Economics 209A
Theory and Application of Non-Cooperative Games
(Fall 2013)

Games w/ pure information externalities
Social learning

- Agents use their information to identify a payoff-maximizing action so the choice of action reflects that information.

- By observing an agent’s action, it is possible to learn something about his information and make a better decision.

- In social settings, where agents can observe one another’s actions, it is rational for them to learn from one another.

- *Social learning* occurs when individuals learn by observing the behavior of others.
What have we learned from Social Learning?

The striking uniformity of social behavior is an implication of social learning:

– Despite the asymmetry of information, agents rationally ‘ignore’ their own information and ‘follow the herd’.

– Despite the available information, so-called *herd behavior* and *informational cascades* often result in an inefficient choice.

– Mass behavior is fragile, in the sense that small shocks may cause behavior to shift suddenly and dramatically.
The canonical model of social learning

- A set of players $N$, a finite set of actions $\mathcal{A}$, a set of states of nature $\Omega$, and a common payoff function $U(a, \omega)$

where $a \in \mathcal{A}$ is the action chosen and $\omega \in \Omega$ is the state of nature.

- Player $i$ receives a private signal $\sigma_i(\omega)$, a function of the state of nature $\omega$, and uses this private information to identify a payoff-maximizing action.
The canonical assumptions

- Bayes-rational behavior
- Incomplete and asymmetric information
- Pure information externality
- Once-in-a-lifetime decisions
- Exogenous sequencing
- Perfect information
Direct methodological extensions

– Caplin & Leahy (AER 1994), Chamley & Gale (ECM 1994)
– Avery & Zemsky (AER 1999), Chari & Kehoe (JET 2004)
– Çelen & Kariv (GEB 2004), Smith & Sørensen (2008)
– Bala & Goyal (RES 1998), Gale & Kariv (GEB 2004), Acemoglu et al. (2008)
The model of BHW (JPE 1992)

- There are two decision-relevant events, say $A$ and $B$, equally likely to occur *ex ante* and two corresponding signals $a$ and $b$.

- Signals are informative in the sense that there is a probability higher than $1/2$ that a signal matches the label of the realized event.

- The decision to be made is a prediction of which of the events takes place, basing the forecast on a private signal and the history of past decisions.
• Whenever two consecutive decisions coincide, say both predict $A$, the subsequent player should also choose $A$ even if his signal is different $b$.

• Despite the asymmetry of private information, eventually every player imitates her predecessor.

• Since actions aggregate information poorly, despite the available information, such herds/cascades often adopt a suboptimal action.
Anderson and Holt (*AER* 1997) investigate the social learning model of BHW experimentally.

They report that “rational” herds / cascades formed in most rounds and that about half of the cascades were incorrect.

The model of Smith and Sørensen (ECM 2000)

- Two phenomena that have elicited particular interest are *informational cascades* and *herd behavior*.
  - Cascade: players 'ignore' their private information when choosing an action.
  - Herd: players choose the same action, not necessarily ignoring their private information.

- Smith and Sørensen (2000) show that with a continuous signal space herd behavior arises, yet there need be no informational cascade.
The model of Çelen and Kariv (GEB 2004)

Signals

- Each player \( n \in \{1, \ldots, N\} \) receives a signal \( \theta_n \) that is private information.

- For simplicity, \( \{\theta_n\} \) are independent and uniformly distributed on \([-1, 1]\).

Actions

- Sequentially, each player \( n \) has to make a binary irreversible decision \( x_n \in \{0, 1\} \).
Payoffs

- $x = 1$ is profitable if and only if $\sum_{n \leq N} \theta_n \geq 0$, and $x = 0$ is profitable otherwise.

