Trade, Domestic Frictions, and Scale Effects (Redux)

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Ideas and Scale Effects

• Idea-based growth models naturally lead to scale effects
  – Jones (95), Kortum (97):

\[ TFP \sim L^e \]

• Jones (Handbook, 05): "(the weak form of) scale effects is so inextricably tied to idea-based growth models that rejecting one is largely equivalent to rejecting the other."
Scale Effects versus Data

- Our baseline calibration: $\varepsilon = 1/4$
Scale Effects: Denmark

\[
\frac{TFP_{DNK}}{TFP_{US}}
\]

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Trade and Scale Effects

- Quantitative trade models are also idea-based models
  - Eaton and Kortum (01, 02), Krugman (80), Melitz (03)-Chaney (08)

- Gains from trade (GT) are generated by the same forces as those generating scale effects

\[
TFP \sim L^\varepsilon \times \lambda^{-\nu} = (L/\lambda)^\varepsilon
\]

- Small countries gain more from trade than large ones
  - this could help to reconcile model with data
The Gains from Trade

Log of R&D−adjusted country size (relative to U.S.)

Log of Gains from Trade (GT)
Does Trade Openness Help to Match the Data?

![Graph showing the relationship between Log of R&D-adjusted country size (relative to U.S.) and Log of Real Wage (relative to U.S.).]
The Role of Trade: Denmark

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This Paper: The Role of Domestic Frictions

- Explore and **quantify** an additional candidate solution
  - countries are not fully integrated domestically

- **Quantitative** model of trade and domestic geography
  - we extend EK(02) to include domestic frictions
  - key innovation: countries are collections of regions
Outline

• General EK-type Model of Regions
  • no domestic trade costs
  • symmetric regions
  • country-level gravity
  • the gains from trade

• Scale Effects

• Quantitative results
  • general case
  • alternative geographies
EK Model of Regions

- $M$ subnational economies or "regions" indexed by $m$
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- $N$ countries indexed by $n$
- Let $\Omega_n$ be the set of regions belonging to country $n$
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- Labor is the only factor of production, available in quantity $L_n$ in country $n$
EK Model of Regions

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- There is a continuum of final goods in the interval $[0, 1]$, and preferences are CES with elasticity of substitution $\sigma$.
- Technologies are linear with good-specific productivity draws in region $m$.
  - Fréchet distribution with parameters $\theta > \sigma - 1$ and $T_m$. 
- Perfect competition.
- Iceberg trade costs $d_{mk} \geq 1$ to export from $k$ to $m$.
- Triangular inequality: $d_{mk} \leq d_{ml} + d_{lk}$ for all $m, l, k$. 

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Labor Mobility

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- Each worker in country $n$ draws an efficiency parameter $z_m$ in every region $m \in \Omega_n$
  - Fréchet distribution with parameters $\kappa > 1$ and $A_m$
Equilibrium

- Let \( \Psi_m \equiv \{ z \text{ s.t. } z_m w_m / P_m \geq z_k w_k / P_k \text{ for all } k \in \Omega_n \} \)
Equilibrium

- Let $\Psi_m \equiv \{z \text{ s.t. } z_m w_m / P_m \geq z_k w_k / P_k \text{ for all } k \in \Omega_n\}$
- The share of workers that choose to live in region $m$ is

$$\pi_m \equiv \int_{\Psi_m} dF_{n(m)}(z) = \frac{A_m (w_m / P_m)^{\kappa}}{\sum_{m \in \Omega_n} A_m (w_m / P_m)^{\kappa}}$$

while the expected real income of workers in country $n$ is

$$V_n \equiv \left(\sum_{m \in \Omega_n} A_m (w_m / P_m)^{\kappa}\right)^{1/\kappa}$$
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- Avg real income equalized across regions within each country

$$\frac{w_m}{P_m} \frac{E_m}{\pi_m L_n} = \gamma V_n$$

where $E_m \equiv L_{n(m)} \int_{\Psi_m} z_m dF_{n(m)}(z)$
Since $X_m = w_mE_m$, demand equal supply of labor in region $m$ requires

$$w_mE_m = \sum_k \frac{T_m w_m^{-\theta} d_{km}^{-\theta}}{\sum_l T_l w_l^{-\theta} d_{kl}^{-\theta}} w_k E_k$$
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- Given wages, we can then solve for trade shares, price indices, real wages, and the allocation of workers to regions within countries
From Regions to Countries

