delta, \tau, \gamma, \kappa, \theta, \sigma]$.

**Proof:** We start with a set of equilibrium variables, given $\Phi$. We need to show that equations (57)-(60) are satisfied given $\Phi$ and yield the same $w, X, M$ (but a different $\bar{\omega}$). Notice that with the definitions above the system yields exactly the same solutions for $w, X, M$ and $L^p$). As a result of the definitions (61) to (63), we have the resulting changes in parameters $\bar{f}^e, \bar{A}^e$ and in the equilibrium $\bar{\omega}$.
### Table A1. The Rise of the East: Changes from Baseline to China in Autarky

<table>
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<th>Country</th>
<th>Linear model</th>
<th>Roy Model</th>
<th>Exogenous-entry Model</th>
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<tbody>
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<td>( X/P )</td>
<td>( r )</td>
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