Information

- Perfect information

$$\mathcal{I}_n = \{\theta_n, (x_1, \ldots, x_{n-1})\}$$

- Imperfect information

$$\mathcal{I}_n = \{\theta_n, x_{n-1}\}$$
The decision problem

The optimal decision rule is given by

\[ x_n = 1 \text{ if and only if } \mathbb{E} \left[ \sum_{i=1}^{N} \theta_i \mid \mathcal{I}_n \right] \geq 0. \]

Since \( \mathcal{I}_n \) does not provide any information about the content of successors’ signals, we obtain

\[ x_n = 1 \text{ if and only if } \theta_n \geq -\mathbb{E} \left[ \sum_{i=1}^{n-1} \theta_i \mid \mathcal{I}_n \right]. \]
The cutoff process

- For any $n$, the optimal strategy is the cutoff strategy

$$x_n = \begin{cases} 1 & \text{if } \theta_n \geq \hat{\theta}_n \\ 0 & \text{if } \theta_n < \hat{\theta}_n \end{cases}$$

where

$$\hat{\theta}_n = -\mathbb{E} \left[ \sum_{i=1}^{n} \theta_i \mid \mathcal{I}_n \right]$$

is the optimal history-contingent cutoff.

- $\hat{\theta}_n$ is sufficient to characterize the individual behavior, and $\{\hat{\theta}_n\}$ characterizes the social behavior of the economy.
A three-agent example
A three-agent example

\[ x = 0 \]

\[ x = 1 \]
A three-agent example under perfect information
A three-agent example under imperfect information
The case of perfect information

The cutoff dynamics follows the cutoff process

\[ \hat{\theta}_n = \begin{cases} 
\frac{-1 + \hat{\theta}_{n-1}}{2} & \text{if } x_{n-1} = 1 \\
\frac{1 + \hat{\theta}_{n-1}}{2} & \text{if } x_{n-1} = 0 
\end{cases} \]

where \( \hat{\theta}_1 = 0 \).
A sequence of cutoffs under perfect information
A sequence of cutoffs under perfect information
Informational cascades

- $-1 < \hat{\theta}_n < 1 \forall n$ so any player takes his private signal into account in a non-trivial way.

Herd behavior

- $\{\hat{\theta}_n\}$ has the martingale property by the Martingale Convergence Theorem a limit-cascade implies a herd.
The case of imperfect information

The cutoff dynamics follows the cutoff process

\[ \hat{\theta}_n = \begin{cases} 
-\frac{1+\hat{\theta}_{n-1}^2}{2} & \text{if } x_{n-1} = 1 \\
\frac{1+\hat{\theta}_{n-1}^2}{2} & \text{if } x_{n-1} = 0 
\end{cases} \]

where \( \hat{\theta}_1 = 0 \).
A sequence of cutoffs under imperfect and perfect information
A sequence of cutoffs under imperfect and perfect information
Informational cascades

- $-1 < \hat{\theta}_n < 1 \ \forall n$ so any player takes his private signal into account in a non-trivial way.

Herd behavior

- $\{\hat{\theta}_n\}$ is not convergent and the divergence of cutoffs implies divergence of actions.

- Behavior exhibits periods of uniform behavior, punctuated by increasingly rare switches.
Takeaways

- The dynamics of social learning depend crucially on the extensive form of the game.

- Longer and longer periods of uniform behavior, punctuated by (increasingly rare) switches.

- A succession of fads: starting suddenly, expiring easily, each replaced by another fad.

- Why do markets move from ‘boom’ to ‘crash’ without settling down?
The social network model

- Agents are bound together by a social network, a complex of relationships that brings them into contact with other agents.

- Markets are characterized by agents connected by complex, multilateral information networks.

- The network is represented by a family of sets \( \{N_i\} \) where \( N_i \) denotes the set of agents \( j \neq i \) who can be observed by agent \( i \).

- Agents choose actions simultaneously and revise their decisions as new information is received.
Related literature

• The theoretical paper most closely related is Bala and Goyal (1998). The models differ in two ways:
  
  – Boundedly rational agents

  – Observing payoffs as well as actions.

• A model of *social experimentation* (with a multi-armed bandit) rather than social learning.
Asymptotic properties

• The welfare-improvement principle
  
  – Agents have perfect recall, so expected utility is non-decreasing over time. This implies that equilibrium payoffs form a submartingale.