- Let $\tilde{X}_{nj} \equiv \sum_{k \in \Omega_j} \sum_{m \in \Omega_n} X_{mk}$ denote country-level trade flows.

- Let $\tilde{\omega}_j \equiv \sum_{m \in \Omega_n} X_{m}$ denote total income and expenditure in country $n$.

- Let $\tilde{\omega}_{j} \equiv \tilde{\omega}_j / L_n$ denote the average nominal income per worker in country $n$ (country wage).
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- Let $\tilde{X}_{nj} \equiv \sum_{k \in \Omega_j} \sum_{m \in \Omega_n} X_{mk}$ denote country-level trade flows
- Let $\tilde{X}_n \equiv \sum_{m \in \Omega_n} X_m$ denote total income and expenditure in country $n$
- Let $\tilde{w}_j \equiv \tilde{X}_n / L_n$ denote the average nominal income per worker in country $n$ (country wage)
No Domestic Trade Costs (Prop. 1)

• $d_{mk} = 1$ for all $m, k \in \Omega_n$
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- Set $\tau_{nn} \equiv 1$ and define $\tau_{ni} \equiv d_{mk}$ for $m \in \Omega_n$ and $k \in \Omega_i$ for $n \neq i$
No Domestic Trade Costs (Prop. 1)

- \( P_m = P_k \) and \( w_m = w_k \), for all \( m, k \in \Omega_n \)
No Domestic Trade Costs (Prop. 1)

- \( P_m = P_k \) and \( w_m = w_k \), for all \( m, k \in \Omega_n \)
- Population shares do not depend on trade

\[
\pi_m = \frac{A_m^{\theta/(\kappa+\theta)} T_m^{\kappa/(\kappa+\theta)}}{\sum_{k \in \Omega_{n(m)}} A_k^{\theta/(\kappa+\theta)} T_k^{\kappa/(\kappa+\theta)}}
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  \]
- Country-level trade shares and prices indices are as in EK
  \[
  \lambda_{ni} = \frac{\tilde{T}_i \tilde{w}_i^{-\theta} \tau_{ni}^{-\theta}}{\sum_j \tilde{T}_j \tilde{w}_j^{-\theta} \tau_{nj}^{-\theta}} \quad \text{and} \quad \tilde{P}_n = \mu^{-1} \left( \sum_i \tilde{T}_i \tilde{w}_i^{-\theta} \tau_{ni}^{-\theta} \right)^{-1/\theta}
  \]
  with
  \[
  \tilde{T}_i \equiv \left( \sum_{m \in \Omega_i} A_m^{\theta/(\kappa+\theta)} T_m^{\kappa/(\kappa+\theta)} \right)^{(\kappa+\theta)/\kappa}
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$$\lambda_{ni} = \frac{\tilde{T}_i \tilde{w}_i^{-\theta} \tau_{ni}^{-\theta}}{\sum_j \tilde{T}_j \tilde{w}_j^{-\theta} \tau_{nj}^{-\theta}}$$ and $$\widetilde{P}_n = \mu^{-1} \left( \sum_i \tilde{T}_i \tilde{w}_i^{-\theta} \tau_{ni}^{-\theta} \right)^{-1/\theta}$$

with

$$\tilde{T}_i \equiv \left( \sum_{m \in \Omega_i} A_m^{\theta/(\kappa+\theta)} T_m^{\kappa/(\kappa+\theta)} \right)^{(\kappa+\theta)/\kappa}$$

- Country-level welfare

$$V_n = \gamma \mu \tilde{T}_n^{1/\theta} \lambda_{nn}^{-1/\theta}$$
Symmetric Regions (Prop. 2)

A1. [Symmetry] \( A_m = A_{m'} \) and \( T_m = T_{m'} \) for all \( m, m' \in \Omega_n \), and \( d_{mk} = d_{m'k'} \) for all \( m, m' \in \Omega_n \) and \( k, k' \in \Omega_i \).

- Same results as under no domestic trade costs, but

\[
\tau_{nn} \equiv \left( \frac{1}{M_n} + \frac{M_n - 1}{M_n} \delta_n^{-\theta} \right)^{-1/\theta}
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where \( \delta_n \equiv d_{mk} \) for \( m \neq k \) with \( m, k \in \Omega_n \)
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Country-Level Gravity (Prop. 3)

- Frictionless domestic trade or A1 are sufficient for the model to exhibit a log-linear gravity equation, but not necessary
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**A2. [International Hub and Spoke]** For all $j \neq n$, if $k \in \Omega_j$ and $m \in \Omega_n$ then $d_{mk} = \delta_m \tau_{nj} \delta_k$
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• Under A2, country-level trade flows are given by (for \( n \neq j \))
\[
\tilde{X}_{nj} = \mu^\theta \frac{\tau_{nj} \tilde{X}_n \tilde{X}_j}{\tilde{P}_n^{-\theta} \tilde{\Xi}_j^{-\theta}}
\]

where
\[
\tilde{\Xi}_j \equiv \left( \sum_{k \in \Omega_j} \frac{X_k}{\tilde{X}_j} \frac{T_k (w_k \delta_k)^{-\theta}}{X_k} \right)^{1/\theta} \quad \text{and} \quad \tilde{P}_n \equiv \left( \sum_{m \in \Omega_n} \frac{X_m}{\tilde{X}_n} \left( \frac{P_m}{\delta_m} \right)^\theta \right)^{1/\theta}
The Gains from Trade (Prop. 4)

- With frictionless domestic trade or A1, country-level GT are (ACR, 12)
  
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  - define \( \hat{x} \equiv x'/x \)
  - define \( \psi_{mk} \equiv X_{mk}/X_m \) as region-level trade shares
The Gains from Trade (Prop. 4)

- With $d_{mk} = \infty$ for $m \neq k$ and $d_{mk} = 1$ for $m = k$, $m, k \in \Omega_n$,

$$GT_n = \left( \sum_{m \in \Omega_n} \pi_m \hat{\psi}^{-\kappa/\theta} \right)^{-1/\kappa},$$

where

$$\hat{\psi}_{mm} = \frac{\hat{w}_m^{-\theta}}{\sum_{k \in \Omega_n} \psi_{mk} \hat{w}_k^{-\theta}},$$

and $\hat{w}_m$ is given by the solution to the system, for $m \in \Omega_n$

$$X_m \hat{w}_m^{\kappa} \left( \sum_{l \in \Omega_n} \psi_{ml} \hat{w}_l^{-\theta} \right)^{\kappa-1/\theta} = \sum_{k \in \Omega_n} \frac{\psi_{km} \hat{w}_m^{-\theta}}{\sum_{l \in \Omega_n} \psi_{kl} \hat{w}_l^{-\theta}} X_k \hat{w}_k^{\kappa} \left( \sum_{l \in \Omega_n} \psi_{kl} \hat{w}_l^{-\theta} \right)^{\kappa-1/\theta}.$$
The Gains from Trade: U.S. and Canada

- Goal: Compare GT computed using ACR formula and Prop. 4

- Data for domestic trade from Anderson and van Wincoop (03)