• The imitation principle
  
  – In a connected network, asymptotically, all agents must get the same average (unconditional) payoffs.
**Convergence** Let \( \{X_{it}, \mathcal{F}_{it} : i = 1, ..., n, t = 1, 2, ...\} \) be an equilibrium. For each \( i \), define \( V_{it}^* : \Omega \to \mathbb{R} \) by

\[
V_{it}^* = E[U(X_{it}, \cdot) | \mathcal{F}_{it}].
\]

Then \( \{V_{it}^*\} \) is a submartingale with respect to \( \{\mathcal{F}_{it}\} \) and there exists a random variable \( V_{i\infty}^* \) such that \( V_{it}^* \) converges to \( V_{i\infty}^* \) almost surely.
**Connectedness** Let \( \{ X_{it}, F_{it} \} \) be the equilibrium and let \( V_{it}^* \) be the equilibrium payoffs. If \( j \in N_i \) and \( j \) is connected to \( i \) then \( V_{i\infty}^* = E[V_{j\infty}^*|F_{i\infty}] \).
**Imitation** Let $i$ and $j$ be two agents such that $j \in N_i$ and $j$ is connected to $i$. Let $E^{ab}$ denote the measurable set on which $i$ chooses $a$ infinitely often and $j$ chooses $b$ infinitely often. Then $V_{i\infty}^a(\omega) = V_{i\infty}^b(\omega)$ for almost every $\omega$ in $E^{ab}$. 
• Apart from cases of disconnectedness and indifference, diversity of actions is eventually replaced by uniformity.

• This is the network-learning analogue of the herd behavior found in the standard social learning model.

• The convergence properties of the model are general but many important questions about learning in networks remain open.

• Identify the impact of network architecture on the efficiency and dynamics of social learning.
A three-person example

- The network consists of three agents indexed by $i = A, B, C$. The neighborhoods $\{N_A, N_B, N_C\}$ completely define the network.

- Uncertainty is represented by two equally likely events $\omega = -1, 1$ and two corresponding signals $\sigma = -1, 1$.

- Signals are informative in the sense that there is a probability $\frac{2}{3}$ that a signal matches the event.

- With probability $q$ an agent is informed and receives a private signal at the beginning of the game.
• At the beginning of each date $t$, agents simultaneously guess $a_{it} = -1, 1$ the true state.

• Agent $i$ receives a positive payoff if his action $a_{it} = \omega$ and zero otherwise.

• Each agent $i$ observes the actions $a_{jt}$ chosen by the agents $j \in N_i$ and updates his beliefs accordingly.

• At date $t$, agent $i$’s information set $I_{it}$ consists of his private signal, if he observed one, and the history of neighbors’ actions.
Complete  Star  Circle

\[ \text{\begin{align*}
A & \rightarrow B \\
C & \leftarrow A & A & \rightarrow B \\
C & \leftarrow A & C & \leftarrow B
\end{align*}} \]
Learning dynamics

- Learning is ‘simply’ a matter of Bayesian updating but agents must take account of the network architecture in order to update correctly.

- If all agents choose the same action at date 1, no further information is revealed at subsequent dates (an absorbing state).

- We can trace out possible evolutions of play when there is diversity of actions at date 1.

- The exact nature of the dynamics depends on the signals and the network architecture.
Complete network

\[ N_A = \{B, C\}, \quad N_B = \{A, C\}, \quad N_C = \{A, B\} \]

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Star network

\[ N_A = \{B, C\}, \quad N_B = \{A\}, \quad N_C = \{A\} \]

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**Circle network**

\[ N_A = \{B\}, \quad N_B = \{C\}, \quad N_C = \{A\} \]

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**Takeaways**

- Convergence to a uniform action tends to be quite rapid, typically occurring within two to three periods.

- Significant differences can be identified in the equilibrium behavior of agents in different networks.

- Even in the three-person case the process of social learning in networks can be complicated.