- Set $\theta = 4$ and $\kappa = 1.3$

- Results
  - $GT_{US}$: 0.67% (Prop 4) vs 0.77% (ACR)
  - $GT_{CAN}$: 6.35% (Prop 4) vs 6.48% (ACR)
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Scale Effects

- Natural relation between technology levels and population (EK, 01)
  - also in Krugman (80) and Melitz (03)-Chaney (08)

**A3. [Technology Scales with Population]** \[ T_m = \phi_n \pi_m L_n \] for all \( m \in \Omega_n \).
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  **A3. [Technology Scales with Population]** \( T_m = \phi_n \pi_m L_n \) for all \( m \in \Omega_n \).

- Ensure that \( A_m \)'s affect the labor allocation across regions but not their productivity:

  **A4. [Normalization of \( A \)'s]** \( \sum_{m \in \Omega_n} A_m = 1 \) for all \( n \).
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**A4. [Normalization of \( A \)'s]** \( \sum_{m \in \Omega_n} A_m = 1 \) for all \( n \).

- Under A3 and A4, we have country-level scale effects

\[ \tilde{T}_n = \phi_n L_n \]
Scale Effects: No Domestic Trade Costs

- Real wage

\[ V_n = \mu \left( \phi_n L_n \right)^{1/\theta} \lambda_{nn}^{-1/\theta} \]
Scale Effects: No Domestic Trade Costs

- Real wage

\[ V_n = \mu (\phi_n L_n)^{1/\theta} \lambda_{nn}^{-1/\theta} \]

- Conditional on trade shares and innovation intensity, real income levels increase with country size with an elasticity \(1/\theta\)
Scale Effects: Domestic Trade Costs under Symmetry

- Real wage

\[ V_n = \mu \times \phi_n^{1/\theta} \times L_n^{1/\theta} \times \tau_{nn}^{-1} \times \lambda_{nn}^{-1/\theta} \]

- R&D Intensity
- Pure Scale Effect
- Domestic Frictions
- Gains from Trade
Scale Effects: Domestic Trade Costs under Symmetry

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- R&D Intensity \quad Pure Scale Effect \quad Domestic Frictions \quad Gains from Trade

- Economies of scale depend on how \( \tau_{nn} \) is affected by country size, \( L_n \)
Scale Effects: Domestic Trade Costs under Symmetry

- Real wage

\[ V_n = \mu \times \phi_{n}^{1/\theta} \times L_n^{1/\theta} \times \tau_{nn}^{-1} \times \lambda_{nn}^{-1/\theta} \]

- R&D Intensity
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- Gains from Trade

- Economies of scale depend on how \( \tau_{nn} \) is affected by country size, \( L_n \)

- Assume \( L_n = \bar{L}M_n \) and \( \delta_n = \delta \). Then,

\[ \tau_{nn} \equiv \left( \frac{1}{L_n} + \frac{L_n - 1}{L_n} \delta^{-\theta} \right)^{-1/\theta} \]
Scale Effects: Domestic Trade Costs under Symmetry

• Real wage

\[ V_n = \mu \times \phi_n^{1/\theta} \times L_n^{1/\theta} \times \tau_{nn}^{-1} \times \lambda_{nn}^{-1/\theta} \]

R&D Intensity \hspace{1cm} Pure Scale Effect \hspace{1cm} Domestic Frictions \hspace{1cm} Gains from Trade

• Economies of scale depend on how \( \tau_{nn} \) is affected by country size, \( L_n \)

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\[ \tau_{nn} \equiv \left( \frac{1}{L_n} + \frac{L_n - 1}{L_n} \delta^{-\theta} \right)^{-1/\theta} \]

• Income-size elasticity is \( (1/\theta)(\delta/\tau_{nn})^{-\theta} \leq 1/\theta \)
A5. [Uniform Trade Costs and Innovation Intensity] $\delta_n = \delta$ for all $n$, $\tau_{ni} = \tau$ for all $n \neq i$ and $\phi_i = \phi$ for all $i$.

- Assume A1, A3, and A5. If $\tau > \delta$ then larger countries have lower import shares, higher wages, and lower price levels. If $\tau = \delta$ then larger countries have lower import shares, but wages and prices do not vary with country size.
Equivalences Across Models

• RRS: symmetric costs + domestic frictions

\[ \tau_{ni}^{RRS} = \nu_{ni} = \nu_{in} \quad , \quad \tilde{T}_i^{RRS} = \phi_i L_i \]

• EK: asymmetric costs with importer specific effect, no dom. fric.

\[ \tau_{ni}^{EK} = F_n \nu_{ni} \quad , \quad F_n = 1 / \tau_{nn}^{RRS} \quad , \quad \tilde{T}_i^{EK} = \tilde{T}_i^{RRS} \]

• W: asymmetric costs with exporter specific effect, no dom. fric.

\[ \tau_{ni}^{W} = F_i \nu_{ni} \quad , \quad F_i = 1 / \tau_{ii}^{RRS} \quad , \quad \tilde{T}_i^{W} = \tilde{T}_i^{RRS} (\tau_{ii}^{RRS})^{-\theta} \]
The RRS, EK, and W models generate the same equilibrium wages and trade flows, but

- the model W implies $\tilde{T}_i^W/L_i$ systematically higher for small countries
  \[ \frac{\tilde{T}_i^W}{L_i} = \phi_i (\tau_{ii}^{RRS})^{-\theta} \]

- the model EK implies prices systematically higher for small countries
  \[ p_n^{EK} = \tau_{nn}^{-1} p_n^{RRS} \]
Calibration: Key Parameters

- $\kappa = 1.3$
  - Suarez-Serrato and Zidar (2014)

- $\theta = 4$
  - trade: $\theta \in [2.5; 5.5]$ (Simonovska and Waugh, 13; Head and Mayer, 13)
  - growth: $\theta = 4.8$ (Jones, 02)
  - scale: $\theta = 3.3$ (Alcala and Ciccone, 04)

- Technology: $T_m = \phi_n \pi_m L_n$
  - $L_n$: equipped labor (K-RC, 05), avg 96-01
  - $\phi_n$: share of R&D employment (WDI), avg 96-01
  - $\pi_m$: implied by the model’s eq (*)

- Number of regions: $M_n$
  - number of metro areas in the data, for 26 OECD countries ($M = 287$)

- $A_m$ for region $m \in \Omega_n$, for all $n$
  - exactly match pop share of region $m$ in country $n$ observed in the data (*)
Calibration of Trade Costs $d_{mk}$

- Trade flows between regions are not available
- Impose $d_{mm} = 1$ and, for $m \neq k$,

\[
d_{mk} = \beta_0 I_{mk} \beta_1^{1-I_{mk}} \text{dist}_{mk}^{\beta_2 I_{mk} + \beta_3 (1-I_{mk})}
\]

- $\text{dist}_{mk}$: distance btw region $m$ and $k$ computed from longitude and latitude data for each metropolitan area in our sample
- $I_{mk}$: dummy variable that equals one if $m$ and $k$ belong to the same country

- Calibrate:
  - $\beta_0$ to match the share of intra-regional trade, in total domestic trade, for the United States, from the CFS, for 2007 = 0.40
  - $\beta_1$ to match the avg int’ bil trade sh, STAN, avg 96-01 = 0.0156
  - $\beta_2 = \beta_3 = 0.27$ from gravity evidence
Fit of Calibrated Model

\[ R^2 = 1 - \frac{\sum_{n,i} (\lambda_{ni}^{\text{data}} - \lambda_{ni}^{\text{model}})^2}{\sum_{n,i} (\lambda_{ni}^{\text{data}})^2} = 0.96 \]