- Because of the lack of common knowledge, inferences agents must draw in order to make rational decisions are quite subtle.
Experimental design

- Each experimental session consisted of 15 independent rounds and each round consisted of six decision-turns.

- The network structure and the information treatment \( (q = \frac{1}{3}, \frac{2}{3}, 1) \) were held constant throughout a given session.

- The ball-and-urn social learning experiments paradigm of Anderson and Holt (1997).

- A serious test of the ability of a structural econometric model based on the theory to interpret the data.
### Selected data
(star network under high-information)

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Herd behavior

Herd behavior is characterized by two related phenomena:

- **Stability**: the proportion of subjects who continue to choose the same action.

- **Uniformity**: a score function that takes the value 1 if all subjects act alike and takes the value 0 otherwise.

Uniformity will persist and lead to herd behavior if stability takes the value 1 at all subsequent turns.
Combining theory and experiments

In combining theory and experiments, we have two objectives in mind:

→ There is much that can be learned about the theory from the data, quite apart from any notion of “testing” the theory – whether the theory is useful in interpreting the data, and what extensions of the theory are required to make it compatible with the data.

← Any attempt to explain it in purely “behavioral” terms would require a large number ad hoc assumptions, which would render the “explanation” rather uninformative, and without a theoretical framework, it is impossible to draw general conclusions that go beyond the particular setting of the experiment.
Any attempt to use theory to explain experimental data must answer a number of questions about how to proceed:

– Do we assume that all subjects are identical or do we allow for heterogeneity?

– Do we assume a single equilibrium is played in each repetition of a game or do we allow for the possibility that different equilibria are played in different instances of the same game?

– Do we allow for mistakes or behavioral biases from the outset or assume full rationality?
These are several interesting approaches, all worth exploring; however, as a first step, we should assume that a single equilibrium is being played and that all players are (fully) rational and symmetric.

The advantage of these assumptions is that they provide a very parsimonious account of the data, recommended by Occam’s Razor, and they maximize our chance of falsifying the theory, in Popper’s sense.
Quantal response equilibrium (QRE)

- Mistakes are made and this should be taken into account in any theory of rational behavior.

- The payoff from a given action is assumed to be a weighted average of the theoretical payoff and a logistic disturbance.

- The "weight" placed on the theoretical payoff is determined by a regression coefficient.

- The recursive structure of the model enables to estimate the coefficients of the QRE model for each decision-turn sequentially.
The logit equilibrium can be summarized by a choice probability function following a binomial logit distribution:

\[
Pr (a_{it} = 1|I_{it}) = \frac{1}{1 + \exp (-\beta_{it}x_{it})}
\]

where \( \beta_{it} \) is a coefficient and \( x_{it} \) is the difference between the expected payoffs from actions 1 and \(-1\).

The regression coefficient \( \beta \) will be positive if the theory has any predictive power.
• Use the estimated coefficient from turn $t$ to calculate the theoretical payoffs from the actions at turn $t + 1$.

• The behavioral interpretation is that subjects have rational expectations and use the true mean error rate.

• The parameter estimates are highly significant and positive, showing that the theory does help predict the subjects’ behavior.

• A series of specification tests shows that the restrictions of the QRE model are confirmed by the data.
The beta time-series under full-information

- Complete
- Star (A)
- Star (B&C)
- Circle
The beta time-series under high-information

Turn

Beta

Complete
Star (A)
Star (B&C)
Circle

Turn

1 2 3 4 5 6
The beta time-series under low-information

- **Turn**
- **Beta**
- **Complete**
- **Star (A)**
- **Star (B&C)**
- **Circle**
The beta time-series in the circle network

- Full
- High
- Low
Concluding remarks

- Use the theory to interpret data generated by experiments of social learning in three-person networks.

- The family of three-person networks includes several architectures, each of which gives rise to its own distinctive learning patterns.

- The theory, modified to include the possibility of errors, adequately accounts for large-scale features of the data.

- A strong support for the use of models as the basis for structural estimation and the use of QRE to interpret experimental data.