- OLS elasticity of simulated int’ trade shares on (with source & dest. FE)
  - calibrated int’ bil trade costs = 3.96 (s.e. 0.079)
  - int’ bil distance = $-1.07$ (s.e. 0.021)
Calibrated Domestic Trade Costs and Country Size

- Country-level index (AY, 13):  
\[ \tau_{nn} = \sum_{m \in \Omega_n} \pi_m \left( \sum_{k \neq m, k \in \Omega_n} \pi_k d_m^{-\theta} \right)^{-1/\theta} \]

\[ M_{DNK} = 1 \quad \text{and} \quad \frac{\tau_{DNK, DNK}}{\tau_{US, US}} = 0.50 \]

\[ M_{JPN} = 36 \quad \text{and} \quad \frac{\tau_{JPN, JPN}}{\tau_{US, US}} = 0.70 \]
Calibrated vs Head-Ries Domestic U.S. Trade Costs

- Region-level Head-Ries index: $d_{mk}^{hr} \equiv \left( \frac{X_{mk}}{X_{kk}} \frac{X_{km}}{X_{mm}} \right)^{-\frac{1}{2\theta}}$
The Role of Domestic Frictions: Real Wages

- Data: real PPP-adjusted GDP (PWT 7.1) per unit of equipped labor
- Income-size elasticities:
  
  data (black) = -0.006; scale effects (green) = 0.25; RRS (pink) = 0.13
## The Role of Domestic Frictions: Denmark

<table>
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<th>$\frac{TFP_{DNK}}{TFP_{US}}$</th>
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<th>dom. fric.</th>
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<td>Data</td>
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Alternative Geographies: Symmetry

\[
\frac{w_n/P_n}{w_{US}/P_{US}} = \left( \frac{\phi_n L_n}{\phi_{US} L_{US}} \right)^{1/\theta} \left( \frac{GT_n}{GT_{US}} \right) \left( \frac{\tau_{nn}}{\tau_{US,US}} \right)^{-1}
\]

R&D-adjusted size \hspace{1cm} trade \hspace{1cm} domestic frictions

\[
\tau_{nn} \equiv \left( \frac{1}{M_n} + \frac{M_n - 1}{M_n} \delta_n^{-\theta} \right)^{-1/\theta}
\]

• Domestic trade costs: $\delta_n = \delta = 2.7$
  
  - model’s eq relation: $\tau^\theta_{nn} = M_n \sum_{m \in \Omega_n} X_{mm}/\tilde{X}_{nn}$
  
  - data on U.S. inter-regional trade data (CFS, 07) for 100 metro areas

• International trade costs
  
  - $\tau_{ni} = \beta_1 \text{dist}_{ni}^{\beta_3}$
  
  - $\beta_1$ matches avg int’ bil trade sh and $\beta_3 = 0.27$
"Gravity" domestic frictions refer to $\log \hat{\tau}_{nn} = \frac{1}{\theta} (\hat{S}_n - \hat{H}_n)$ where $\hat{S}_n$ and $\hat{H}_n$ are fixed effects from running gravity on normalized trade shares.
Real Wages: Data vs Models

• Size elasticities: data (black) = -0.01; no dom.fric. (red) = 0.20; sym.dom.fric. = 0.09; asym.dom.fric. (pink) = 0.13
### Other Variables: Data vs Models

<table>
<thead>
<tr>
<th>Size elasticities</th>
<th>import share</th>
<th>nominal wage</th>
<th>prices</th>
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<tr>
<td>data</td>
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</table>
Gains from Trade

- ACR formula vs general case
  - Denmark: 1.222 vs 1.222
  - Sweden: 1.186 vs 1.189
  - USA: 1.022 vs 1.021
  - Avg: 1.211 vs 1.211
Final Remarks

- Do small countries have better institutions?
  - R&D-adjusted scale is not correlated with measures of institutional quality, schooling, and measures of patents per (equipped) worker

- Multinational Production and Non-Traded Goods (working paper)

- Diffusion of ideas across and within countries